THE OPTIMUM DESIGN OF COMPRESSED THIN-WALLED COLUMNS OF OPEN CROSS-SECTION

A.I. MANEVICH
Department of Theoretical Mechanics and Strength of Materials. Ukrainian University of Chemical Technology. Gagarina av. 8, Dniepropetrovsk, 49005, Ukraine

S.V. RAKSHA
Department of Applied Mechanics. Dniepropetrovsk State Technical University of Railway, ak. Lazaryana str. 2, Dniepropetrovsk, 49010, Ukraine

The structural optimization problem for compressed thin-walled columns of open cross-section (channel and lipped channel) is considered on the base of the linear stability theory and the nonlinear interactive buckling theory. Optimal dimensionless parameters are presented as a function of a single leading parameter of load and geometry P^* and imperfection amplitudes.

1. INTRODUCTION

The minimum weight design problem for compressed thin-walled columns, in particular, of channel profile, with respect to their stability was investigated in a few works already in 60-70-s (see [1], [2]). Since the mechanism of failure of compressed thin-walled bars can be different depending on their slenderness, various theoretical models of stability were used – the linear theory for elastic material, models accounting for elastic-plastic deformations, semiempirical models taking into account premature local buckling of elements using the «effective width» approach and so on. Approximate analytical and numerical solutions of the problem were derived (note that even linear local buckling problem has not a closed analytical solution as it is necessary to consider the column as assemblage of interacting plates).

In the last decades it has become clear that optimal parameters of thin-walled structures may be strongly affected by the nonlinear interaction of different modes of buckling. The Koiter and Skaloud's prediction (1962) about a danger of using the linear stability theory at solving the optimization problem for thin-walled structures because of their high imperfection sensitivity connected with closeness of critical stresses for different buckling modes was confirmed in investigations by J.M.T. Thompson, G.M. Lewis, W.J. Supple, V. Tvergaard, A.I. Manevich, C. Massonet, R. Maquoi.[see reviews 2-5]. But the problem is studied qualitatively but not quantitatively since (as applied to compressed columns) all conclusions were obtained either for the Neut's idealized model or for hollow square cross-section..

In the paper results of solution of the minimum-weight problem for centrally compressed thin-walled members of channel and lipped channel cross-section are presented. The solutions are based both on the linear stability theory and the nonlinear theory of coupled buckling taking into account the interaction of the overall bending modes with local modes. An efficient method of nonlinear programming is used for solving the optimization problem. This paper is a continuation of the work [6] in which a exact solution of the structural optimization problem for centrally compressed columns of channel cross-section has been derived in the framework of the linear buckling theory.

2.STATEMENT OF THE PROBLEM AND THE METHOD OF SOLUTION

Let a thin-walled column of open cross-section shown in Fig.1 is compressed centrally by a force P being simply supported on its edges. We assume the usual formulation of the minimum-weight problem for thin-walled columns with respect. To their stability. The length L , force P, the material properties (elasticity modulus E, Poisson's ratio ν) and initial imperfections are considered given. The profile thickness t, widths of the flange, web and shelves b_1, b_2, b_3 are design variables (thickness of all elements is considered to be constant implying cold-formed members).

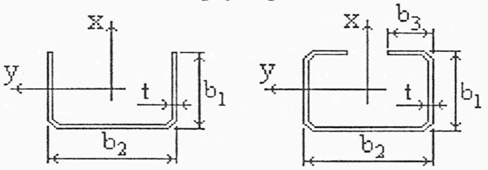

Fig.1. Cross-sections of the columns and design variables

The optimization problem is stated as a nonlinear programming one. Constraints of the problem are buckling constraints taking into account the overall modes (bending and torsional-bending) and local modes, see. Fig.2. For the lipped channel there are possible two types of local modes, depending on the lip width: 1) the flange-lip contact lines has displacements (distortional» mode, fig. 2,c); 2) this line is immovable (local mode, fig.2,d).

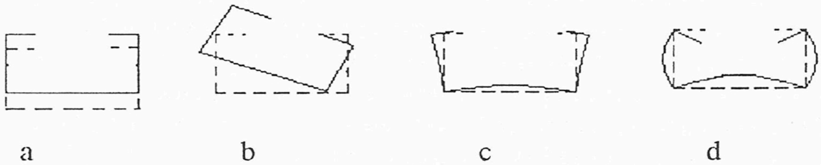

Fig. 2.Buckling modes for lipped channels

In the *linear optimization* (LO) all these buckling constraints are independent conditions. In the *nonlinear optimization* (NO) the interaction between the overall bending and local modes was taken into account, and *coupled buckling constraint* has the form $\sigma \leq \sigma_*$, where σ is the average stress over cross-section, σ_* is the limit value of σ in the interactive buckling (the constraint on the linear torsional-bending mode was retained). Strength constraints are not imposed, i.e. the assumption was assumed that the material is elastic and the yield limit is sufficiently high. As it will be seen from the obtained solution this assumption is valid (for usually used steels and alloys) in a range of low values of the load parameter P^* (see below). For other P^* values the presented solution determines an idealized optimal configuration; having optimal parameters obtained on the base of more complicated models accounting for plastic deformations one can estimate the effect of plasticity on optimal project.

The linear overall buckling stresses were calculated according to the Vlasov's theory [7]. The general equation for critical stresses in the case of profiles with one axis of symmetry is decomposed into two equations:

$$\sigma = \sigma_y, \quad (1 - a_x^2/r^2)\sigma^2 - (\sigma_x + \sigma_\omega)\sigma + \sigma_x\sigma_\omega = 0. \tag{1}$$

where:
a_x - the shear center coordinate,
σ_x, σ_y - the Euler's critical stresses for bending with respect to x- and y- axes,
σ_ω is the critical stress of the torsional buckling,

$$\sigma_x = \pi^2 EI_x/(A \cdot L^2), \quad \sigma_y = \pi^2 EI_y/(A \cdot L^2) \tag{2}$$

$$\sigma_\omega = (\pi^2 EI_\omega/L^2 + GI_d)/(A \cdot r^{2)}, \quad r^2 = (I_x + I_y)/A + a_x^2,$$

I_x, I_y, I_d and I_ω, are the moments of inertia at bending with respect to the two axes, at torsion and the warping constant. Critical stresses for the torsional-bending mode (Fig.2,b) are equal to the minor root of the second Eq. (1).

The linear local buckling stresses were calculated according to [8]. The column was considered as an plates assemblage, and the exact solution of the buckling problem was constructed by conjugation of the solutions for all the constituent plates, with exact conjugation conditions at the contact lines. For each plate the solution of the differential equation of stability was assumed in the form $w = w(\eta)\sin m\pi\xi$, $\eta = b/L$; $\xi = z/L$ (z is the longitudinal coordinate).

Boundary conditions on the free edges and conjugation conditions on the contact lines result in a characteristic equation which determines the local critical stresses (after minimization in m). For the lipped channel profile the account for «distorsional buckling» modes (Fig. 2, c) demands calculation of the in-plane stiffness matrices (in addition to the bending stiffness matrices) in other to satisfy all the conjugation conditions on the flange-lip contact lines. All details of the calculations are dropped here.

The coupled buckling limit stresses were calculated in the framework of the Koiter's asymptotic method in the first approximation (in potential energy quadratic and cubic terms are retained). This theory is adequate if the linear critical stresses for different modes are sufficiently close. The limit stresses were determined from the set of equations (summation on index «s» is not carried out)

$$a_s(1 - \lambda/\lambda_s)\zeta_s + a_{ijs}\zeta_i\zeta_j = a_s\zeta_s^* \lambda/\lambda_s \quad (s=1,2) \tag{2}$$

together with the condition of vanishing its jacobian: J=0. Here λ is the load factor, λ_s is the critical value of λ for mode «s», ς_s and ς_s^* are the amplitudes of displacement and initial deflection in this mode (which were normalized by the condition of equality of the maximal deflection to the thickness t). Coefficients a_s, a_{ijs} are determined by the known integrals containing the linear buckling modes. As a load factor we took the dimensionless average stress $\sigma^* = (\sigma_x/E)10^3$.

Because of closeness of the critical stresses for a cluster of short-wave local modes it is necessary to take into account many local modes. With the aim of simplification of the solution we considered the interaction of each local mode with the overall mode separately. In process of the optimization search the number of the most dangerous local mode changes. Note that dependencies of local buckling stresses (and coupling buckling limit stresses) on the half-wave number m can have two local minima, one of them being determined by the web slenderness, another - by the flange slenderness. In view of these features the local buckling and limit stresses were calculated for a wide range of half-waves numbers, i.e. instead of one constraint we consider a set of constraints (as a rule, for values m=2–35).

The nonlinear programming (NP) problem was solved by the linearized method of reduced gradient (LMRG) [4]. This method, realizing the idea of changing an independent variables set in a vicinity of the admissible domain boundary by means of the linear operations with the sensitivity matrices, effectively overcomes familiar difficulties arising at solving the NP-problem (in particular, connected with zigzag-type motion in a boundary vicinity). Our experience of many years employment of this algorithm have shown its high efficiency and reliability. As a rule, a few tens iterations were required to achieve the optimum with relative error of order 10^{-3} .

For generality of analysis all the constraints were formulated in dimensionless parameters of load, weight and stress:

$$P^* = \frac{P}{L^2 \cdot E} 10^6, \quad G^* = \frac{A}{L^2} 10^3, \quad \sigma^* = \frac{\sigma}{E} 10^3, \tag{3}$$

where A is the cross-section area (the scaling multipliers are introduced in order to deal with parameters of order of unity). The cross-section also was characterized with nondimensional geometric parameters b_2 / L, b_1 / b_2, b_3 / b_2, t / b_2 .

2. RESULTS OF THE SOLUTION

In the linear optimization all optimal nondimensional parameters are determined by specifying the single parameter P^*; in the nonlinear optimization the initial imperfections should be additionally specified. We assume the following values of the imperfection amplitudes (divided by thickness t): (0;0); (0.5;0); (0.5; 0.1) (the first figure –the overall mode, the second – a local mode; for each local mode imperfections amplitude were assumed to be the same). The solution for zero imperfections corresponds to the LO.

We consider the range of parameter P^* (0; 2) in which the assumption about elastic deformations of the material may be justified, according to results of given solution (for usual materials). In this range ten values of P^* have been chosen for which the optimization problem has been solved.

In Fig. 3 there are presented optimal profiles of channel cross-sections obtained at the LO and NO (the imperfections amplitudes (0.5, 0)) for P^*=0.5 (a) and P^*=1.5 (b); in Fig.4 the optimal cross-sections of lipped channel for the same values P^* are depicted (dimensions are given for length L=1 m).

The LO profiles have nearly constant values of widths ratio in entire range of P^* considered. For channel cross-section $b_1 / b_2 = 0.42 - 0.43$; for lipped channel $b_1 / b_2 = 0.37 - 0.39$, $b_3 / b_2 = 0.33 - 0.36$. The thickness parameter t / b_1 depends upon P^* for channel as follows (for lipped channel values t / b_1 are nearly the same).

Parameter P^*	0.5	1.0	1.5	2.0
t / b_2	0.049	0.056	0.061	0.065

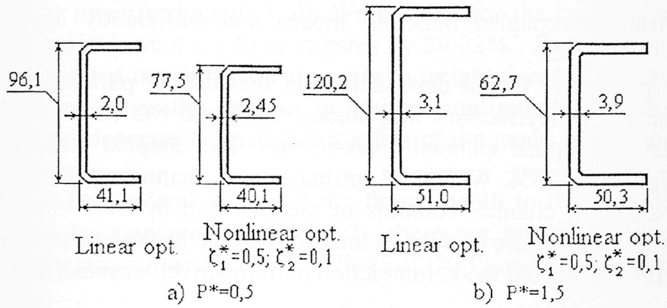

Fig. 3. Optimal cross-sections of channel at LO and NO for two values of P^* (L=1 m).

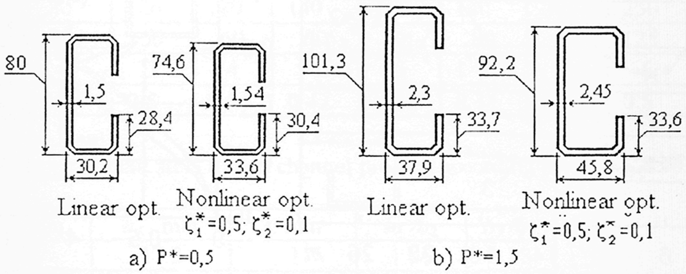

Fig. 4. Optimal cross-sections of lipped channel at LO and NO for two values of P^* (L=1 m).

In the NO the widths ratios and the relative thickness depend upon the imperfections amplitudes, as it is seen from Table 1 (for P^*=1).

Table 1. Optimal parameters for channel and lipped channel at P^*=1 for various combinations of overall and local imperfections.

Imperfections	Optimal dimensionless parameters						
	Channel			Lipped channel			
	b_2/L	b_1/b_2	t/b_2	b_2/L	b_1/b_2	b_3/b_2	t/b_2
(0; 0)	0.110	0.425	0.024	0.093	0.367	0.341	0.021
(0,5; 0)	0.099	0.490	0.029	0.090	0.459	0.349	0.022
(0,5;0,1)	0.088	0.529	0.037	0.086	0.512	0.369	0.024

The account for the mode interaction in optimization results in perceptible decrease of web width b_2 and increase of thickness t. The LO columns have equal critical stresses for the torsional-bending mode and one of the local modes. For overall bending mode (with respect to y-axis, Fig.1) critical stresses are found to be higher by 10–15 %. The NO columns have equal critical stresses for the torsional-bending mode and one of the coupled modes. The critical stresses for the local modes are higher than for the overall ones by 50–80%. In Fig. 5 curve 1 represents the spectrum of local critical stresses for the LO channel column at $P^*=2$, curve 2 – that of the NO column at $P^*=2$ and imperfections amplitudes (0.5; 0), and curve 3 – coupled buckling stresses for the NO columns (when each local mode is accounted for separately). We see a very dense spectrum of coupled buckling modes and sufficiently high local buckling stresses.

Weight parameter G^* in dependence on the loading parameter P^* is shown in Fig.6 for the LO (imperfections amplitudes (0;0)) and NO (imperfections (0.5;0) for both channel and lipped channel cross-section. The coupled buckling increases the optimal weight by 5-10%. Weight of optimal lipped channel columns is lower by 16% in comparison with channel columns in the LO, and by 24% – in the NO. So the lipped channel profiles are preferable for two reasons: a) the lips increase local critical stresses; b) they lower the mode interaction and imperfection sensitivity.

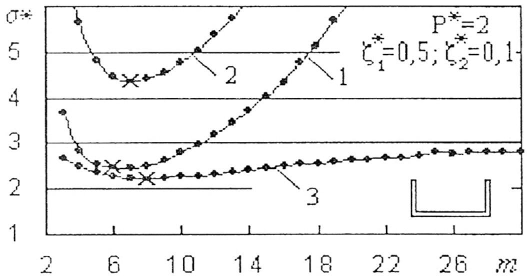

Fig. 5. Spectrum of local- and coupled buckling stresses for optimal columns in the LO and NO.

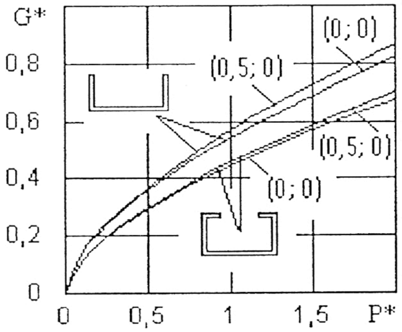

Fig.6. Weight parameter G^* versus load parameter P^* for channel and lipped channel.

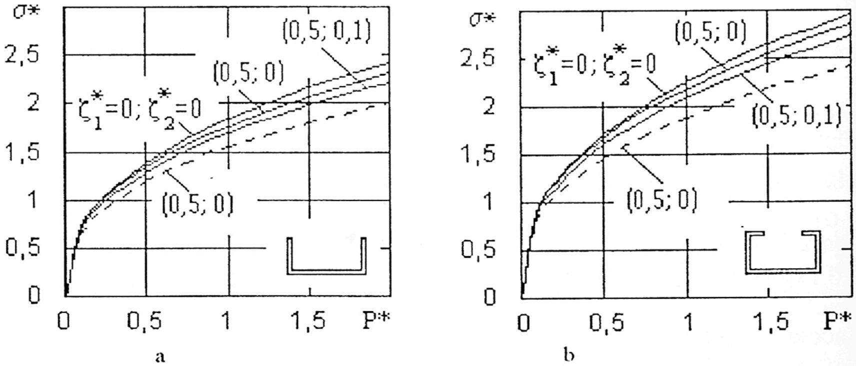

Fig. 7. Dimensionless ultimate stresses versus the load parameter P^* for two profiles at various

combinations of overall and local imperfections amplitude.

Dimensionless critical stresses (averaged over the cross-section) in dependence on parameter P^* for the both profiles are presented in Fig. 7, a, b, for various combinations of initial imperfections amplitudes (solid lines).

It is interesting that at imperfections (0.5; 0.1) and (1.0; 0) (the last case is not presented in Fig.1) critical stresses nearly the same, so we can conclude that local imperfections have larger effect on optimal parameters than overall ones. The dashed lines represent the ultimate stresses for the LO columns when the coupled buckling is taken into account at imperfections (0.5; 0). We see that for the columns obtained at LO the mode interaction lowers the limit stresses by 20-22% . But after optimization with respect to the coupled instability the decrease in weight does not exceed 5-8% for the same imperfections. Thus the changes in optimal configurations obtained in the nonlinear optimization decrease the dangerous effect of the mode interaction by a few times.

Let us compare the optimal values of the flange width to the web width ratios with those specified by standards. In Table 2 there are presented dimensions of channel cross-sections according to GOST 8278-83 (ГОСТ) and corresponding values of b_1/b_2.

Table 2. Dimensions of bent steel channel profiles according to GOST 8278-83.

b_2, mm	60	80	100	120	140	160	180	200	250	300
b_1, mm	32	50	50	60	60	80	80	80	125	100
t, mm	3	4	3	4	4	5	5	5	6	8
b_1/b_2	**0.53**	0.625	**0.5**	**0.5**	**0.43**	**0.5**	**0.44**	**0.4**	**0.5**	0.33

Table 3. Dimensions of bent steel lipped channel profiles according to GOST 8282-83.

b_2, mm	b_1, mm	b_3, mm	t, mm	b_1/b_2	b_3/b_2
62	66	17.5	3	1.06	0.28
65	32	8	1	**0.49**	0.12
65	32	8	1.6	**0.49**	0.12
100	50	10	2	**0.50**	0.10
100	80	35	5	0.80	**0.35**
120	55	18	5	**0.46**	0.15
160	50	20	3	0.31	0.12
160	60	32	4	**0.37**	0.20
300	60	50	5	0.20	0.16
400	160	50	3	**0.40**	0.12
400	160	60	4	**0.40**	0.15
550	65	30	4	0.12	0.05
410	65	30	4	0.16	0.07

The solution obtained enables us to indicate the following range of optimal values of this ratio (for different values of P^* and real values of imperfections amplitudes): b_1/b_2= 0.425 –0.53. Those figures in the last line of Table 2 which fall in this interval are distinguished by bold shrift. We see that only for two standard profiles ratio b_1/ b_2 does not fall in the optimal range obtained for compressed columns.

In Table 3 there are presented dimensions of lipped channel cross-sections according to GOST 8282-83 and corresponding values of ratio b_1/ b_2 and b_3 / b_2. The ranges of optimal values of these ratios for compressed columns are: b_1/ b_2=0.37–0.52, b_3 /b_2=0.33–0.37. By bold shrift in Table 3 are also indicated those cases when the corresponding ratio falls in the optimal interval. As it is seen from Table 3, the most of standard values of widths ratios for lipped channels are far from the optimal values obtained for compressed columns.

REFERENCES

[1] Оптимальное проектирование конструкций. Библиографический указатель. Часть 1.АН СССР, СО, Институт гидродинамики. Новосибирск, 1975, с. 140-157.

[2] Życzkowski M., Gajewski A. Optimal structural design with stability constraints. In: COLLAPSE. The buckling of structures in theory and practice. Ed. J.M.T. Thompson, G.W. Hunt. Cambridge Univ. Press, 1983.

[3] Hutchinson J.W., Koiter W.T. Postbuckling theory. Appl. Mech. Reviews, 1970, **23**, N 12, 1353-1366.

[4] Tvergaard V. Buckling behaviour of plate and shell structures. In: Theoretical and Applied Mechanics (ed. W.T. Koiter). North-Holland Publ. Comp., 1977.

[5] Manevich A.I. Stability and optimal design of stiffened shells (in Russian). Kiev-Donetsk, 1979. 152 p.

[6] Маневич А.И., Ракша С.В. Оптимальные центрально сжатые стержни открытого профиля. В сб.: Theoretical Foundations in Civil Engineering .Proc. of the Polish-Ukrainian seminar, Warsaw, 2000 (to be published).

[7] Власов В.З. Тонкостенные упругие стержни. Изд-е 2-е. М.:ГИФМЛ. 1959, 568 с.

[8] Маневич А.И., Ракша С.В. Местное и связанное выпучивание тонкостенных стержней при сжатии и изгибе в двух плоскостях. В сб.: Theoretical Foundations in Civil Engineering.4,V.1, part 2 (Proc. Of the Polish-Ukrainian seminar, Warsaw, July 1996), Dnepropetrovsk, 1996, с. 270-275.

[9] Койтер В.Т. Устойчивость и закритическое поведение упругих систем. Механика. Период. сб. перев. иностр. статей, 1960, №5, 99-110.