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Two-criteria optimization of H-section bars–beams under bending and compression

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Abstract

The two-criteria optimum design problem for thin-walled members with two axes of symmetry (of H-type) subjected to compression and bending is solved. Components of the objective function vector are the compressive force and the bending moment in the web plane. Pareto-optima and "compromise" optimal projects have been obtained and compared with standard profiles.

Keywords: Structural optimization; Multi-criteria optimization; Thin-walled bars

1. Introduction

In papers [1,2] the optimum design problem for channel and lipped channel profiles subjected to compression and/or bending was considered in the framework of the theory of multi-criteria optimization of structures (vector-valued optimization). In this work our consideration is expanded to H-section as a typical cross-section with two axes of symmetry. The main idea remains the same as in the previous papers. Usually the weight minimization problems for structural elements with respect to stability were considered for separate loading cases—for compressive force, bending moment and others. But, in practice, structural members often undergo the action of various loads, separately or in a combination. Therefore the question arises: what configuration of the cross-section can be considered as an optimal one with account of such multifarious character of loading? This problem is formulated and solved in the framework of QJ;the theory of multi-criteria optimization. Only requirements of stability (overall and local) are taken into account (the material is suggested to be elastic).

We would like to emphasize one important difference between our approach and usual applications of the vector-valued optimization. Usually this theory is employed in order to account essentially different objective functions, e.g., weight and cost. We take into account different loads, so the components of the objective function vector are the compressive force and the bending moment in the web plane.

2. Statement of the problem and method of solution

We seek optimal parameters of a simply supported H-cross-section bar–beam under action of compressive force P and/or bending moment M in the web plane (Fig. 1). Two equivalent (dual) approaches for the optimization problem are possible: (1) to minimize the weight of the structure for given loading; (2) to maximize the load for given weight (volume, or cross-section area). As we consider the beam for various loads, it is preferable to use here the second approach. So the cross-section area A and the length of the beam L are considered given, as well as the material properties (Young's modulus E, Poisson's ratio ν). Design variables are dimensions of the cross-section elements—width and thickness of the web b_w, t_w and flanges b_f, t_f.

The optimization problem is solved with respect to stability. We take into account the overall buckling (flexural and torsional) under compressive force, and flexural–torsional buckling (or lateral buckling) under bending moment. Critical forces for the *overall* flexural and torsional buckling P_f, P_t and critical moment for

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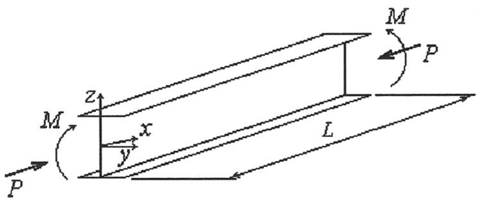

Fig. 1. H-section beam and its loading.

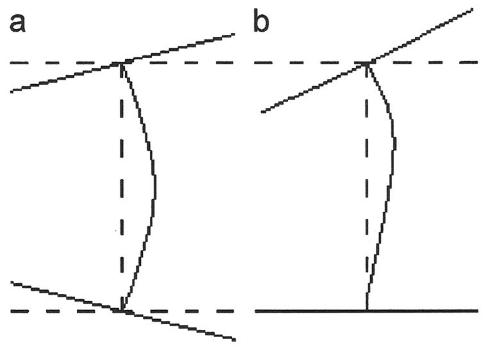

Fig. 2. Typical local buckling modes under compression (a) and bending (b).

flexural–torsional buckling M_{ft}, as well as their critical combinations, were calculated according to the linear theory of thin-walled bars [3].

Along with overall buckling we accounted for the *local* buckling. The linear local buckling loads (compressive force P_l, bending moment M_l and their combinations) were obtained from solution of the buckling problem for the beam–column considered as an assemblage of strips–plates. Typical shapes of local buckling modes under compression and bending are shown in Fig. 2. Note that we took into account a wide spectrum of short-length local modes with numbers of half-waves m from 2 up to 35.

Strength constraints were not imposed, i.e. the assumption was adopted that the yield limit was sufficiently high. This assumption is warranted for not too large values of the weight parameter G^* (see below).

All constraints were formulated in the following dimensionless parameters of the load, weight and stress:

$$P^* = \frac{P}{L^2 \cdot E} 10^6, \quad M^* = \frac{M}{L^3 \cdot E} 10^8,$$
$$G^* = \frac{A}{L^2} 10^3, \quad \sigma^* = \frac{\sigma}{E} 10^3. \tag{1}$$

The cross-section was characterized with dimensionless geometric parameters b_w/L, b_f/b_w, t_w/b_w, and t_f/b_f. All computations were performed in these dimensionless parameters. So the solution of the optimization problem provides optimal dimensionless parameters. To obtain values of dimensional optimal parameters one should specify additionally L value.

The optimization problem was solved as a nonlinear programming one by the linearized method of reduced gradient [4].

3. Results of the solution. The single-criterion optimization

3.1. Beams under the bending moment

First we considered the single-criterion optimization for beams under bending moment in the symmetry plane. The problem was formulated as follows: for given G^* determine b_w/L, b_f/b_w, t_w/b_w, t_f/b_f yielding to maximum of the minimal value of two critical moments M_{ft}, M_l (for overall and local buckling). Optimal dimensionless parameters were computed for the range of parameter G^* (0; 0.5) in which the assumption about elastic deformation of the material is found to be justified.

Optimal bars turn out to be equally stable with respect to flexural–torsional and local buckling (since the solution is based on the linear buckling theory). It has been revealed that two dimensionless parameters—b_f/b_w and t_f/t_w—weakly depend upon G^* and practically are nearly constant: $b_f/b_w = 0.5$–0.6, $t_f/t_w = 2.35$–2.55.

Two other dimensionless parameters—thickness to width ratios for web and flange t_f/b_f and t_w/b_w—depend upon the weight parameter G^*, as is shown in Fig. 3.

Typical optimal cross-sections are shown in Fig. 4 for two G^*-values (dimensional parameters in mm are given

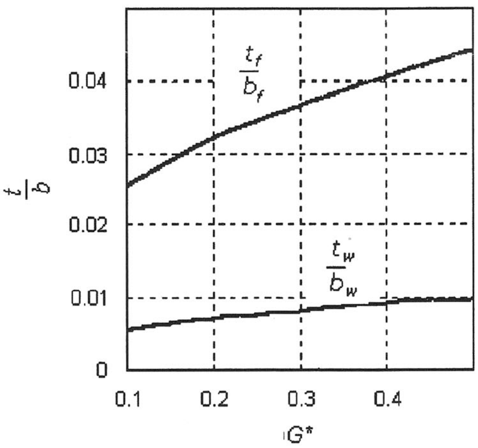

Fig. 3. Optimal thickness to width ratios for web and flange under bending.

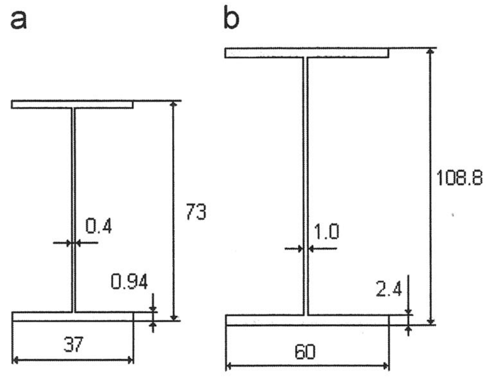

Fig. 4. Optimal profiles of beam under bending for $L = 1$ m (dimensions in mm): (a) $G^* = 0.1$ and (b) $G^* = 0.4$.

for $L = 1$ m). These profiles differ from the standard ones with very thin web.

The dependence of the dimensionless critical moment M^* on weight parameter G^* can be approximated with the following simple formula (with high accuracy):

$$M^* = 30.865G^{*2}. \tag{2}$$

So, if the bending moment is given, one can easily compute the parameter G^*, and then to determine all optimal parameters of the cross-section using the above results.

3.2. Bars under compression

The single-criterion optimization of bars under compression is formulated as follows: for given G^* determine parameters b_w/L, b_f/b_w, t_w/b_w, and t_f/b_f yielding to maximum of minimal value of three critical forces P_f, P_t, and P_l.

Similarly to the case of optimal bent beams, two dimensionless parameters—b_f/b_w and t_f/t_w—practically are constant: $b_f/b_w \approx 1.84$, $t_f/t_w \approx 0.72$, independently of G^* value. Note that these values are quite different comparing to the case of bending. Flanges become thinner, the height of webs smaller. Two other dimensionless parameters t_f/b_f and t_w/b_w depend upon the weight parameter G^*, as is shown in Fig. 5.

Optimal bars under compression turn out to be equally stable with respect to three modes: flexural, torsional and local buckling. Therefore the optimal profiles noticeably differ from those for bent beams, as shown in Fig. 6, where optimal cross-sections of compressed H-sections for two values of G^* at $L = 1$ m are given.

The dependence of the dimensionless force P^* on weight parameter G^* can be approximated with high accuracy, similarly to (2), with the following power function:

$$P^* = 5.405G^{*5/3}. \tag{3}$$

It is interesting to compare optimal H-bars under compression with optimal channel sections studied in [1]. For optimal channel sections in [1] the following relationship between P^* and G^* was obtained: $P^* = 2.794G^{*1.668}$.

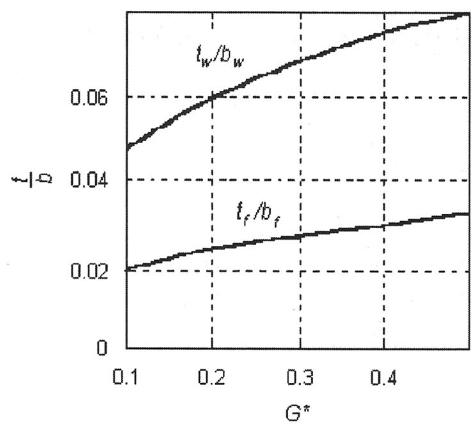

Fig. 5. Thickness to width ratios for web and flange of optimal compressed bars.

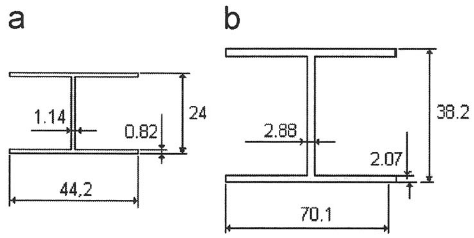

Fig. 6. Optimal profiles of compressed H-section bars with $L = 1$ m (dimensions in mm): (a) $G^* = 0.1$ and (b) $G^* = 0.4$.

Comparing this relation with (3) one can conclude that optimal H-section bar sustains the compressive force approximately two times as much as optimal channel section bar.

4. Results of the solution. The two-criteria optimization

4.1. Pareto-curves

Configurations of the optimal cross-sections for bending and compression are found to be rather different, and therefore solution of the two-criteria optimization problem is of particular interest. There were constructed Pareto-optimal curves presenting the optimal profiles for various combinations of the force P^* and bending moment M^* for a given value of the weight parameter G^*. The Pareto-curves were obtained by means of minimization in P^* with a constraint on M^*, which was gradually moved from the value corresponding to optimal bar under compression up to the value for optimal beam under bending moment.

The Pareto-curves are presented in Fig. 7 on the plane "normalized force P^*-normalized moment M^*" for several values of G^* (solid lines). They show the correspondence between the critical force and the moment for optimal profiles. When the point moves along the curve (for a given G^*), relationship between the critical force and the moment changes, from that for optimal bars under pure compression to optimal beams for pure bending. Dashed lines relate to the single-criterion optimal projects. Each point of the right dashed curve gives a critical combination of force and the moment for optimal H-sections at certain G^* value, for max P-projects. The left dashed line is similar curve for max M-projects.

Note that the shapes of Pareto-curves noticeably differ from those for channel cross-section, obtained in [1].

4.2. Compromise optimal projects

Finally, we have constructed "compromise" optimal projects. The optimization problem was solved with the global criterion in the form:

$$F = \alpha\frac{P^*}{P^*_{\max}} + (1-\alpha)\frac{M^*}{M^*_{\max}}, \tag{4}$$

where α is a "weight" coefficient which is chosen by designer, and critical force and moment P^* and M^* are

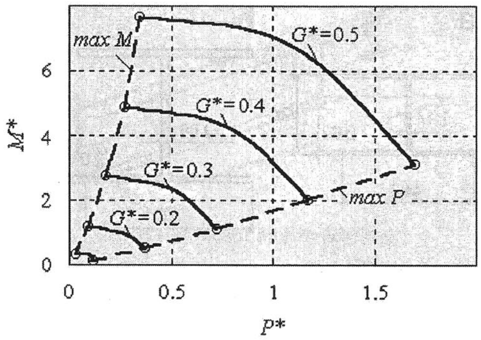

Fig. 7. Pareto-curves on the plane P^*-M^*.

Table 1
Compromise optimal H-sections for different values of coefficient α ($G^* = 0.5$, $L = 1$ m)

α	P^*	M^*	Dimensions (mm)				Ratios	
			b_f	b_w	t_f	t_w	b_f/b_w	t_f/t_w
0	0.35	7.64	65	115	2.8	1.1	0.56	2.53
0.25	0.54	7.5	66	95	3.1	1.0	0.70	3.17
0.5	1.58	4.1	70	39	2.9	2.6	1.8	1.08
0.75	1.71	3.11	75	41	2.4	3.3	1.84	0.732
1.0	1.71	3.11	75	41	2.4	3.3	1.84	0.732

normalized by division on their maximal values P^*_{max} and M^*_{max} obtained in the single-criterion optimizations in P^* and M^*, respectively, for given G^*. Some results of computations for various α values (0; 0.25; 0.5; 0.75; 1) are presented in Table 1 for $G^* = 0.5$, $L = 1$ m (values $\alpha = 0$ and $\alpha = 1$ relate to the single-criterion optimizations).

We see that the variation of α noticeably influences the optimal parameters. At small α they are close to those derived in the single-criterion optimization in M^*. If $\alpha > 0.7$, the optimal projects coincide with ones obtained in the single-criterion optimization in P^*, i.e. for $\alpha = 1$.

The obtained optimal dimensionless parameters b_f/b_w, t_f/b_f, and t_w/b_w have been compared with their values for standard H-sections. It is convenient to represent cross-sections as points in the plane of dimensionless parameters b_f/b_w and t_f/b_f (Fig. 8). Different symbols correspond to

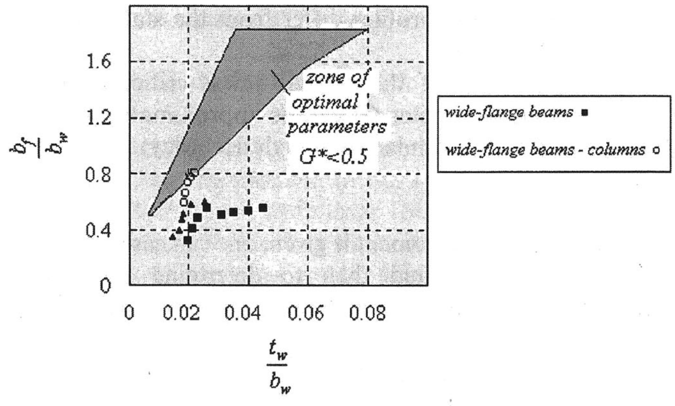

Fig. 8. Comparison of the obtained optimal projects with the standard H-sections.

different types of standard profiles. We can indicate here the zone, which contains optimal cross-sections for various G^*. Some standard profiles (wide flange light profiles) fall into the optimal zone or close to it (in the case of prevailing bending moment (for $\alpha < 0.4$)). But the most standard profiles are rather far from the optimal H-sections for any α values, in distinction from the standard channel sections [1]. As a rule, standard profiles have more thick webs and flanges. We may conclude that these standard profiles are not optimal independent of what combination of compression force and bending moment acts on the bar.

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