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Instantaneous Reactive Power in Systems with Stochastic Electric Power Processes

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***Abstract* — The paper is dedicated to the estimation of instantaneous reactive power in a DC traction power system. Currently, the integral and frequency methods are mostly used to define reactive power and do not give high accuracy in the calculations of power balance or cannot describe the essence of physical processes. The problem is complicated in DC electric transport systems due to random impulse character of voltages and currents. As a comparison, the calculation of reactive power according to various approaches was performed for the DE1 electric locomotive. It is suggested that the instantaneous reactive power should be used for the analysis of electromagnetic exchange processes in DC power systems. The paper shows the definition and main formulas for defining the instantaneous reactive power taking into consideration the random character of voltage and current. Numerical calculations together with statistical analysis were performed for two variants of experimentally recorded voltages and currents: the first one – on the bus-bar of traction substation, which supplies the section with the Pendolino trains in Poland and the second one – on the current collector of the VL8 electric locomotive in Ukraine. They confirm the validity of the aforementioned method.**

Keywords — instantaneous power, reactive power, railway transport, power system, substation, locomotive.

I. Introduction

The estimation of consumed and recovered power is compulsory to solve the tasks of power supply, energy saving and improvement of energy quality in any electric power system. Nevertheless, in the modern electrical power engineering the conceptions and expressions for calculation of reactive power Q in electric circuits under non-sinusoidal conditions are the most ambiguous questions, which are discussed in researches as [1]-[14]. There are numerous approaches to defining Q, but the integral and frequency methods are the most commonly used [10]-[12]. The integral methods of reactive power estimation do not provide high accuracy in the calculations of power balance and it is impossible to evaluate the active power losses using them [10]-[12]. On the other hand, the frequency methods cannot describe the essence of physical processes in the electric circuits. Their expressions are formal and give the discrepant results in most cases [12].

II. Theoretical Background: Actual and Instantaneous Powers

The problem is considerably complicated in DC electric transport systems, because the voltages and currents are the stochastic processes regardless of the mode, in which electric rolling stock (ERS) operates on feeder zone [15]-[18]. The fact of ambiguity on the value and sign of Q is proved by numerical calculations performed on example of the DE1 electric locomotive according different suggestions in [1]-[3], [10]-[12] and results are shown in Table 1.

TABLE I. Comparison of Reactive Powers Calculated According to Different Methods

№	Active power P [MW]	Total power S [MVA]	Reactive power				
			Budeanu $Q_B\times10^4$ *[var]*	*Fryze* $Q_F\times10^6$ *[var]*	*differential* Q_d *[var]*	*integral* $Q_i\times10^4$ *[var]*	*generalized* $Q_g\times10^4$ *[var]*
1	2.574	3.285	-4.982	2.041	$1.034\cdot10^5$	-3.872	6.328
2	1.932	2.381	3.53	1.392	$-1.792\cdot10^4$	2.655	2.181
3	3.208	3.906	-4.679	2.229	$-1.438\cdot10^5$	-2.81	6.357
4	1.754	2.263	3.346	1.442	$-1.239\cdot10^4$	2.516	1.766
5	2.694	3.441	4.556	2.14	$7.98\cdot10^3$	3.284	1.619
6	1.636	2.067	1.146	1.265	$-2.825\cdot10^4$	1.301	1.917
7	3.059	3.801	2.684	2.257	$2.227\cdot10^5$	2.464	7.408
8	1.798	2.278	-2.03	1.399	$-5.918\cdot10^4$	-2.421	3.785
9	2.202	2.783	2.642	1.701	$2.395\cdot10^4$	2.193	2.292
10	1.709	1.982	1.531	1.005	$4.83\cdot10^4$	1.002	2.2
11	2.616	3.389	2.63	2.154	$-2.572\cdot10^3$	2.803	8.49
12	1.846	2.242	2.314	1.273	$1.45\cdot10^5$	4.924	2.673
13	2.233	2.791	-2.264	1.674	$-3.875\cdot10^4$	-1.927	2.732
14	1.829	2.129	-1.089	1.09	$-5.97\cdot10^4$	-1.066	2.523
15	1.302	1.531	2.8	1.047	$4.855\cdot10^4$	3.855	1.372

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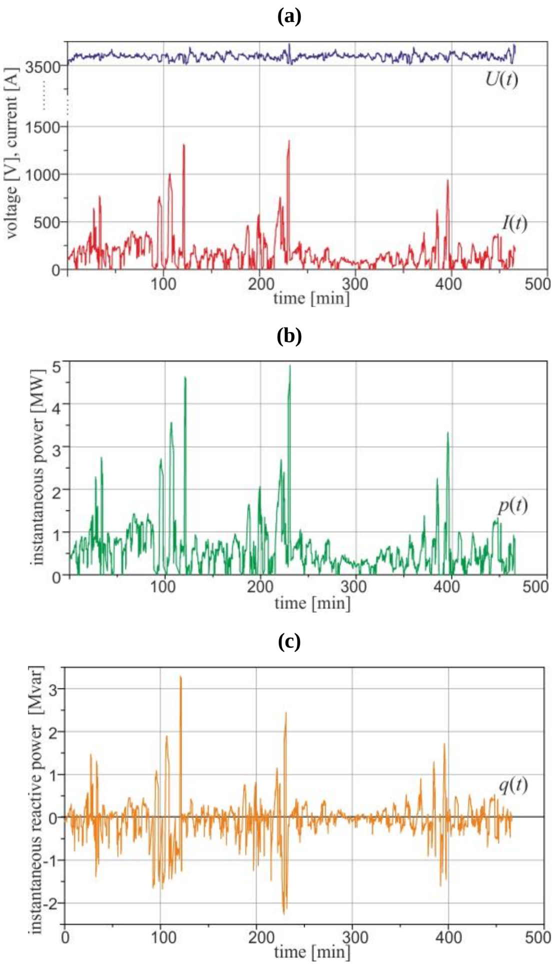

Fig. 1. The time records of the voltage $U(t)$ and current $I(t)$ (a), instantaneous power $p(t)$ (b) and instantaneous reactive power $q(t)$ (c) on DC bus-bar of a traction substation.

As a physical quantity, an instantaneous reactive power $q(t)$ has the most complete information about energy processes in the electric circuit with an arbitrary shape of voltage $U(t)$ and current $I(t)$. Thus, electrical engineers and experts frequently follow to the classical interpretation of powers for linear AC circuits. According to it, the sign of instantaneous power $p(t)$ determines the presence of exchange processes (that is the reactive power) in the circuit. If $p(t)>0$, the reactive power does not exist in the circuit. If $p(t)$ changes its sing to negative, then the exchange processes stimulate the existing reactive power. But we cannot agree with this mistaken opinion taking into consideration Figs. 1a and 1b, which show the time records of the voltage $U(t)$, current $I(t)$ and instantaneous power $p(t)=U(t)\cdot I(t)$ of a feeder line with Pendolino trains in Poland.

As can be seen, the instantaneous power $p(t)$ does no change its sign, thus the exchange processes between the traction substation and ERS are absent at the first view. However, such exchange processes exist as long as the power circuits of the substation, traction supply system and ERS have powerful non-linear reactive elements as the inductances of armature, main and field coils, inductive shunts, smoothing reactors of a substation, capacitances and inductances of passive filters and other.

Therefore, commonly used approaches are unsuitable to the circuits with non-sinusoidal voltages and currents. The exchange electromagnetic processes may be described using the instantaneous reactive power (IRP) in much the same way with the instantaneous values of current and voltage [12], [19].

III. Analytical Expression of the Instantaneous Reactive Power

Let us represent a traction substation as an active two-terminal circuit with the voltage $U(t)$ and an electric rolling stock as a passive two-terminal circuit, which are connected by traction power system (Fig. 2). The equivalent electric scheme of the passive two-terminal circuit consists of the parallel connection of active and reactive elements. The active element has conductance G with the active component $I_a(t)$ of the current and shows the active power losses in the load. The reactive element represents the consumption of non-active power component that is the stored and distorted powers. It consists of the susceptance B with the reactive component $I_r(t)$ of current. The element B does not consume energy, it stores and gives it back to the source. Hence, the total current is the sum of two components and equals $I(t)=I_a(t)+I_r(t)$.

Let us consider a decomposition of input voltage $U(t)$ and current $I(t)$ using the Gram-Schmidt orthogonalization process [20]. According to it, if $\{\bar{A}_1, \bar{A}_2, ..., \bar{A}_k, ..., \bar{A}_n\}$ is a finite or countable system of the linear independent vectors in the Hilbert space (basis), there the orthogonal system $\{\bar{B}_1, \bar{B}_2, ..., \bar{B}_k, ..., \bar{B}_n\}$ can exist and create the same linear variety with $\bar{A}_1 = \bar{B}_1$. The system $\{\bar{B}_1, \bar{B}_2, ..., \bar{B}_k, ..., \bar{B}_n\}$ may be defined by the recurrent formula as

$$\bar{B}_k = \bar{A}_k - \sum_{i=1}^{k-1} \frac{\bar{A}_k \bar{B}_i}{\bar{B}_i \bar{B}_i} \bar{B}_i , \tag{1}$$

where $k=2,3,\ldots,n$; $\bar{A}_k \bar{B}_i$ and $\bar{B}_i \bar{B}_i$ are the scalar products of the vectors.

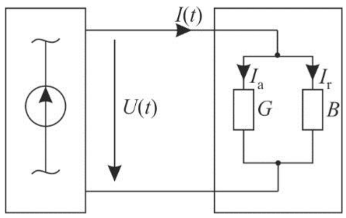

Fig. 2. Representation of a feeder zone of traction power system.

Let $\bar{A}_1 \equiv U$ and $\bar{A}_2 \equiv I$, then it is possible to write the general form according to (1) as

$$\bar{B}_2 = \bar{A}_1 - \frac{\bar{A}_2 \bar{B}_1}{\bar{B}_1 \bar{B}_1} \bar{B}_1 . \tag{2}$$

Then, the reactive component $\bar{I}_r$ of current is orthogonal to the voltage $\bar{U}$ and may be written as

$$\bar{I}_r = \bar{I} - \frac{\bar{I}\bar{U}}{\bar{U}\bar{U}} \bar{U} \tag{3}$$

or in the integral form at the time interval [0…τ] as

$$I_{\mathrm{r}}(t) = I(t) - \frac{\frac{1}{\tau}\int_0^{\tau} I(t)U(t)dt}{\frac{1}{\tau}\int_0^{\tau} U^2(t)dt} U(t) . \quad (4)$$

The numerator of (4) is an active power P, which is consumed by ERS. The denominator of (4) is a squared rms value of input voltage U_{rms}^2. Thus, (4) has to be written as

$$I_{\mathrm{r}}(t) = I(t) - \frac{P}{U_{\mathrm{rms}}^2} U(t) . \quad (5)$$

After multiplication of all summands of (5) to the input voltage $U(t)$, we get the expression of the IRP $q(t)$ as

$$q(t) = p(t) - \frac{P}{U_{\mathrm{rms}}^2} U^2(t) , \quad (6)$$

where the instantaneous power of the load (ERS) is equal to $p(t)=U(t)\cdot I(t)$. But the last expression of IRP is not directly applicable and its main coefficients cannot be calculated by using common expressions. Let us determine them.

As for the voltage $U(t)$ and current $I(t)$ are the stationary stochastic processes (Fig. 1a), thus the active power P and the root mean square value of voltage U_{rms} in (6) have to be determined using the terms of auto- and cross-correlation functions of the random functions theory [21]-[23].

As it is known in [24], the active power of the regenerative braking process is written as an arithmetical mean value of the instantaneous power $p(t)$ as

$$P = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T U(t)\cdot I(t)dt , \quad (7)$$

where T is a duration of separate phases of stochastic processes $U(t)$ and $I(t)$.

Consider the expression of the cross-correlation function $K_{UI}(\tau)$ of voltage and current. The function is non-random and is equal to the mathematical expectation of the multiplication of two centred random functions (processes) of $\overset{\circ}{U}(t)$ and $\overset{\circ}{I}(t)$ at the different and very close moments of time t and t+τ in the period [0, T] as

$$K_{UI}(\tau) = M\left[\overset{\circ}{U}(t)\overset{\circ}{I}(t+\tau)\right] = M\left\{[U(t)-m_U][I(t+\tau)-m_I]\right\} =$$
$$= \frac{1}{T}\int_0^T \left\{[U(t)-m_U][I(t+\tau)-m_I]\right\} dt = \frac{1}{T}\int_0^T U(t)I(t+\tau)dt -$$
$$-\frac{1}{T}\int_0^T U(t)m_I dt - \frac{1}{T}\int_0^T I(t+\tau)m_U dt + \frac{1}{T}\int_0^T m_U m_I dt , \quad (8)$$

where m_U and m_I are mathematical expectations of the stationary random functions of voltage $U(t)$ and current $I(t)$ respectively.

When τ is equal to zero, (8) is written as

$$K_{UI}(\tau=0) = \frac{1}{T}\int_0^T U(t)I(t)dt - m_U m_I . \quad (9)$$

From (7)-(9) the active power is defined by the cross-correlation function $K_{UI}(\tau)$ of voltage and current at the time shift τ=0 as

$$P = K_{UI}(\tau=0) + m_U m_I . \quad (10)$$

Let us write the expression of auto-correlation function $K_U(\tau)$ of voltage to determine U_{rms}. The function is non-random and is equal to the mathematical expectation of the multiplication of the centred random function $\overset{\circ}{U}(t)$ taken at the different and very close moments of time t and t+τ in the period [0, T] as

$$K_U(\tau) = M\left[\overset{\circ}{U}(t)\overset{\circ}{U}(t+\tau)\right] = M\left\{[U(t)-m_U][U(t+\tau)-m_U]\right\} =$$
$$= \frac{1}{T}\int_0^T \left\{[U(t)-m_U][U(t+\tau)-m_U]\right\} dt = \frac{1}{T}\int_0^T U(t)U(t+\tau)dt -$$
$$-\frac{1}{T}\int_0^T U(t)m_U dt - \frac{1}{T}\int_0^T U(t+\tau)m_U dt + m_U^2 , \quad (11)$$

where m_U is a mathematical expectation of the stationary random function of voltage $U(t)$.

Analogically, (8) and (9), expression (11) may be rewritten at the time τ=0 as

$$K_U(\tau=0) = \frac{1}{T}\int_0^T U^2(t)dt - m_U^2 = U_{\mathrm{rms}}^2 - m_U^2 \quad (12)$$

and the expression of rms voltage is

$$U_{\mathrm{rms}}^2 = K_U(\tau=0) + m_U^2 . \quad (13)$$

Finally, substitute (10) and (13) into (6) to get the general expression of instantaneous reactive power:

$$q(t) = U(t)I(t) - \frac{K_{UI}(\tau=0) + m_U m_I}{K_U(\tau=0) + m_U^2} \cdot U^2(t) \quad (14)$$

IV. Results of a Numerical Calculation

The results of a numerical calculation are shown in Figs. 1b, 1c, 3, 4b, 4c and 5-7. They are based on the experimentally recorded voltage $U(t)$ and current $I(t)$ on a bus-bar of a 3.3 kV DC traction substation supplied by 110 kV AC power line (in Poland) and on the current collector of the VL8 electric locomotive (in Ukraine). Calculations are performed using equations (10), (13) and (14). The statistical, probabilistic and correlation analyses of $U(t)$, $I(t)$, $p(t)$ and $q(t)$ with main coefficients are presented in Figs. 4-7 and some of them are used in the numerical calculations. As it ensues from these figures, the instantaneous power $p(t)$ has a positive sign, that is $p(t)>0$, in all devices of the electric traction system. This indicates the absence of reactive power according to the classical interpretation of powers for linear AC circuits. Therefore, the IRP should be estimated for the full description of exchange processes. The aforementioned reactive elements in the traction system and the changeable character of calculated $q(t)$ in Fig. 1c confirm the presence of electromagnetic exchanging processes between the traction substation (as a source) and ERS (as a consumer). These exchange processes are absent in the traction mode only in case, when $q(t)=0$.

The evaluation of the IRP $q(t)$ allows providing the optimal compensation of reactive power Q from the position of minimal power losses P in the supply system. For full compensation the IRP $q_c(t)$ of compensating device may correspond to the IRP $q(t)$ of DC system and be opposite in phase, that is $q_c(t)=-q(t)$.

Analysis of time records and histograms allows to write the next findings for a traction substation. The voltage varies in range from 3507 to 3725 V and distributes under the normal law with mathematical expectation of 3589.7 V. The traction current changes within wide limits from 100 to 1355 A, but its mathematical expectation equals only 189.3 A. The instantaneous power varies from 0.1 to 4.9 MW. This power and current do not follow the Gaussian law, because the most probably values have right-skewed distribution. The IRP changes from -2.26 to 3.29 Mvar and is not normally distributed. Its mathematical expectation is approximately

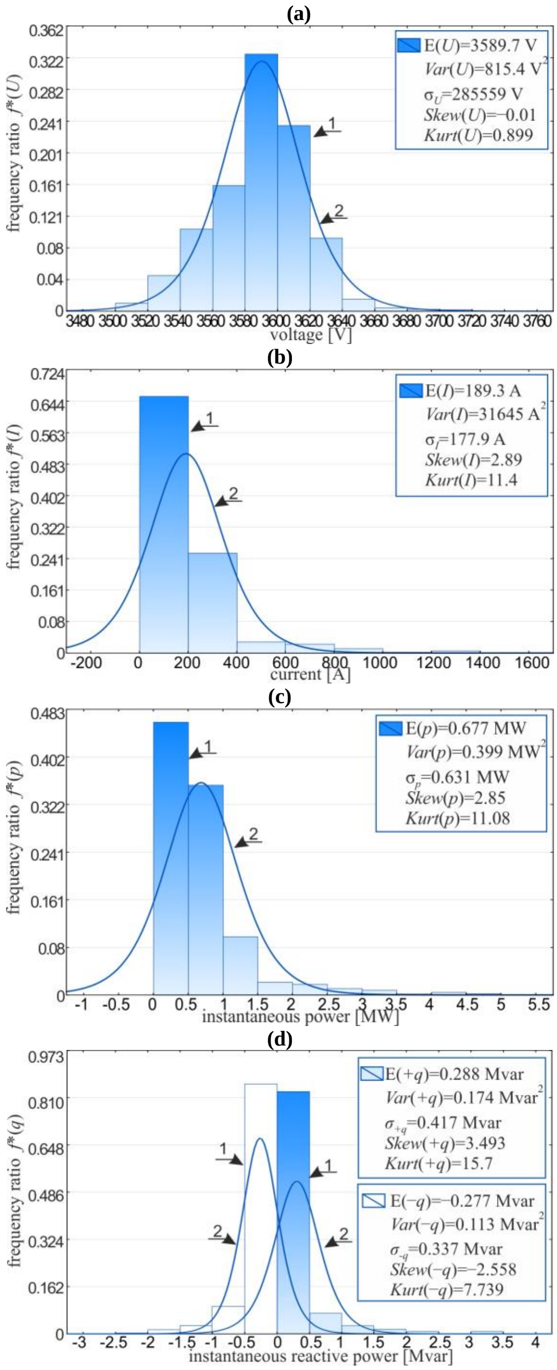

Fig. 3. Statistical (1) and theoretical (2) distributions of voltage (a), current (b), instantaneous power (c) and instantaneous reactive power (d) on a DC bus-bar of a traction substation.

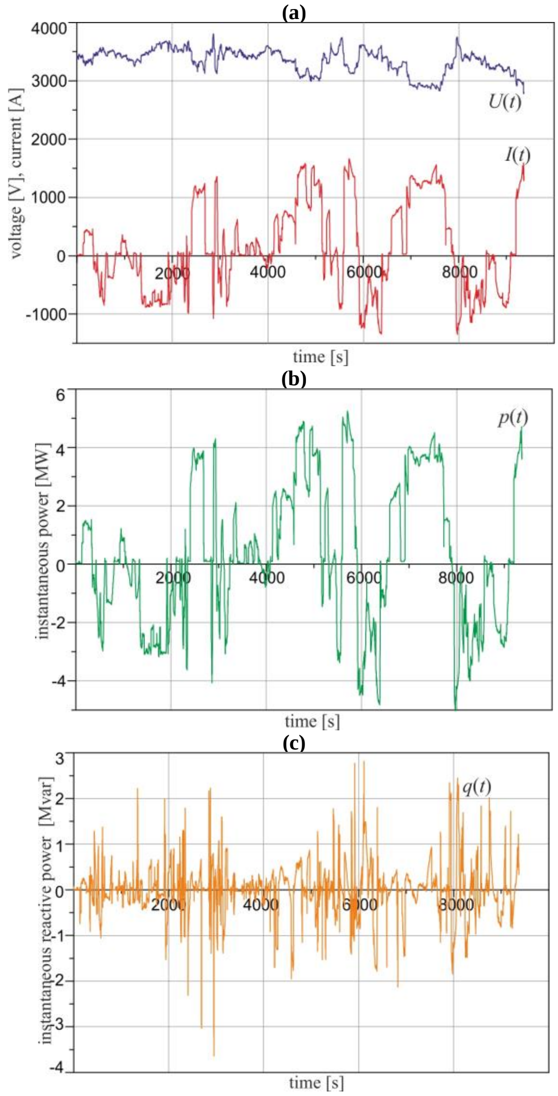

Fig. 4. The time records of the voltage $U(t)$ and current $I(t)$ (a), instantaneous power $p(t)$ (b) and instantaneous reactive power $q(t)$ (c) on the current collector of an electric locomotive.

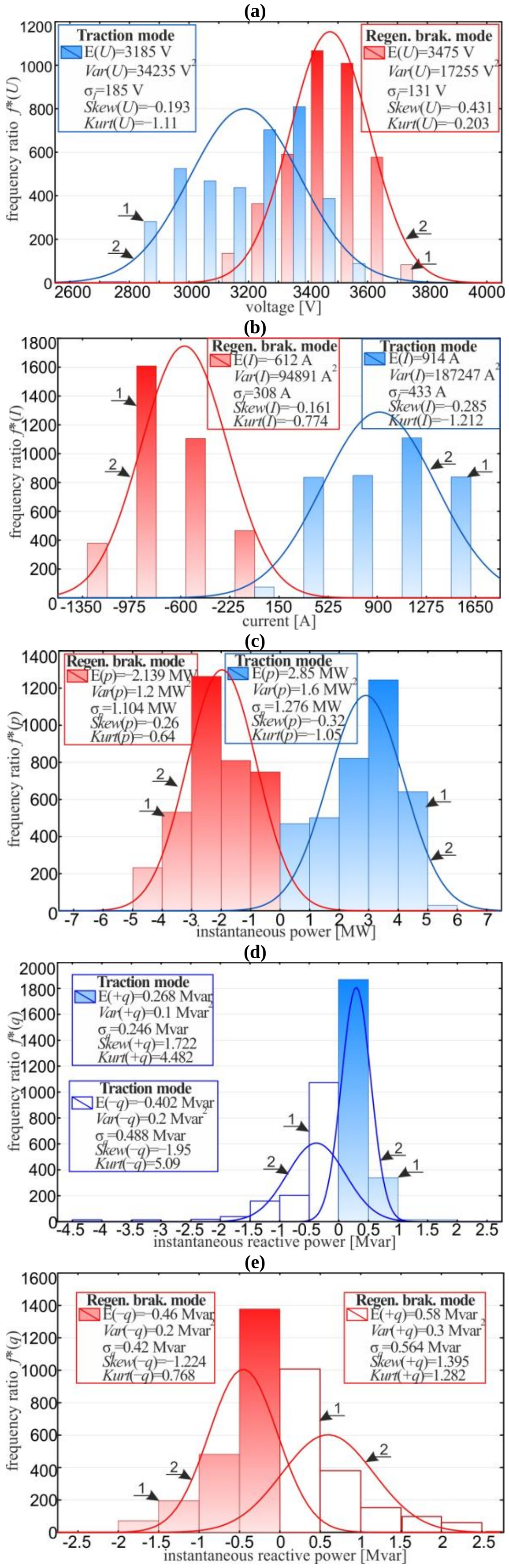

Fig. 5. Statistical (1) and theoretical (2) distributions of voltage (a), current (b), instantaneous power (c), instantaneous reactive power in traction (d) and regenerative braking (e) modes on the current collector of an electric locomotive.

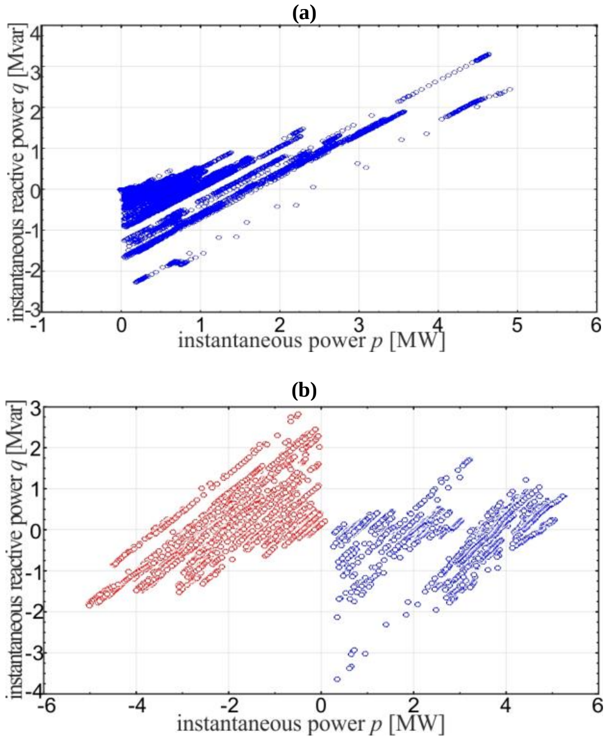

Fig. 6. The scatterplots of the instantaneous reactive power against the instantaneous power of a traction substation (a) and an electric locomotive (b).

equal to 0, but the variance is 0.22 Mvar^2, the standard deviation is 0.46 Mvar, skewness is 0.97 and kurtosis is 9.81. Fig. 3d shows the statistical characteristics for different signs of the IRP $q(t)$. Analogically, the main statistical coefficients are shown in Fig. 5 for the VL8 locomotive.

In practice, it is helpful and often necessary to estimate a probabilistic dependence, i.e. the correlation of the system of a few random values; in our case they are $q(t)$ and $p(t)$. The most clearly this dependence follows from a scatterplot in which the couples of investigated random quantities are shown by points in Fig. 6. It confirms the presence of positive correlation between the powers $q(t)$ and $p(t)$. This means that increase of the instantaneous power $p(t)$ influences the tendency of $q(t)$, which increases at an average. The scatterplot characterizes not only dependence between $q(t)$ and $p(t)$, but their probabilistic dispersion too. Such probabilistic dependence, that is correlation, is shown for a system of random quantities of $q(t)$, $U(t)$ and $I(t)$ by surface plot in Fig. 7. It confirms that $q(t)$ has a positive correlation with $I(t)$ and has weak negative correlation with $U(t)$. In this case the traction substation is supplied by 110 kV AC power line with low resistance, the Pendolino trains mostly consume lower currents so the output voltage of traction substation is kept in very close limits (Fig. 1a). In case of the VL8 electric locomotives these characteristics can sharply increase (Fig. 4a), because of high consumed powers during a long period of time and high voltage drops [16], which can vary in wide range from 3014 to 3989 V [15]. Such voltage variation can have bimodal or plateau (multimodal) distribution; thus, it significantly increases the reactive power and has negative influence on electric traction system and power quality indices. Therefore, it is very important to instantaneously estimate and compensate the reactive power in the traction power supply system, ipso facto, improve the λ and $tg\varphi$ coefficients too.

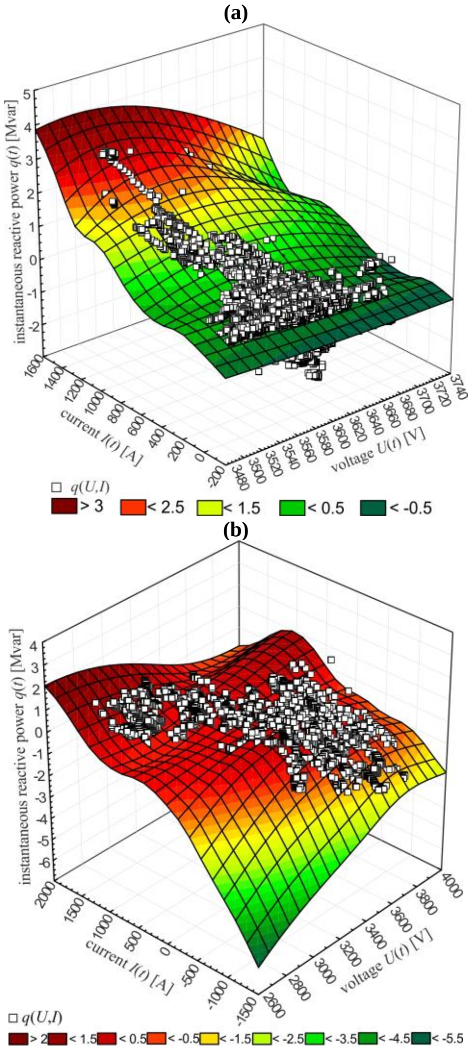

Fig. 7. The surface plot of the instantaneous reactive power against the voltage and current of a traction substation (a) and an electric locomotive (b).

V. Conclusions

Lots of research describes the estimation and ways for compensation of reactive power only in certain systems and conditions, but there are no any "universal" expression for calculation this power. The theories of instantaneous powers were developed and discussed by H. Akagi, A. Nabae, J. Willems, L. Czarnecki, L. Rossetto, F. Peng, J. Lai and for the non-linear circuits with deterministic values of voltages and currents. These theories are inapplicable and cannot be adapted for non-linear circuits of DC electric traction systems, because the voltages and currents in these systems are random processes and frequently have non-stationary character. On the other hand, traditional techniques of reactive power definition do not fully describe the exchange processes of electromagnetic energy between the traction substation and ERS, because there is a contradiction between the sign of instantaneous power and the fact of reactive power presence. Therefore, the paper gives a new expression of instantaneous reactive power, which takes into account the random character of the voltages and currents and is based on the decomposition into orthogonal components of the traction current according to the Gram-Schmidt process. The numerical, statistical, probabilistic and correlation analyses confirm the validity and applicability of the proposed method for estimation of energy exchange processes in DC electric traction systems.

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