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Modeling the Dynamic Response of Railway Track

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Abstract. The authors developed a model based on the dynamical theory of elasticity, which allows us to describe the dynamic equilibrium of strains of the railway track layers. The model of the stress-strain state for the railway track consists in a combination of the geometry equations of the outline of a part in the system's space, involved into interaction at a given time and the equations of the dynamic equilibrium of its deformation. Considering the obligation to register the course of the process in time, for each time step of integration the outline of the strains propagations front is determined. The strains propagations front is described by the equations of propagation in the material of the elastic spatial wave of oscillations. It allows determining complete spatial-temporal account of the deflection dynamics of the rail support, which enables the execution of calculations, including for conditions of fast-speed and high-speed trains. With the application of the developed model, a number of scientific and practical problems were solved.

1. Introduction

Engineering and scientific problems of railway track calculations are based on the application of mathematical models. Despite their large scale, development of new models or adaptation of existing ones remains a vital task. This is explained not only by the emergence of new designs of elements of the railway track and the rolling stock, not only by the change in operating conditions, including enhancing the speed of movement. The main reason can be considered the constant development of the physical and mathematical apparatus applying to describe such models, and the increase in the power of computer technology, which allows using more sophisticated algorithms.

For a long time, the stress-strain state of the railway track was described by static models, or quasi-static ones, which in one form or another took into account the dynamics of loading on the track. This required the application of a number of hypotheses and assumptions, but significantly reduced the volume of calculations. For many needs, this approach could be considered adequate today [1]. But there is a class of problems that requires considering the dynamics of not only rolling stock, but also the railway track reaction. One of the directions for the development of railway transport, which encourages the transition to a fundamentally new dynamic model of the railway track, is certainly enhancing the speed of movement, including on the sections of international transport corridors [2, 3].

The main commonly accepted approach that is used to model (simulate) the dynamic interaction of rolling stock and track is the application of systems of equations compiled according to the Lagrange-d'Alembert principle [4]. It is known that the basis of this solution is the dynamic equilibrium of motion (oscillations) of bodies with constant masses brought to a point (center of mass), which are interconnected by elastic and dissipative linkings. Unlike rolling stock, in case for the railway track



this representation does not quite correspond to its design [5, 6]. The main reason is that this approach requires the replacing the deformation processes of the layers for the mutual movements of individual objects. Depending on the task, the representation of the railway track can vary from reducing to one element – evenly elastic cushion, to the cyclic repetition of multi-element blocks. However, after a certain limit, excessive detail, as well as substantial simplification, does not increase the adequacy of the model. In any case, one of the main issues in creating such models remains the determination of the element mass of the railway track and the physical characteristics of the connections between them. As a rule, they are accepted with certain conditions through analytical calculations or laboratory tests. The latter is best corresponds the layered objects such as ballast [7-10].

One of the ways in solving this contradiction is a complex solution of the problems for rolling stock and the track interaction, when one model of the railway track is used to simulate the passage of rolling stock and obtain its action on the track, and the other – to describe the dynamic processes already directly in the track from a certain load. Such approaches are used in [11-17] and many others. These references are selective examples to highlight the proposed classification. Factoring in the large number of well-developed works in which the mentioned elements of railway track simulation are applied in one form or another, this sequence of references (literary sources) does not mean their relevance, and the list was compiled according to the requirements of completeness. In paper [18] the authors have analyzed the various variants of detailing the railway track.

For example, in paper [14] to study the vibration of sleepers, including places with existing depressions, the railway track was first presented as a rail combined with a wheel (a common approach, given the high stiffness of the wheel-rail connection and low reduced rail weight compared to weight of the wheel), and a sleeper that is connected to the rail on one side and the under sleeper base on the other, with elastic-dissipative linkings. Then, the finite element method is used to detail the stress-strain state of the track.

In paper [15], a greater detail of the railway track was chosen: the sleeper (object with mass) has an elastic-dissipative linking with the rail and with ballast (it is represented as an object with mass reduced to one sleeper), ballast has elastic-dissipative linking with the base (ground); in addition, the ballast objects still have adjacent elastic-dissipative linkings between themselves. Two-stage simulation is used to study the vibrational effect of rolling stock on the ground: rolling stock – track – sub-track base and track – ground.

Thus, there is a possibility to create fundamentally new mathematical models, aimed primarily at obtaining a tool for studying the dynamic features of the railway track.

2. Methodology

As a tool for solving such problems, the authors developed a model based on the dynamical theory of elasticity [19], which allows us to describe the dynamic equilibrium of strains of the railway track layers. Considering the obligation to register the course of the process in time, for each time step of integration the outline of the strains propagations front is determined. For bodies in a solid state, the strains propagations front is described by the equations of propagation in the material of the elastic spatial wave of oscillations. Thus, the model of the stress-strain state for the railway track consists in a combination of the geometry equations of the outline of a part in the system's space, involved into interaction at a given time and the equations of the dynamic equilibrium of its deformation.

Taking into account that the geometry of strains propagations is described by equations of motion for a spatial wave, the author in a number of published works used the terms "wave model" or "model based on the wave theory of stress propagation". It turned out that such terminology is not successful, since in many cases the developed model is mistakenly accepted as a tool for studying exclusively the propagation of vibrations from the railway track. In addition, the geometry of wave propagation is only one of several involved positions, and such terminology does not accurately reflect the essence of a model. Therefore, now the authors use the more appropriate name – the spatial model of dynamic strains for the railway track based of the elasticity theory.

It allows determining complete spatial-temporal account of the deflection dynamics of the rail support, which enables the execution of calculations, including for conditions of fast-speed and high-

speed trains. In the developed model, the railway track is described as a set of objects $\omega_i \in \Omega$, (figure 1).

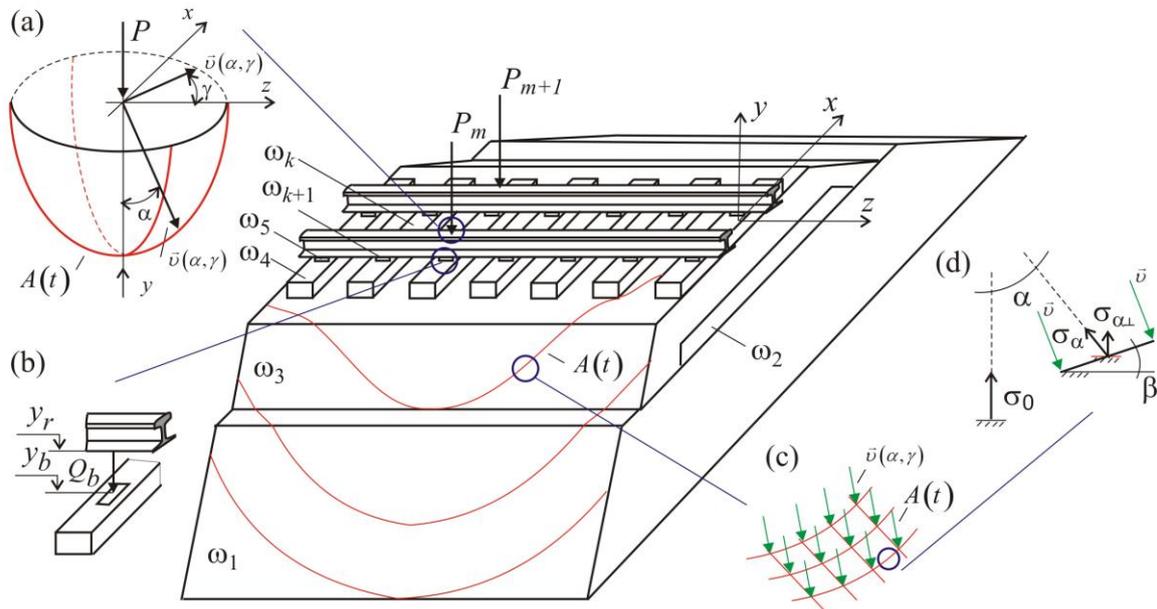


Figure 1. Calculation scheme for the spatial model of dynamic strains of the railway track on the basis of the elasticity theory: a – the strains propagations front; b – fluctuations of rails are determined from the condition of mutual rail deflection and deflection of the rail support; c – the location of the ends of the vectors determines the surface of the wave front for the current time step; d – the direction of stresses for the location of element.

For a railway track system, objects are geometrically restricted elements that have homogeneous (or conditionally homogeneous) physical properties. Depending on the task that has a solution, the list and detail of such objects may have options. For the calculation of the railway track as a whole it will be rails, sleepers, fasteners, ballast and roadbed. The number of objects such as sleepers and fastenings will depend on the length of the railway track section introduced into the model. If it is necessary, to monitor the stress in the experimental section of the track in the full cycle of their formation from the moving load (wheel) from the moment of its location even beyond the impact and by the time of location already outside the impact [1] can be considered a section by length of 15-20 sleepers.

For certain tasks, ballast and sometimes roadbed, cannot be defined as homogeneous medium. In these cases, such elements can be divided into a subset of objects with the desired detail. For this approach, the main quality of an object is the propagation velocity of an elastic wave in it. Characteristics such as Young's modulus (E), Poisson's ratio (μ) and density (ρ) are set the same within the object. This is a key factor for dividing the system into separate objects. A set of these characteristics makes it possible to determine the wave propagation velocity both longitudinal (C_l) and transverse (C_t) directions, and in arbitrary (C_α) – equation (1).

To create the outline of the wave propagation at each time step of calculations (Δt), a set of vectors ($\{\vec{v}\}$) with angular steps corresponding to the desired detail is used – equation (2), (figure 1(a), 1(c)). The vector approach, first, allows you to take into account the space restrictions of the propagation by the geometry of the object. Second, it allows you to consider the transition from one object to the next by adjusting the velocity of elongation of the corresponding vectors. Thus, the location of the ends of the vectors determines the surface of the wave front for the current time step in a multi-object system ($A(t)$).

$$\left. \begin{aligned} C_\alpha &= C_r C_l (C_r^2 \cos^2 \alpha + C_l^2 \sin^2 \alpha)^{-1/2}; \\ C_l &= \left(\frac{E(1-\mu)}{\rho(1+\mu)(1-2\mu)} \right)^{1/2}; \\ C_r &= \left(\frac{E}{2\rho(1+\mu)} \right)^{1/2}. \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} A(t) &= \{ \nabla \vec{v} \in \omega_k \}; \\ \vec{v} &= \Delta t C_\alpha; \\ \alpha &\in [0; 0.5\pi]; \\ \gamma &\in [0; 2\pi]. \end{aligned} \right\} \quad (2)$$

With every time step the stresses zone increases, dividing the rail support into conditional segments (B_i) with surface area (S_i) – equation (3). It also gives the opportunity to obtain a real mass of substance (material) that involves in the interaction (m_i), and excludes the need to attract a "reduced" mass

$$\left. \begin{aligned} B_i &= A(t) \setminus A(t - \Delta t); \\ S_{i-1} &= B_i \cap B_{i-1}; \\ m_i &= \rho_i B_i. \end{aligned} \right\} \quad (3)$$

Each such segment is divided into separate elements – space, limited by four adjacent vectors – (figure 1(c)).

The stresses on the initial (σ_i) and final (σ_{i+1}) surfaces of such an element must be in dynamic equilibrium - the difference between their potentials corresponds to the deformation of the material (u_i) given its mass and the acceleration – equation (4).

$$m_i \frac{d^2 u_i}{dt^2} = \sigma_i S_i - \sigma_{i+1} S_{i+1}. \quad (4)$$

As stresses in equation (4) we mean those, acting in the direction (α) that determines the location of this element – (figure 1(d)). To form a single equation for the dynamic equilibrium of a segment, the functions of bring the stresses in an arbitrary direction to those that coincide with operative direction of the applied force (σ_0) – equation (5).

$$\left. \begin{aligned} \sigma_{\alpha(\perp)} &= \sigma_0 \xi \cos^3 \alpha; \\ \xi &= (\varphi^2 - \varphi^2 \sin^2 \alpha + \sin^2 \alpha) \varphi^{-2}; \\ \varphi &= \left(\frac{1-2\mu}{2(1-\mu)} \right)^{1/2}. \end{aligned} \right\} \quad (5)$$

The main part of the developed model is a description of the stress-strain state of the rail support to which forces acting on brackets from the rail deflection are applied. These forces are defined in the simulation process as a connection that binds the rail deflection and the rail support strain. Changing the position of the wheel along the length of the rail does not directly lead to a change in the

coordinates of the forces application on the rail support. It redistributes the pressure on the immovable brackets; with the movement time of wheels in the system new loaded brackets will be "added" and, accordingly, "withdrawn" unloaded ones. Fluctuations of rails are determined from the condition of mutual rail deflection (y_r), which is based on the brackets, and deflection of the rail support in the place of brackets (y_b) from forces, transmitted to these brackets from the rails (Q_b) – equation (6), (figure 1(b)).

$$\left. \begin{aligned} y_r(z, t) &= y_b(z, t); \\ y_r &= f(Q_b, \sigma_{\alpha, \gamma} : (\alpha, \gamma) \equiv z); \\ Q_b &= f\left(\frac{d^4 y_r}{dz^4}\right). \end{aligned} \right\} \quad (6)$$

The developed mathematical model for the system of objects that is the railway track (in accordance with (figure 1)), can be presented in the form of equations (7). Dynamic strain of the segment is subordinate to the system of equations (1). Its solution determines the strain (deformations) of any point of the under-rail space, taking into account dissipation of substance (D_k), which prevents the occurrence of super-fast strains.

$$\left\{ \begin{aligned} \frac{d^2 \sigma_{0(1)}}{dt^2} \sum_{\alpha} \frac{m_{\alpha} \xi_{\alpha} v_{\alpha} \cos^4 \alpha}{E_{\alpha}} &= f(P_m, Q_b) - \sigma_{0(1)} \sum_{\alpha} \sum_{\gamma} S_{\beta} \xi_{\alpha} \cos^2(\alpha - \beta) + D_k \frac{d\sigma_{0(1)}}{dt}; \\ \frac{d^2 \sigma_{0(2)}}{dt^2} \sum_{\alpha} \frac{m_{\alpha} \xi_{\alpha} v_{\alpha} \cos^4 \alpha}{E_{\alpha}} &= \sigma_{0(1)} \sum_{\alpha} \sum_{\gamma} S_{\beta} \xi_{\alpha} \cos^2(\alpha - \beta) - \sigma_{0(2)} \sum_{\alpha} \sum_{\gamma} S_{\beta} \xi_{\alpha} \cos^2(\alpha - \beta) + D_k \frac{d\sigma_{0(2)}}{dt}; \\ &\dots \\ \frac{d^2 \sigma_{0(i)}}{dt^2} \sum_{\alpha} \frac{m_{\alpha} \xi_{\alpha} v_{\alpha} \cos^4 \alpha}{E_{\alpha}} &= \sigma_{0(i-1)} \sum_{\alpha} \sum_{\gamma} S_{\beta} \xi_{\alpha} \cos^2(\alpha - \beta) - \sigma_{0(i)} \sum_{\alpha} \sum_{\gamma} S_{\beta} \xi_{\alpha} \cos^2(\alpha - \beta) + D_k \frac{d\sigma_{0(i)}}{dt}; \\ &\dots \\ \frac{d^2 \sigma_{0(n)}}{dt^2} \sum_{\alpha} \frac{m_{\alpha} \xi_{\alpha} v_{\alpha} \cos^4 \alpha}{E_{\alpha}} &= \sigma_{0(n-1)} \sum_{\alpha} \sum_{\gamma} S_{\beta} \xi_{\alpha} \cos^2(\alpha - \beta) - \sigma_{0(n)} \sum_{\alpha} \sum_{\gamma} S_{\beta} \xi_{\alpha} \cos^2(\alpha - \beta) + D_k \frac{d\sigma_{0(n)}}{dt}. \end{aligned} \right. \quad (7)$$

The change of stresses within one object is subordinated to the system of equations (7). Each such equation, included in the system, describes the transfer of pressure from one wall of a segment to another, factoring in the areas, dependence between the stresses by the elements that makes up the surface of the segment, and the inertial motion of masses of the material under deformation. If such a transition occurs between different objects, the contact surface is divided into segments, (figure 2). This requires appropriate adjustments:

$$\left\{ \begin{aligned} &\dots \\ \{\sigma_{\omega 1(i)}\} &= f(A_{\omega 1}(t - \Delta t), A_{\omega 1}(t) + A'(t)); \\ \{\sigma_{\omega 2(i)}\} &= f(A'(t), A_{\omega 2}(t)); \\ \{\sigma_{\omega 1(i+1)}\} &= f(A_{\omega 1}(t), A_{\omega 1}(t + \Delta t)); \\ &\dots \end{aligned} \right. , \quad (8)$$

where $\{\sigma_{(i)}\} = f(A(t - \Delta t), A(t))$ – the abbreviated representation of the i -th equation of the system (7).

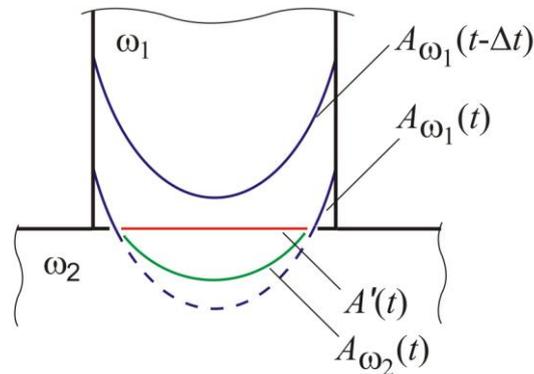


Figure 2. Separation of space into segments at the contact of two objects.

3. Conclusions

In the work analytical dependences were obtained and key points of the mathematical tool for the creating models of the stress-strain state of the railway track according to the principle of combining the geometry equations of the system space part outline were formulated. This system was involved in the interaction for a given moment of time and equations of the dynamic equilibrium of its deformation. Solutions to the following problems associated with the implementation of the mathematical model were obtained: definition of equations for outline of interaction front distribution, description of the spatial surface expansion of the area interaction through the vector set, correction for the front outline of the stresses distribution by objects of limited space (sleepers, substrates, etc.) and semi-limited one (ballast, roadbed, etc.), correction of the stresses front distribution when transition from one object to the next with other physical properties, determination of the equations of the dynamic equilibrium for deformed zones of space, description of the stresses distribution functions for the remote zone in space, principles of pressure transfer from one object to the next through the contact surface, etc.

A fundamentally new model of the track interaction was developed, which allows determining the stress-strain state of the railway track with a complete space-time record in the dynamics of the rail-base depression, which will provide to perform calculations for both railways with the usual speed of trains, and for conditions of high-speed running.

With the application of the developed model, a number of scientific and practical problems were solved. In particular, the levels of motion speed in accordance with the track design were determined, in which the rail-base does not have time to realize strains along the entire length in the formation of the rail depression, which leads to fundamental changes in the stress-strain state [20]. Values of the stiffness and dissipation factors for the railway track according to the design and motion speed for representing the railway track in the models of rolling stock, described by the systems of equations based on the Lagrange-d'Alembert equation [18] were obtained. Features of the stress-strain state for the railway track with the dual gauge design were analyzed [3].

4. References

- [1] Kurhan D 2016 Determination of Load for Quasi-static Calculations of Railway Track Stress-strain State *Acta Technica Jaurinensis*. **9(1)** pp 83-96
- [2] Kurhan M and Kurhan D 2018 Problems of providing international railway transport *MATEC Web Conf.* 230 (2018) 01007
- [3] Kurhan M B and Kurhan D M 2018. *Scientific-technical support of the railway Ukraine - European Union* (Dnipro, Ukraine)

- [4] Darenskiy A, Vitolberg V, Fast D, Klymenko A and Leibuk Y. 2017. A mathematical model of the rail track presented as a bar on elastic and dissipative supports under the influence of moving loads *MATEC Web Conf.* 116 (2017) 03002
- [5] Potapov D, Panchenko S, Leibuk Y, Tuley Y and Plis P 2018 Effect of joint and isolated irregularities of the track on the wear of rails in curves *MATEC Web Conf.* 230 (2018) 01012
- [6] Boiko V, Molchanov V, Tverdomed V and Oliinyk O. 2018 Analysis of Vertical Irregularities and Dynamic Forces on the Switch Frogs of the Underground Railway *MATEC Web Conf.* 230 (2018) 01001
- [7] Fischer S and Németh A 2018 Special laboratory testing method for evaluation particle breakage of railway ballast material *Science and Transport Progress.* **2(74)** pp. 87-102
- [8] Juhász E and Fischer S 2018 Investigation of railroad ballast particle breakage *Pollack Periodica.* 14(2) pp. 3-14
- [9] Pshinko O, Patlasov O, Andrieiev V, Arbuzov M, Hubar O, Hromova O and Markul R 2018 Research of railway crashed stone use of 40–70 mm fraction *Proceedings of 22rd International Scientific Conference. Transport Means 2018.* pp. 170-178
- [10] Sysyn M, Gerber U, Kovalchuk V and Nabochenko O 2018 The complex phenomenological model for prediction of inhomogeneous deformations of railway ballast layer after tamping works *Archives of Transport.* **47(3)** pp. 91-107
- [11] Connolly D P and Forde M C 2015 Use of conventional site investigation parameters to calculate critical velocity of trains from rayleigh waves *Transportation Research Record: Journal of the Transportation Research Board.* **2476** pp. 32-36
- [12] Kouroussis G, Vogiatzis K E and Connolly D P 2017 A combined numerical/experimental prediction method for urban railway vibration *Soil Dynamics and Earthquake Engineering.* **97** pp. 377-386
- [13] Meli E and Ridolfi A 2015 An innovative wheel–rail contact model for railway vehicles under degraded adhesion conditions *Multibody System Dynamics.* **33** pp. 285-313
- [14] Esmailia M, Mosayebia S and Zakeria J 2017. Ground-borne vibrations caused by unsupported railway sleepers in ballasted tracks *Procedia Engineering.* **199** pp. 2645-50
- [15] Kouroussis G, Olivier B, Romero A, Galvin P and Connolly D P 2018. A fast numerical assessment of railway-induced ground vibration in urban conditions *Proceedings of the 25th International Congress on Sound and Vibration* (Hiroshima, Japan, July 8–12, 2018)
- [16] Dong K, Connolly D P, Laghrouche O, Woodward P K and Alves Costa P 2018. The stiffening of soft soils on railway lines *Transportation Geotechnics.* **17(A)** pp. 178-191
- [17] Mekonnen A and Teklebrhan A 2018. Modelling the influence of sub-grade material parameters on the response of railway line under moving train loads *Journal of Vibroengineering.* **20(3)** pp. 1501-10
- [18] Kurhan M B and Kurhan D M 2017. Railway track representation in mathematical model of vehicles movement *Science and Transport Progress.* **6(72)** pp. 40-48
- [19] Kolsky H 1953 *Stress waves in solids* (Oxford)
- [20] Kurhan D M 2015. Features of perception of loading elements of the railway track at high speeds of the movement *Science and Transport Progress.* **2 (56)** pp. 136-145