Kinetostatics of Wheel Vehicle in the Category of Spiral-Screw Routes

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ABSTRACT. Deterministic mathematical model of kinetostatics of wheel vehicle in terms of different modes of spatial motion in the context of curved route is proposed. Earth-based coordinate system is introduced which pole and axial orientation are determined by the convenience of route description as well as vehicle-related coordinates which pole axial orientation are determined within inertial space with the help of natural trihedral. Turn of the natural trihedral within inertial coordinates is described by means of quaternion matrices in the context of Rodrigues-Hamilton parameters. Rodrigues-Hamilton parameters are in matrix form in direct accordance with specified hodograph. Kinetostatics of wheel vehicle is considered in terms of spatial motion with an allowance for three-dimensional aerodynamic forces, gravity, and tangential and centrifugal inertial forces. In the context of spiral-screw lines deterministic mathematical model of wheel vehicle kinetostatics is proposed in the form of hodograph in terms of uniform motion, accelerated motion, and decelerated motion within following route sections: straight and horizontal; in terms of vertical grade; in terms of horizontal plane. Analytical approach to determine animated contact drive-control forces of wheel vehicle for structural diagrams having one and two support points involving of a driving-driven wheel characteristic is proposed based on kinetostatics equations. Mathematical model of wheel vehicle kinetostatics in terms of spatial motion is constructed on the basis of nonlinear differential Euler-Lagrange equations; it is proposed to consider physically implemented motion trajectories of wheel vehicles in the context of spiral-screw lines; hodograph determines spatial displacement; Rodrigues-Hamilton parameters determines spatial turn; Varignon theorem is applied to identify components of drive (control) force. The obtained results make it possible to solve a wide range of problems connected with dynamic design of wheel vehicles involving controllability, and estimation of dynamic load of both system and support surface.

Introduction. In the context of uniform**,** accelerated, and decelerated motion, modes of front-drive, rear-driven, and four-wheel drive vehicle in terms of spatial curved route within junctions and turns, grades and straights, problems connected with estimation of dynamic load of structure and road surface [1,2,3] as well as stability and controllability [4,5,6] are topical. Solving of problems of dynamic design [7,8] of wheel vehicle help determine equivalent contact loads on supporting points taking into consideration characteristic of driver wheel, synthesize required control components, and identify relevant torque of driver wheel to provide desired motion mode of a vehicle in terms of specified route [9].

Problem definition. Inertial and geometrical parameters of wheel vehicle; configuration of supporting points taking into consideration characteristic of driver wheel for front-drive, rear-driven, and four-wheel drive vehicle; external force effect on a vehicle (gravity force and aerodynamic force); route geometry (straight, turn, grade, junction, manoeuvring); mode of a vehicle motion (uniform, accelerated, decelerated) are supposed as preselected.

Equivalent contact driving force (internal resulting constraint reaction of support surface) providing desired mode of a vehicle motion in terms of preselected route should be identified. It is required to distribute equivalent contact driving force on supporting points involving of drive wheel characteristic for front-drive, rear-driven, and four-wheel drive structural schemes of wheel vehicle.

Mathematical model of wheel vehicle kinetics. Spatial deterministic mathematical model of wheel vehicle kinetics in the context of different modes of motion within curved route is based upon nonlinear differential Euler-Lagrange equations in the form of quaternion matrices [10]. In this context, weight-specified material point (*m*) with application of aerodynamic forces, gravity, inertial forces, and unknown contact driving forces (controlling forces) providing desired mode of motion in terms of predetermined spatial curved route is taken as dynamic model of a vehicle.

Following coordinates (Earth-based coordinate system which pole and axial orientation are determined by the convenience of route description as well as vehicle-related coordinates which pole axial orientation are determined within inertial space with the help of natural trihedral) are introduced.

In the context of the taken problem, definition mathematical model bears following simplifications: centre of vehicle masses coincides with a pole of related coordinates; matrix of inertia of a vehicle degenerates into zero matrix; that is gyrodynamics of a vehicle is not considered.

Dynamics of vehicle advance is described with one quasivelocity (V_τ) being projection of a vector of linear velocity of vehicle mass centre on the tangent to motion trajectory (route) and two quasiaccelerations (W_τ, W_n): tangential (W_τ) and normal (centripetal) W_n.

In the context of the assumptions, Euler-Lagrange equations describing kinetics of wheel vehicle take the simple form:

$$\begin{Vmatrix} 0 \\ W_\tau \\ W_n \\ 0 \end{Vmatrix} = gA^t \cdot {}^tA^t \begin{Vmatrix} 0 \\ 0 \\ 0 \\ -1 \end{Vmatrix} + \frac{qS}{m} R_d \cdot {}^tR_d \cdot \begin{Vmatrix} 0 \\ C_{1d} \\ C_{2d} \\ C_{3d} \end{Vmatrix} + \frac{1}{m} \begin{Vmatrix} 0 \\ N_\tau \\ N_n \\ N_b \end{Vmatrix}, \tag{1}$$

where m – is vehicle mass;

g – is gravity acceleration;

q – is velocity pressure;

S – is specific area;

C_{1d}, C_{2d}, and C_{3d} – are aerodynamic coefficients;

W_τ, and W_n – are quasiaccelerations;

A – is quaternion matrix in terms of Rodrigues-Hamilton parameters determining orientation of natural trihedron within Earth-based coordinate system;

R_d – is quaternion matrix determining orientation of aerodynamic axes relative to natural ones;

N_τ, N_n, and N_b – are driving forces.

Mathematical model of vehicle motion within route section in the context of various modes. Spatial curve supporting trajectory-route as well as motion mode of a vehicle within the route is categorically determined in Earth-based coordinate system by means of hodograph [11]:

$$\bar{r}(t) = \bar{i}r_1 + \bar{j}r_2 + \bar{k}r_3, \tag{2}$$

where $\bar{i}$, $\bar{j}$, are – $\bar{k}$ basis vectors of Earth-based coordinate system.

It is proposed to determine hodograph combined with physically implemented trajectories of vehicle motion in the context of spiral and screw lines [9]:

$$\bar{r}(t) = \|\rho_0\rho_1\rho_2\rho_3\| \begin{Vmatrix} 1 \\ t \\ t^2 \\ t^3 \end{Vmatrix} (\bar{i}\cos\omega t + \bar{j}\sin\omega t) + \bar{k}\|h_0h_1h_2h_3\| \begin{Vmatrix} 1 \\ t \\ t^2 \\ t^3 \end{Vmatrix}, \tag{3}$$

where $\rho_i h_i (i = 0, 1, 2, 3)$ are running parameters determined on prescribed boundary conditions;

ω – is average turn rate equal to $\omega = \frac{\varphi_0}{t_0}$. In this context φ_0 is complete turn angle; and t_0 is specified time for turn.

Components of hodograph are determined as follows:

$$r_1 = \|\rho_0\rho_1\rho_2\rho_3\| \begin{Vmatrix} 1 \\ t \\ t^2 \\ t^3 \end{Vmatrix} \cos\omega t, \quad r_2 = \|\rho_0\rho_1\rho_2\rho_3\| \begin{Vmatrix} 1 \\ t \\ t^2 \\ t^3 \end{Vmatrix} \sin\omega t, \quad r_3 = \|h_0h_1h_2h_3\| \begin{Vmatrix} 1 \\ t \\ t^2 \\ t^3 \end{Vmatrix}. \tag{4}$$

It should be noted that highly developed world countries continue research to determine new forms of transit curves providing smooth changes in curve [2]. Transit curves in the form of cubic parabola, sinusoid, lemniscate, three- or four-leafed roses, and pseudospiral, which is circular curve, logarithmic spiral, Euler spiral etc. in special cases, are known. Transit curves proposed from heuristic or criteria viewpoints should correspond to true guide path of transport vehicle both in the context of constant velocity and variable one [2].

Depending upon values of running parameters hodograph proposed in the context of spiral and screw lines makes it possible simulate various particular cases of implemented trajectories and modes of vehicle motion within the routes.

a) Route section is straight ($\omega = 0$) and horizontal ($h_0 = 0$, $h_1 = 0$, $h_2 = 0$, $h_3 = 0$); mode of motion is uniform ($V_{1A} = V_{1B}$, $V_{2A} = 0$, $V_{2B} = 0$, $V_{3A} = 0$, $V_{3B} = 0$). Then $r_2(t) = 0$, $r_3(t) = 0$,

$$r_1(t) = r_{1A} + V_{1A}t; \tag{5}$$

that is hodograph is:

$$\bar{r}(t) = \bar{i}(r_{1A} + V_{1A}t). \tag{6}$$

In this context $0 \le t \le t_0$ where t_0 is time to travel predetermined route section ($r_{1B} - r_{1A}$); that is:

$$t_0 = \frac{r_{1B} - r_{1A}}{V_{1A}}. \tag{7}$$

b) Route section is straight and horizontal; mode of motion is not uniform:

- decelerated ($V_{1A} > V_{1B}$);

- accelerated ($V_{1A} < V_{1B}$).

Then hodograph is:

$$\bar{r}(t) = \bar{i}\left(r_{1A} + V_{1A}t + \frac{1}{4}\frac{V_{1B}^2 - V_{1A}^2}{r_{1B} - r_{1A}} t^2 \right). \tag{8}$$

c) Route section is in vertical plane ($\bar{i}$, $\bar{k}$); it has:

- rise ($r_{3A} = 0$, $r_{3B} > 0$),

- grade ($r_{3A} = 0$, $r_{3B} < 0$).

Mode of motion is determined with the help of boundary conditions: $V_{1A} = V_{1B}$, $V_{2A} = 0$, $V_{2B} = 0$, $V_{3A} = 0$, $V_{3B} = 0$.

Then hodograph is:

$$\bar{r}(t) = \bar{i}(r_{1A} + V_{1A}t) + \bar{k}\left(3 - 2\frac{V_{1A}}{r_{1B} - r_{1A}}t\right)\left(\frac{V_{1A}}{r_{1B} - r_{1A}}\right)^2 r_{3B}t^2. \tag{9}$$

Appropriate profile of the route section (motion trajectory) within vertical plane ($0xz$) is obtained in the form of combination of square parabola and cubic parabola:

$$z = 3x^2 - 2x^3, \tag{10}$$

where

$$x = \frac{r_1(t) - r_{1A}}{r_{1B} - r_{1A}}, \qquad z = \frac{r_3(t)}{r_{3B}}. \tag{11}$$

In the context of route section under consideration:

- Variation range of variables is: $0 \le x \le 1$, $0 \le z \le 1$;

- Extremum points are: $x_1^э = 0$, $x_2^э = 1$;

- Extreme values are: $z_{\min}(0) = 0$, $z_{\max}(1) = 1$;

- Bending point is: $\left(\frac{1}{2}, \frac{1}{2}\right)$;

- Concavity interval is: $0 < x < \frac{1}{2}$;

- Convexity interval is: $\frac{1}{2} < x < 1$.

d) Route section is within horizontal plane (i, j) providing $\varphi_0 = \frac{\pi}{2}$ angle turn; mode of motion is determined with the help of boundary conditions:

$$r_{1A} > 0, \quad r_{2A} = 0, \quad r_{3A} = 0, \quad r_{1B} = 0, \quad r_{2B} > 0, \quad r_{3B} = 0;$$
$$V_{1A} = 0, \quad V_{2A} > 0, \quad V_{3A} = 0, \quad V_{1B} > 0, \quad V_{2B} = 0, \quad V_{3B} = 0.$$

Then hodograph is:

$$\bar{r}(t) = \left[r_{1A} + \frac{12}{\pi^2}\left(\frac{V_{2A}}{r_{1A}}\right)^2 (r_{2B} - r_{1A})t^2 - \frac{16}{\pi^3}\left(\frac{V_{2A}}{r_{1A}}\right)^3 (r_{2B} - r_{1A})t^3 \right] \cdot \left[\bar{i}\cos\left(\frac{V_{2A}}{r_{1A}}t\right) + \bar{j}\sin\left(\frac{V_{2A}}{r_{1A}}t\right) \right]. \quad (12)$$

In this context kinematical connection is available:

$$V_{2A} \cdot r_{2B} = V_{1B} \cdot r_{1A}. \quad (13)$$

Relevant plan of the route section (motion trajectory) within horizontal plane is obtained in polar coordinate system in the form of quadratic Archimedean spiral and cubic one:

$$\frac{r(\varphi) - r_{1A}}{r_{2B} - r_{1A}} = \frac{12}{\pi^2}\varphi^2 - \frac{16}{\pi^3}\varphi^3, \quad (14)$$

where

$$\varphi = \frac{V_{2A}}{r_{1A}}t, \quad r^2(\varphi) = r_1^2(\varphi) + r_2^2(\varphi). \quad (15)$$

In the context of the route section under consideration, variables domain is: $0 \le \varphi \le \frac{\pi}{2}$, $r_{1A} \le r \le r_{2B}$; that is $r(0) = r_{1A}$, $r\left(\frac{\pi}{4}\right) = \frac{r_{1A} + r_{2B}}{2}$, $r\left(\frac{\pi}{2}\right) = r_{2B}$.

In terms of special case if $V_{2A} = V_{1B}$ then $r_{1A} = r_{2B}$ and $r(\varphi) = r_{1A}$ in the context of any φ; that is motion trajectory takes a form of radial arc if right-angle turn takes place.

Kinematics of a vehicle within predetermined route section. Kinematics of a vehicle under different motion modes is identified with the help of predetermined hodograph $\bar{r}(t)$. Formulas for components of tangential acceleration and normal one are represented by means of laconic vector notation [12]:

$$W_\tau = \frac{\dot{\bar{r}} \cdot \ddot{\bar{r}}}{|\dot{\bar{r}}|}, \quad W_n = \frac{|\dot{\bar{r}} \times \ddot{\bar{r}}|}{|\dot{\bar{r}}|}, \quad W_b = 0, \tag{16}$$

where $\dot{\bar{r}}$ and $\ddot{\bar{r}}$ – are time derivatives 1 and two of vector function – hodograph;

$|\dot{\bar{r}}|$ – is module of time derivative 1 of hodograph that is velocity value of vehicle motion;

$|\dot{\bar{r}} \times \ddot{\bar{r}}|$ – is module of 1 and 2 time derivatives of hodograph;

$\dot{\bar{r}} \cdot \ddot{\bar{r}}$ – is scalar product of 1 and 2 time derivatives of hodograph.

Formulas for velocity components:

$$V_\tau = |\dot{r}|, \quad V_n = 0, \quad V_b = 0. \tag{17}$$

In terms of Earth-based coordinate system and axes of natural trihedron, components of vehicle velocity are of following kinematic relations:

$$\begin{Vmatrix} 0 \\ \dot{r}_1 \\ \dot{r}_2 \\ \dot{r}_3 \end{Vmatrix} = A \cdot {}^tA \begin{Vmatrix} 0 \\ V_\tau \\ V_n \\ V_b \end{Vmatrix}, \quad \begin{Vmatrix} 0 \\ V_\tau \\ V_n \\ V_b \end{Vmatrix} = A^t \cdot {}^tA^t \begin{Vmatrix} 0 \\ \dot{r}_1 \\ \dot{r}_2 \\ \dot{r}_3 \end{Vmatrix}, \tag{18}$$

where A, ${}^tA$, A^t, ${}^tA^t$ are quaternion matrices in the context of Rodrigues-Hamilton parameters: a_0, a_1, a_2, a_3 [5, 10, 11]; in this context Rodrigues-Hamilton parameters describing a turn (orientation) of natural trihedron (vehicle-related dynamical frame of reference) in terms of terrestrial, fixed, inertial coordinates are determined directly according to specified hodograph with the help of following matrix equation:

$$\begin{Vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{Vmatrix} \begin{Vmatrix} a_0^2 \\ a_1^2 \\ a_2^2 \\ a_3^2 \end{Vmatrix} = \frac{1}{|\dot{\bar{r}}|\,|\dot{\bar{r}} \times \ddot{\bar{r}}|} \begin{Vmatrix} |\dot{\bar{r}}|\,|\dot{\bar{r}} \times \ddot{\bar{r}}| \\ |\dot{\bar{r}} \times \ddot{\bar{r}}|\,\dot{r}_1 \\ \ddot{r}_2\left(\dot{r}_1^2 + \dot{r}_3^2\right) - \dot{r}_2\left(\dot{r}_1\ddot{r}_1 + \dot{r}_3\ddot{r}_3\right) \\ |\dot{\bar{r}}|\left(\dot{r}_1\ddot{r}_2 - \dot{r}_2\ddot{r}_1\right) \end{Vmatrix}, \tag{19}$$

where $|\dot{\bar{r}}|^2 = \dot{r}_1^2 + \dot{r}_2^2 + \dot{r}_3^2$,

$$\left|\dot{\bar r}\times\ddot{\bar r}\right|^2=\left(\dot r_2\ddot r_3-\dot r_3\ddot r_2\right)^2+\left(\dot r_3\ddot r_1-\dot r_1\ddot r_3\right)^2+\left(\dot r_1\ddot r_2-\dot r_2\ddot r_1\right)^2 .$$

It should be noted that the matrix

$$\begin{Vmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1\end{Vmatrix}$$

is symmetrical and nondegenerate; moreover, it has following property of orthogonality:

$$\begin{Vmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1\end{Vmatrix}\cdot\begin{Vmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1\end{Vmatrix}=4\begin{Vmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{Vmatrix}, \tag{20}$$

That is inverse matrix is:

$$\frac{1}{4}\begin{Vmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1\end{Vmatrix}. \tag{21}$$

Then required solution may be identified by means of following matrix form:

$$\begin{Vmatrix}a_0^2\\ a_1^2\\ a_2^2\\ a_3^2\end{Vmatrix}=\frac{1}{4\left|\dot{\bar r}\right|\left|\dot{\bar r}\times\ddot{\bar r}\right|}\begin{Vmatrix}1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1\end{Vmatrix}\begin{Vmatrix}\left|\dot{\bar r}\right|\left|\dot{\bar r}\times\ddot{\bar r}\right|\\ \left|\dot{\bar r}\times\ddot{\bar r}\right|\dot r_1\\ \ddot r_2\left(\dot r_1^2+\dot r_3^2\right)-\dot r_2\left(\dot r_1\ddot r_1+\dot r_3\ddot r_3\right)\\ \left|\dot{\bar r}\right|\left(\dot r_1\ddot r_2-\dot r_2\ddot r_1\right)\end{Vmatrix}. \tag{22}$$

$\bar\tau$, $\bar n$, $\bar b$ basis vectors of natural trihedron of space curved route are determined with the help of specified hodograph in terms of the vector form:

$$\bar\tau=\frac{\dot{\bar r}}{\left|\dot{\bar r}\right|},\quad \bar n=\frac{\dot{\bar r}\times\left(\ddot{\bar r}\times\dot{\bar r}\right)}{\left|\dot{\bar r}\right|\left|\dot{\bar r}\times\ddot{\bar r}\right|},\quad \bar b=\frac{\dot{\bar r}\times\ddot{\bar r}}{\left|\dot{\bar r}\times\ddot{\bar r}\right|}. \tag{23}$$

Kinetostatics of ground transport vehicles. Equivalent contact driving (controlling) force of ground transport vehicles is determined by means of kinetic equations.

a) Schematic of vehicle having one supporting point.

In this case internal resulting constraint force of supporting surface is determined with the help of matrix formula:

$$\frac{1}{m}\begin{Vmatrix}0\\N_\tau\\N_n\\N_b\end{Vmatrix}=\begin{Vmatrix}0\\W_\tau\\W_n\\0\end{Vmatrix}+gA^t\cdot{}^tA^t\begin{Vmatrix}0\\0\\0\\1\end{Vmatrix}-\frac{qS}{m}R_d\cdot{}^tR_d\begin{Vmatrix}0\\C_{1d}\\C_{2d}\\C_{3d}\end{Vmatrix}. \qquad (24)$$

b) Schematic of vehicle having two supporting points.

For two-wheel vehicle resulting driving force (N_τ , N_n , N_b) should be distributed on two supporting points involving drive wheel characteristic in the form of required system of two equivalent driving forces ($\bar{F}_1$, $\bar{F}_2$). In this context the two structural schemes to locate supporting points (0_1, 0_2) are possible:

- Tandem scheme; and

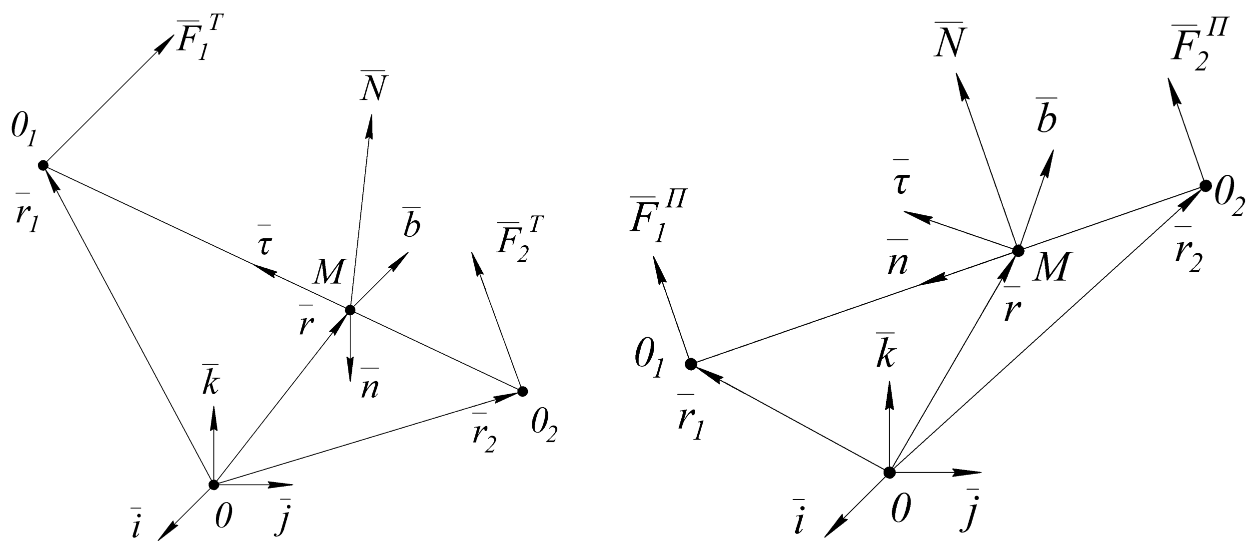
- Parallel one.

The schemes are shown in Fig.1 where geometrical parameters are set within coordinates related to a vehicle; $\bar{\tau}$ is driving direction.

The formulated problem of dynamic design of two-wheel ground transport vehicle is static problem; its solution should involve Varignon theorem [12]:

$$\bar{r}_1\times\bar{F}_1+\bar{r}_2\times\bar{F}_2=\bar{r}\times\bar{N}\,, \qquad (25)$$

where $\bar{r}_1=\bar{r}+\bar{\tau}\,l_1$, $\bar{r}_2=\bar{r}-\bar{\tau}\,l_2$.



a) b)

Fig.1. Tandem structural scheme (a) and parallel one (b) to locate supporting points

Specifically, bringing pole 0 in coincidence with supporting point two 0_2 results in:

$\bar{r} = \bar{\tau}\, l_2$, $\bar{r}_2 = 0$, $\bar{r}_1 = (l_1 + l_2)\bar{\tau}$ for tandem scheme;

$\bar{r} = \bar{n}\, h_2$, $\bar{r}_2 = 0$, $\bar{r}_1 = (h_1 + h_2)\bar{n}$ for parallel scheme

where l_1, h_1 is $M0_1$ section (distance from centre of mass to supporting point one);

l_2, h_2 is $M0_2$ section (distance from centre of mass to supporting point two).

Then within axes of natural trihedron, Varignon theorem is represented in the form of determinants:

$$\begin{vmatrix} \bar{\tau} & \bar{n} & \bar{b} \\ l_1 + l_2 & 0 & 0 \\ F_{1\tau}^T & F_{1n}^T & F_{1b}^T \end{vmatrix} = \begin{vmatrix} \bar{\tau} & \bar{n} & \bar{b} \\ l_2 & 0 & 0 \\ N_\tau & N_n & N_b \end{vmatrix} \quad \text{for tandem scheme;}$$

$$\begin{vmatrix} \bar{\tau} & \bar{n} & \bar{b} \\ 0 & h_1 + h_2 & 0 \\ F_{1\tau}^{\Pi} & F_{1n}^{\Pi} & F_{1b}^{\Pi} \end{vmatrix} = \begin{vmatrix} \bar{\tau} & \bar{n} & \bar{b} \\ 0 & h_2 & 0 \\ N_\tau & N_n & N_b \end{vmatrix} \quad \text{for parallel scheme.}$$

Whence it follows:

- In terms of tandem scheme: $F_{1\tau}^T$ is uncertain, $F_{1n}^T = \dfrac{l_2}{l_1 + l_2} N_n$, $F_{1b}^T = \dfrac{l_2}{l_1 + l_2} N_b$;

- In terms of parallel scheme: $F_{1\tau}^{\Pi} = \dfrac{h_2}{h_1 + h_2} N_\tau$, F_{1n}^{Π} is uncertain, $F_{1b}^{\Pi} = \dfrac{h_2}{h_1 + h_2} N_b$.

Bringing 0 pole in coincidence with supporting point one 0_1 results in:

$\bar{r} = -l_1 \bar{\tau}$, $\bar{r}_1 = 0$, $\bar{r}_2 = -(l_1 + l_2)\bar{\tau}$ for tandem scheme;

$\bar{r} = -h_1 \bar{n}$, $\bar{r}_1 = 0$, $\bar{r}_2 = -(h_1 + h_2)\bar{n}$ for parallel scheme.

Then within axes of natural trihedron, Varignon theorem is:

$$\begin{vmatrix} \bar{\tau} & \bar{n} & \bar{b} \\ l_1 + l_2 & 0 & 0 \\ F_{2\tau}^T & F_{2n}^T & F_{2b}^T \end{vmatrix} = \begin{vmatrix} \bar{\tau} & \bar{n} & \bar{b} \\ l_2 & 0 & 0 \\ N_\tau & N_n & N_b \end{vmatrix} \quad \text{for tandem scheme;}$$

$$\begin{vmatrix} \bar{\tau} & \bar{n} & \bar{b} \\ 0 & h_1 + h_2 & 0 \\ F_{2\tau}^{\Pi} & F_{2n}^{\Pi} & F_{2b}^{\Pi} \end{vmatrix} = \begin{vmatrix} \bar{\tau} & \bar{n} & \bar{b} \\ 0 & h_2 & 0 \\ N_\tau & N_n & N_b \end{vmatrix} \quad \text{for parallel scheme.}$$

Whence it follows:

- In terms of tandem scheme: $F_{2\tau}^{T}$ is uncertain, $F_{2n}^{T} = \frac{l_1}{l_1 + l_2} N_n$, $F_{2b}^{T} = \frac{l_1}{l_1 + l_2} N_b$;

- In terms of parallel scheme: $F_{2\tau}^{\Pi} = \frac{h_1}{h_1 + h_2} N_\tau$, F_{2n}^{Π} is uncertain, $F_{2b}^{\Pi} = \frac{h_1}{h_1 + h_2} N_b$.

To verify the results obtained and cope with uncertainty in the process of the problem solving apply static invariant one [6]:

$$\bar{F}_1 + \bar{F}_2 = \bar{N}. \tag{26}$$

Within axes of natural trihedron for tandem scheme, the equation

$$F_{1n}^{T} + F_{2n}^{T} = N_n, \quad F_{1b}^{T} + F_{2b}^{T} = N_b \tag{27}$$

is performed identically while following equation

$$F_{1\tau}^{T} + F_{2\tau}^{T} = N_\tau \tag{28}$$

becomes resolvable if technical specifications are involved in terms of driving-driven wheel characteristic.

Example 1. Four-wheel drive structural scheme; that is

$$F_{1\tau}^{T} > 0, \quad F_{2\tau}^{T} > 0 \tag{29}$$

and design parameter is assumed to be constrained

$$\left| \frac{F_{1\tau}^{T}}{F_{2\tau}^{T}} \right| = k. \tag{30}$$

Then $F_{1\tau}^{T} = kF_{2\tau}^{T}$.

From which $kF_{2\tau}^{T} + F_{2\tau}^{T} = N_\tau$,

that is $F_{2\tau}^{T} = \frac{1}{1+k} N_\tau$, $F_{1\tau}^{T} = \frac{k}{1+k} N_\tau$.

Particularly, if driving contact forces are equal within front axle 0_1 and back axle 0_2 then $k = 1$, and hence

$$F_{2\tau}^{T} = \frac{1}{2} N_{\tau}, \quad F_{1\tau}^{T} = \frac{1}{2} N_{\tau}. \tag{31}$$

Example 2. If front drive structural scheme then

$$F_{1\tau}^{T} > 0, \quad F_{2\tau}^{T} < 0. \tag{32}$$

Subsequently $F_{1\tau}^{T} - F_{2\tau}^{T} = N_{\tau}$

or $kF_{2\tau}^{T} - F_{2\tau}^{T} = N_{\tau}$ that is

$$F_{2\tau}^{T} = \frac{1}{k-1} N_{\tau}, \quad F_{1\tau}^{T} = \frac{k}{k-1} N_{\tau}, \tag{33}$$

where $k > 1$.

Example 3. If rear-driven structural scheme then

$$F_{1\tau}^{T} < 0, \quad F_{2\tau}^{T} > 0. \tag{34}$$

Subsequently $-F_{1\tau}^{T} + F_{2\tau}^{T} = N_{\tau}$

or $-kF_{2\tau}^{T} + F_{2\tau}^{T} = N_{\tau}$, то есть

$$F_{2\tau}^{T} = \frac{1}{1-k} N_{\tau}, \quad F_{1\tau}^{T} = \frac{k}{1-k} N_{\tau}, \tag{35}$$

where $k < 1$.

Comparable result is available while considering parallel scheme. The equations

$$F_{1\tau}^{\Pi} + F_{2\tau}^{\Pi} = N_{\tau}, \quad F_{1b}^{\Pi} + F_{2b}^{\Pi} = N_{b} \tag{36}$$

are performed identically. Then the equation

$$F_{1n}^{T} + F_{2n}^{T} = N_{n} \tag{37}$$

is solvable in terms of extra conditions.

In this context lateral contact forces are assumed as those related to technically reasonable conditions:

$$F_{1n}^{\Pi} > 0, \quad F_{2n}^{\Pi} > 0, \quad \frac{F_{1n}^{\Pi}}{F_{2n}^{\Pi}} = \mu, \tag{38}$$

where μ is predetermined coefficient (design parameter).

Then $F_{1n}^{\Pi} = \mu F_{2n}^{\Pi}$.

Where $\mu F_{2n}^{\Pi} + F_{2n}^{\Pi} = N_n$,

That is $F_{2n}^{\Pi} = \frac{1}{1+\mu} N_n, \quad F_{1n}^{\Pi} = \frac{\mu}{1+\mu} N_n$.

In particular, it is assumed that $\mu = 1$ for symmetrical structural scheme. Consequently

$$F_{2n}^{\Pi} = \frac{1}{2} N_n, \quad F_{1n}^{\Pi} = \frac{1}{2} N_n. \tag{39}$$

Summary. Deterministic mathematical model of wheel vehicle kinetics in terms of different modes of spatial motion in the context of curved route has been proposed. The model is based upon nonlinear Euler-Lagrange equations. In the category of spiral-screw lines deterministic mathematical model of wheel vehicle kinematics has been proposed in the form of hodograph in the context of uniform motion, accelerated motion, and decelerated motion within following route sections: straight and horizontal; in terms of vertical grade; in terms of turn in horizontal plane. Analytical approach to determine contact drive-control forces of wheel vehicle for structural schemes having one and two support points involving of a driving-driven wheel characteristic (four-wheel drive scheme, front-wheel drive scheme, and rear-wheel drive scheme) has been proposed on the basis of kinetostatics equations.

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