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МОДЕЛИРОВАНИЕ РАБОТЫ АЭРОТЕНКА

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SIMULATION OF AERATION TANK WORK

Разработана численная модель для оценки эффективности работы аэротенка. Модель основана на применении уравнений гидродинамики и массопереноса. При моделировании очистки сточных вод в аэротенке учитываются биологические процессы. Для численного интегрирования моделирующих уравнений используется метод конечных разностей. Разработанная численная модель позволяет учитывать геометрическую форму аэротенка. Представлены результаты вычислительного эксперимента.

Розроблена численна модель для оцінки ефективності роботи аэротенка. Модель заснована на застосуванні рівнянь гідродинаміки та масопереносу. При моделюванні очищення стічних вод у аеротенці враховуються біологічні процеси. Для чисельного інтегрування моделюючих рівнянь використується метод кінцевих різниць. Розроблена численна модель дозволяє враховувати геометричну форму аеротенка. Представлені результати обчислювального експерименту.

Introduction. Aeration tanks (AT) are widely used for biological wastewater at different enterprises. For practice it is very important to predict the efficiency of AT work under different regimes of work. To solve this problem we need to use mathematical models. These models must be convenient for practical application and do nor consume much computer time.

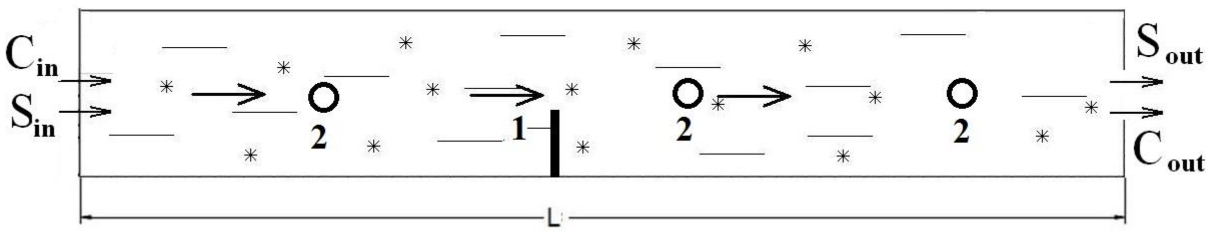

Fig. 1. Aeration tank "*vitesnitel*" (AT of displacement type):
1 – baffle; 2 – point of sludge supply

Literature review. To calculate the wastewater treatment the engineers use empirical models, mass balance models, one – dimensional equations of mass transport and CFD models [1-5]. CFD experiments comprise of two steps. The first step is computation of flow field. Very often this flow field is computed using of Navier-Stokes equations. The second step is simulation of admixture transfer on the basis of computed flow field. Application of Navier-Stokes equations needs much computing time. It is not convenient in case of many calculations during AT design or at stage of AT re-engineering.

Goal. The goal of this work is the development of mathematical model to simulate the process of biological wastewater treatment in "*vitesnitel*" aeration tank (aeration tank of displacement type).

Mathematical model. Process of biological treatment in AT is separated in two stages during simulation. At first stage we consider the process of substrate and sludge movement in the aeration tank. It is so called 'mass transfer' process. To simulate this process we use the following 2-D transport equations (plan model)

$$\frac{\partial C}{\partial t}+\frac{\partial uC}{\partial x}+\frac{\partial vC}{\partial y}=div(\mu gradC), \tag{1}$$

$$\frac{\partial S}{\partial t}+\frac{\partial uS}{\partial x}+\frac{\partial vS}{\partial y}=div(\mu\, grad\; S), \tag{2}$$

where

$C(x,y)=\frac{1}{H}\int_0^H C(x,y,z)dz$ – is the averaged concentration of substrate;

H – is the depth of the aeration tank;

$S(x,y)=\frac{1}{H}\int_0^H S(x,y,z)dz$ – is the averaged concentration of sludge for biological treatment;

u, v – are the flow velocity components in *x, y* direction respectively;

$\mu=(\mu_x,\mu_y)$ – are the coefficients of turbulent diffusion in x, y direction respectively;

t – is time.

The boundary conditions for these equations are as following:

1. at the inlet opening the boundary condition (Fig.1)

$$C=C_{in}, S=S_{in}, \tag{3}$$

where C_{in}, S_{in} – are known concentrations of substrate and sludge respectively.

2. at the outlet opening the boundary condition in the numerical model (Fig.1) is written as follows

$$\begin{aligned}C(i+1,j)&=C(i,j),\\ S(i+1,j)&=S(i,j),\end{aligned} \tag{4}$$

where $C(i+1,j), S(i+1,j)$ – are concentrations at the last computational cell; $C(i,j), S(i,j)$ – are concentrations at the previous computational cell.

Boundary condition (4) means that we neglect the diffusion process at the outlet boundary.

3. at the solid walls the boundary condition is

$$\frac{\partial C}{\partial n}=0,\ \frac{\partial S}{\partial n}=0,$$

where n – is normal vector to the boundary.

The initial condition, for $t=0$, is

$$C=C_0,\ S=S_0,$$

where C_0, S_0 – are known concentrations of substrate and sludge respectively in computational domain.

At the second stage of mathematical simulation we consider the biological process in the aeration tank. To simulate this process in each computational cell inside the aeration tank we use the following simplified model

$$\frac{dC(t)}{dt}=-\frac{\mu(t)}{Y}S(t), \tag{5}$$

$$\frac{dS(t)}{dt}=\mu(t)S(t), \tag{6}$$

where μ – is biomass growth rate,

Y – is biomass yield factor.

To calculate biomass growth rate Monod law is used.

As the initial condition for each equation (5), (6), at each time step, we use the meaning of C, S obtained after computing Eq. 1, 2.

To solve Eq.1, 2 it is necessary to know the flow field in aeration tank. To simulate this flow field we use model of potential flow. In this case the governing equation is

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}=0 \tag{7}$$

where P – is the potential of velocity.

The velocity components are calculated as follows:

$$u=\frac{\partial P}{\partial x},\ v=\frac{\partial P}{\partial y}. \tag{8}$$

Boundary conditions for equation (7) are:

1. At the inlet boundary $\frac{\partial P}{\partial n}=V$, where V is known velocity.

2. At the outlet boundary $P=const$.

3. At the solid boundaries $\frac{\partial P}{\partial n}=0$.

Numerical model. Numerical integration of governing equations is carried out using rectangular grid. Concentration of substrate, sludge and P were determined in the centers of computational cells. Velocity components *u, v* were determined at the sides of computational cells.

To solve equation (7) we used the difference scheme of “conditional approximation”. To use this scheme we wrote Eq.5 in ‘unsteady’ form

$$\frac{\partial P}{\partial t} = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \tag{9}$$

where t – is ‘fictitious’ time.

It’s known that for $t \to \infty$ solution of Eq.9 tends to the solution of Eq.7.

We split the process of Eq. 9 in two steps and difference equations at each step are as follows:

$$\frac{P_{i,j}^{n+\frac{1}{2}} - P_{i,j}^{n}}{\Delta t} = \left[\frac{-P_{i,j}^{n+\frac{1}{2}} + P_{i-1,j}^{n+\frac{1}{2}}}{\Delta x^2}\right] + \left[\frac{-P_{i,j}^{n+\frac{1}{2}} + P_{i,j-1}^{n+\frac{1}{2}}}{\Delta y^2}\right], \tag{10}$$

$$\frac{P_{i,j}^{n+1} - P_{i,j}^{n+\frac{1}{2}}}{\Delta t} = \left[\frac{P_{i+1,j}^{n+1} - P_{i,j}^{n+1}}{\Delta x^2}\right] + \left[\frac{P_{i,j+1}^{n+1} - P_{i,j}^{n+1}}{\Delta y^2}\right]. \tag{11}$$

The calculation on the basis of these formulas is complete if the following condition is fulfilled:

$$\left| P_{i,j}^{n+1} - P_{i,j}^{n} \right| \le \varepsilon ,$$

where ε – is a small number; n – is iteration number.

Difference scheme of splitting (10), (11) is implicit but unknown value of P is calculated, at each step of splitting, using explicit formula of *‘running calculation’*. That is very convenient for programming the difference formulae.

To solve Eq. 9 it is necessary to set initial condition for fictitious time $t = 0$. The initial condition is

$$P = P_0 ,$$

where P_0 – is known value of potential in computational domain.

If we know field of P in computational domain we can compute velocity components at the side of computational cells

$$u_{ij} = \frac{P_{i,j} - P_{i-1,j}}{\Delta x}, \tag{12}$$

$$v_{ij} = \frac{P_{i,j} - P_{i,j-1}}{\Delta y}. \tag{13}$$

Main features of the implicit difference scheme to solve numerically Eq.1, 2 we consider only for equation of substrate transport because Eq.1 and 2 are similar from mathematical point of view. Before numerical integration we split transport equation in two equations. The scheme of splitting is as follows

$$\frac{\partial C}{\partial t} + \frac{\partial u C}{\partial x} + \frac{\partial v C}{\partial y} = 0, \tag{14}$$

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(\mu \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial C}{\partial y}\right). \tag{15}$$

From the physical point of view, equation (14) takes into account substrate movement along trajectories, equation (15) takes into account the process of substrate diffusion in aeration tank. After that splitting the approximation of equation (12) is carried out. Time dependent derivative is approximated as follows:

$$\frac{\partial C}{\partial t} \approx \frac{C_{ij}^{n+1} - C_{ij}^{n}}{\Delta t}.$$

The convective derivatives are represented as:

$$\frac{\partial uC}{\partial x} = \frac{\partial u^{+}C}{\partial x} + \frac{\partial u^{-}C}{\partial x},$$

$$\frac{\partial vC}{\partial y} = \frac{\partial v^{+}C}{\partial y} + \frac{\partial v^{-}C}{\partial y},$$

where $u^{+} = \frac{u + |u|}{2}$, $u^{-} = \frac{u - |u|}{2}$, $v^{+} = \frac{v + |v|}{2}$, $v^{-} = \frac{v - |v|}{2}$.

$$\frac{\partial u^{-}C}{\partial x} \approx \frac{u_{i+1,j}^{-} C_{i+1,j}^{n+1} - u_{i,j}^{-} C_{i,j}^{n+1}}{\Delta x} = L_x^{-} C^{n+1},$$

$$\frac{\partial v^{+}C}{\partial y} \approx \frac{v_{i,j+1}^{+} C_{i,j} - v_{i,j}^{+} C_{i,j-1}}{\Delta y} = L_y^{+} C^{n+1},$$

$$\frac{\partial v^{-}C}{\partial y} \approx \frac{v_{i,j+1}^{-} C_{i,j+1} - v_{i,j}^{-} C_{i,j}}{\Delta y} = L_y^{-} C^{n+1}.$$

At the next step we write the finite difference scheme of splitting:

- at the first step k=1/2:

$$\frac{C_{ij}^{n+k} - C_{ij}^{n}}{\Delta t} + \frac{1}{2}\left(L_x^{+} C^{k} + L_y^{+} C^{k}\right) = 0; \tag{16}$$

- at the second step k=1, c=n+1/2:

$$\frac{C_{ij}^{k}-C_{ij}^{c}}{\Delta t}+\frac{1}{2}\left(L_{x}^{-}C^{k}+L_{y}^{-}C^{k}\right)=0. \tag{17}$$

This difference scheme is implicit and absolutely steady but unknown concentration C is calculated using the explicit formulae at each step (“method of running calculation”).

Further, Eq.(15) is numerically integrated using implicit difference scheme (10), (11). To solve Eq.3, 4 we used Euler method. On the basis of developed numerical model code ‘*BIOTreat’* was developed. FORTRAN language was used to code the solution of difference equations.

Case Study. Developed code ‘*BIOTreat’* was used to solve the following model problem. We consider tree corridor aeration tank with baffle (Fig.1). The aeration tank is filled with sludge (concentration S_0=10) and substrate (concentration C_0=100) at time t=0. All parameters of the problem are dimensionless. During time period from t=0 till t=1 the inlet and outlet openings are closed and no flow in the aeration tank. It means that for this time period only biological treatment takes place and we solve only Eq. 5, 6 of the model. At time t=1 the inlet and outlet openings are open and the transport process starts. Direction of flow is shown in Figure 3 by arrows. At the inlet opening the substrate concentration is equal to C_0=100 and sludge concentration is equal to S_0=10. Also at this time 3 sources of sludge supply inside the aeration tank starts to work with intensity Q_i (Fig.1).

In Figure 2 we present sludge and substrate concentration change near the outlet opening of the aeration tank (point A in Fig.3) and substrate concentration near baffle. From Figure 2 we can see that the process of biological treatment accelerates from t=1. In Figure 3 the concentration field of sludge for time step t=2.7 is shown.

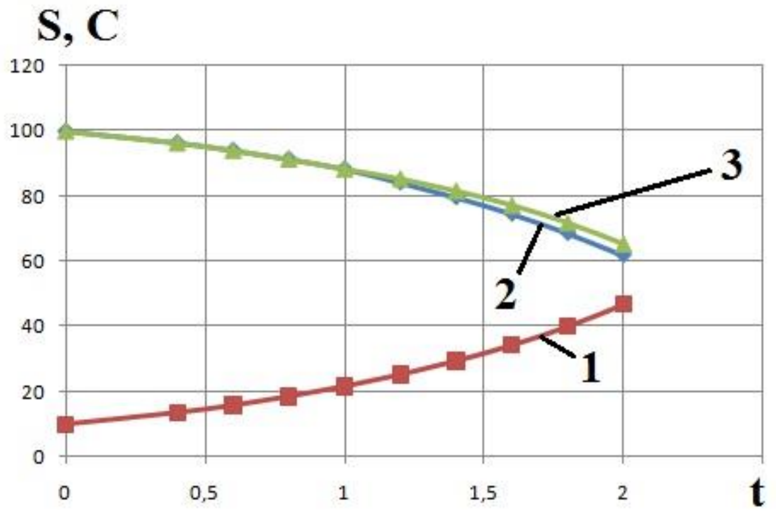

Fig. 2. Sludge and substrate concentration vs time: 1 – sludge concentration at outlet opening; 2 – substrate concentration at outlet opening (point A); 3 – substrate concentration near baffle (point B)

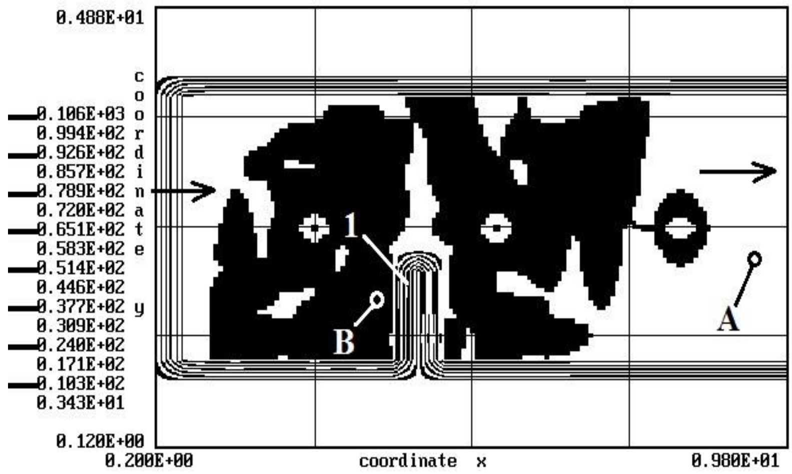

Fig. 3. Field of sludge concentration inside the aeration tank, t=2.7: 1 – baffle; A – check point near outlet opening; B – check point near baffle

Conclusions.

Numerical model was developed to compute wastewater treatment in aeration tank (aeration tank of displacement type). To simulate the process of biological treatment 2-D transport equations of substrate and sludge are used together with simplified models of biological treatment.

The future work in this field will be connected with development of 3D fluid dynamics model which takes into account oxygen transfer in the aeration tank.

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ABSTRACT

The results of the study, dedicated to the process of biological treatment in aeration tank.

The purpose of the study is development of numerical model to compute quickly wastewater treatment in aeration tank.

The method of the research is CFD simulation.

Findings. New numerical model is proposed to compute the process of biological treatment in aeration tank.

The originality. New model was developed for 2D computing of biological treatment in aeration tank.

Practical implications. Developed model allows quick computing of aeration tank work with account of its geological form.

Keywords: aeration tank, wastewater, CFD modeling