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TWO-CRITERIA OPTIMIZATION OF THIN-WALLED BEAMS-COLUMNS UNDER COMPRESSION AND BENDING

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ABSTRACT

A two-criteria optimization problem for thin-walled members of open cross-section with one axis of symmetry (of the channel type) subjected to compression or /and bending in the symmetry plane under stability constraints is solved. The objective function vector components are the axial compressive force and the bending moment. Optimal configurations of thin-walled members for marginal cases of loading - axial compression or pure bending - are obtained, Pareto-optima are constructed and «compromise» optimal projects are derived as a function of a single leading nondimensional parameter of weight. The obtained optimal projects are universal, in distinction from the optimal projects for special loading cases, and applicable under arbitrary combination of compression and bending. Comparison of standard profiles (specified by the design codes) with the optimal projects enables us to indicate «bad» standard profiles which are far from optimal ones under any loads.

KEYWORDS

Structural Optimization, Multi-criteria minimization, Thin-walled Members, Pareto-optima

INTRODUCTION

The minimum weight design problems for thin-walled bars - beams under compression or bending with respect to their stability were investigated in a few works already in 60-70-s on the base of various theoretical models (see review by Zyczkowski & Gajewski (1983)) and were refined later (Yoshida & Maegawa (1979), Manevich & Raksha (2000) and others). Optimal profiles of compressed or bent thin-walled members which are found for the separate loads are essentially different As in exploitation they undergo, as a rule, the action of various loads, separately or in a combination, there arises the question: what configuration of the profile may be considered as optimal with account of such multifarious character of the loading? This problem of the choice of a «compromise» optimal project may be formulated and solved in the framework of the theory of multi-criteria optimization of structures (vector-valued optimization) which was intensively elaborated in last decades.

In the paper the two-criterion optimization problem for thin-walled members of open cross-section with one axis of symmetry (of channel type) subjected to compression and bending (in various combination), under stability constraints is solved. The objective function vector components are axial compressive force and bending moment. An efficient method of nonlinear programming is used for solving the optimization problem.

At the first stage optimal configurations of thin-walled members are derived for the marginal cases of loading - axial compression or pure bending (with using exact linear solutions for local buckling). Optimal dimensionless parameters are presented as a function of a single leading parameters - a nondimensional weight parameter. At the second stage the Pareto-optima are constructed in the plane (P- M_z) where P is the compressive force and M_z is the bending moment. Finally, at the third stage «compromise» optimal projects are derived with using a global criterion, comprising P and M_z. The obtained optimal members are universal, in distinction from the optimal projects for special loading cases, and applicable at arbitrary combination of compression and bending.

Comparison of standard profiles (specified by the design codes) with the obtained optimal projects enables us to indicate «bad» standard profiles which can not be optimal at any loads considered.

STATEMENT OF THE PROBLEM AND THE METHOD OF SOLUTION

Let us consider a thin-walled member of open cross-section shown in Fig.1, simply supported on its edges and subjected to compression and bending (in the symmetry plane) in various (a priori unknown) combinations. The length L , force P, the material properties (elasticity modulus E, Poisson's ratio ν) are considered given. The profile thickness t, widths of the web and flange b_1, b_2 are design variables (thickness of all elements is considered to be constant implying cold-formed members).

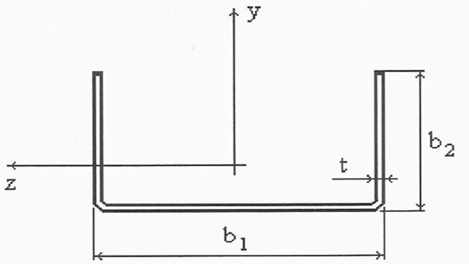


Figure1: Cross-sections of a bar and design variables

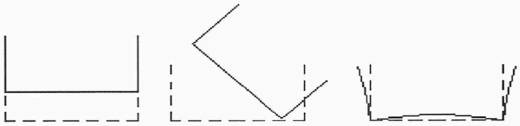


Figure 2: Buckling modes of a channel

The objective function depends upon formulation of the optimization problem We consider sequentially following optimization problems:

1. The optimization with a single criterion - maximum of the critical compressive force P or maximum of the bending moment M_z for given total cross-section area.
2. Constructing Pareto-optimal cross-sections in the framework of the two-criteria optimization problem.
3. Optimization with using a global criterion which comprises P and M_z.

The optimization problems are stated as nonlinear programming ones. In the single-criterion optimization for a compressed column the objective function is $P = max\ min\ (P_f, P_{tf}, P_l)$, where P_f, P_{tf}, P_l *are* critical forces for the flexural (euler) mode in the symmetry plane, for torsional-flexural mode and local mode respectively (see. Fig.2). For a beam under pure bending the objective function is $M_z = max\ min\ (M_{ztf},\ M_{zl})$, где $M_{ztf},\ M_{zl}$ are critical bending moments for torsional-flexural mode (out-of-plane buckling) and for local buckling.

The overall buckling stresses for the bar under compression or bending were calculated according to the theory of thin-walled bars (f.e., Vlasov, 1959). *The local buckling stresses* were calculated according to (Manevich & Raksha (1996)). All the plates that constitute the thin-walled bar-beam are divided on several strips for which the longitudinal stress may be considered as a constant, and the exact solution of the buckling problem is constructed by conjugation of the solutions for all the strips, with exact conjugation conditions at the contact lines (in distinction from the finite strips method which uses power approximations for displacements). For each the strip with local coordinates (x, y) b the solution of the differential equations of stability was assumed in the form $w = w(\eta)\sin m\pi\xi$, $\eta = y / L$, $\xi = x / L$ (x is the longitudinal coordinate). Boundary conditions on free edges and conjugation conditions on the contact lines result in a characteristic equation which determines the local critical stresses (after minimization in m). All details of the calculations are dropped here.

Because of closeness of the critical stresses for a cluster of short-wave local modes it is necessary to take into account many local modes. Note that dependencies of the local buckling stresses on the half-wave number m can have two local minima, one of them being determined by the web slenderness, another - by the flange slenderness. In view of these features the local buckling stresses were calculated for a wide range of half-waves numbers, i.e. instead of one constraint we consider a set of constraints (as a rule, for values m=2–25).

Strength constraints, as a rule, were not imposed, i.e. the assumption was assumed that the material is elastic and the yield limit is sufficiently high. As it will be seen from the obtained solution this assumption is valid in a range of relatively low values of the weight parameter G^* or the load parameter P^* (see below), for usually employed steels and alloys. For other G^* values the presented solutions determine idealized optimal configurations.

The nonlinear programming (NP) problem was solved by the linearized method of reduced gradient (Manevich (1979)).. This method realizing the idea of changing an independent variables set in a vicinity of the admissible domain boundary by means of the linear operations with the sensitivity matrices, effectively overcomes familiar difficulties arising at solving the NP-problem (in particular, connected with zigzag-type motion in a boundary vicinity). Our experience of many years employment of this algorithm has shown its high efficiency and reliability. As a rule, 10 - 20 iterations were required to achieve the optimum with relative error of order 10^{-3} .

For generality of analysis all the objective functions and constraints were formulated in dimensionless parameters of weight, load and stress:

$$G^* = \frac{A}{L^2}10^3,\ P^* = \frac{P}{L^2 \cdot E}10^6,\ M^* = \frac{M}{L^3 \cdot E}10^8,\ \sigma^* = \frac{\sigma}{E}10^3, \tag{3}$$

where A is the cross-section area (the scaling multipliers are introduced in order to deal with parameters of order of unity). The cross-section also was characterized by nondimensional geometric parameters b_2 / b_1, t / b_1.

RESULTS OF THE SOLUTION

All optimal *nondimensional* parameters are determined by specifying the single weight parameter G^* (all *dimensional* parameters are determined when additionally the length L is given). We considered the range of parameter G^* (0; 0.6) in which the assumption about elastic deformations of the material may be justified according to results of given solution (for usual materials). In this range several values of G^* have been chosen for which the optimization problem was solved.

Optimization with a single criterion. Columns under compression or bending.

Solution of the optimization problem for centrally compressed columns of channel cross-section based on the exact solution of the local buckling problem for the column as a plates assemblage was obtained in Manevich & Raksha (2000). Here we present only some results of the solution. For bars under pure bending earlier only approximate solutions have been obtained based on simplified expressions for the local buckling stresses. Here an exact solution for this case is presented.

The optimal columns under compression have equal critical stresses for the torsional-flexural mode and one of the local modes. For overall bending mode (in the symmetry plane, Fig.1) critical stresses are found to be higher by 15–25 %. The optimal beams under pure bending also have equal critical moments for the torsional- flexural mode (out-of-plane buckling) and one of the local modes.

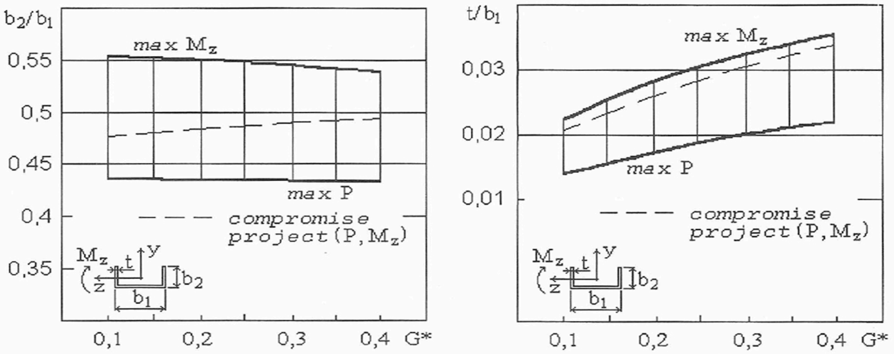
The calculations show that the optimal profiles under compression as well as under bending have nearly constant values of flange width to web width ratio in entire range of G^* considered. For compressed channels b_2 / b_1=0.42 –0.43; .in the case of pure bending b_2 / b_1= 0.53 - 0.56. The thickness parameter t/b_1 depends upon G^* as follows.

TABLE 1

PARAMETER t/b_1 FOR SINGLE-CRITERION OPTIMA

Parameter G*	0.1	0.2	0.3	0.4	0.5
t / b_1					
compression	0.0139	0.0174	0.0200	0.0219	0.0238
bending	0.0223	0.0283	0.0323	0.0357	0.0389

Parameters of cross-sections, optimal for compression (*max* P^*) and pure bending (*max* M_z^*) are presented in Figure 3 versus the cross-section total area parameter G^*. Optimal cross-sections turn out to be rather different for compression and for bending.



a b

Figure 3: Nondimensional geometrical parameters of optimal bars in single-criterion optimization and «compromise» projects.

The load parameters P^* and M_z^* versus the weight parameter G^* for both single-criterion optima are shown in Fig.4. The calculations show that for the single-criterion optima dependencies of the critical force P^* and critical moments in two planes M_z^* and M_y^* upon G^* can be approximated with high accuracy (the error less than 1%) by the power functions. These approximations are presented in Table 2.

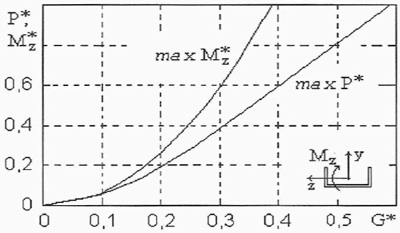


Figure 4: Load parameters P^* and M_z^* versus the weight parameter G^* for the single-criterion optima.

TABLE 2
APPROXIMATIONS OF CRITICAL FORCES AND MOMENTS VIA THE WEIGHT PARAMETER G* FOR SINGLE-CRITERION OPTIMA

Optimal bar under compression	Optimal bar under pure bending
$P^* = 2.794\,G^{*1.668}$	$P^* = 2.453\,G^{*1.676}$
$M_z^* = 3.805\,G^{*1.978}$	$M_z^* = 6.527\,G^{*1.995}$
$M_y^* = 16.334\,G^{*2.0025}$	$M_y^* = 14.184\,G^{*1.956}$

We see that the optimal channel under condition *max* P^* can carry out the moment M_z which approximately by 40 % less than the optimal channel for *max* M_z^*, and, correspondingly, the latter bar can carry out the compressive force by 12 % lesser than the optimal bar for *max* P^*.

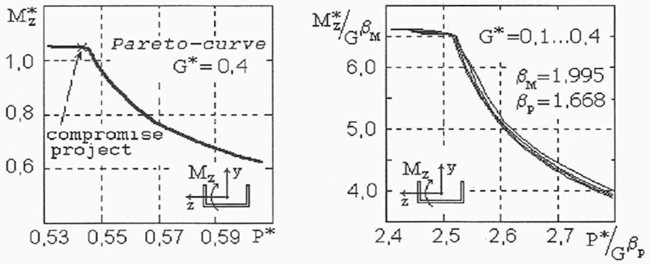
Two-criteria optimization

Pareto-optima

There were constructed the Pareto-optima for two criteria *max* P^*, *max* M_z^* for a set of values G^*. The Pareto-optimal projects were obtained by means of minimization in $\boldsymbol{P}^*$ with constraints on M_z^* which gradually increased M_z^* from the value obtained for the optimal bar under compression up to the value obtained for the optimal bar under bending.

In Figure 5, *a* the Pareto-optimal solutions are presented in the plane P^*-M_z^* for $G^*=0.4$. In order to obtain the Pareto-optima for different values of G^* in a compact form, they were recalculated in parameters $P^* / G^{*1.668}$, $M^* / G^{*1.995}$, with account of the power approximations which are presented in Table 2. Results are presented in Figure 5, *b*. The curves for various G^* almost coincide in these variables, and therefore these curves may be used approximately for any G^* values.

In Figure 6 the dashed lines present optimal projects obtained in the single-criterion optimization, and the solid lines show Pareto-optimal solutions for several G^* values (each point on this lines presents limit values of P^* and M_z^* for a certain optimal project obtained for a given G^*).



a b

Figure 5: Pareto-optimal projects for two criteria - $\boldsymbol{P}^*$, $\boldsymbol{M}_z^*$

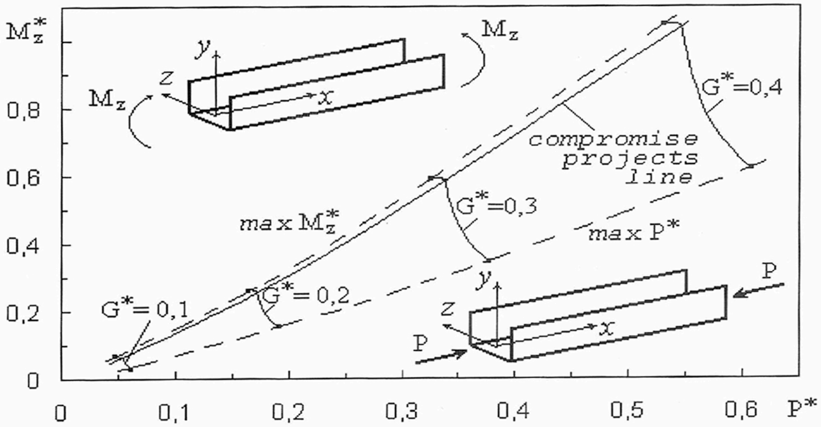


Figure 6: The single-criterion optimal projects (dashed lines), compromised projects (solid line) and the Pareto-optimal solutions for several G^* values.

«Compromise» projects

With the aim of finding a «compromise» project the problem of optimization was solved with the global criterion in the form:

$$F = P^*/P^*_{max} + M_z^*/M^*_{z\,max} \tag{5}$$

where P^*_{max} and $M^*_{z\,max}$ are the values of P^* and M^*_z, obtained in single-criterion optimizations in P^* and M^*_z . The projects obtained are presented in Figure 3 (dashed lines) and indicated by the cross in Figure 5. In Figure 7 cross-sections of the optimal bars for *max* P^*, *max* M^*_z and the «compromised» optimum are compared for given G^*=0.4 and length L=1 m . We see that the compromised optima are close to the single-criterion optima obtained for *max* M^*_z. They can carry out almost the same limit moment, but the limit force is less about 10% in comparison with the project obtained under condition of *max* P^* (for the same G^*).

It is worthwhile to compare the optimal values of nondimensional parameters with those for standard profiles . In Table 3 there are presented dimensions of channel cross-sections according to Russian standards GOST 8278-83 and corresponding values of nondimensional parameters b_2/b_1, t/b_1.

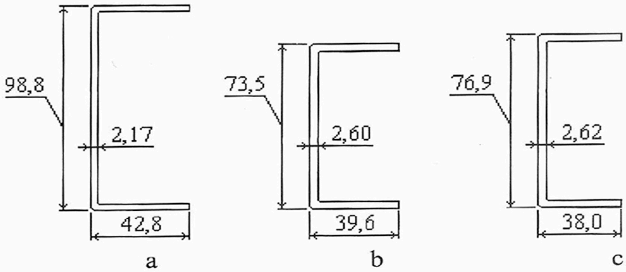


Figure 7: Optimal profiles obtained in single-criterion optimization (a, b), and «compromised» optimum (c) , for G^*=0.4 and length L=1 m.

TABLE 3
DIMENSIONS OF BENT STEEL CHANNEL PROFILES ACCORDING TO GOST 8278-83

b_1, mm	b_2, mm	t, mm	b_2/b_1	t/b_1
60	32	3	0.53	0.05
80	50	4	0.625	0.05
100	50	3	0.5	0.03
120	60	4	0.5	0.033
120	60	5	0.5	0.042
140	60	4	0.43	0.029
160	80	4	0.5	0.025
160	80	5	0.5	0.031
180	80	5	0.44	0.028
200	80	4	0.4	0.02
200	80	5	0.4	0.025
250	125	6	0.5	0.024
300	100	8	0.33	0.027

In Figure 8 domains of optimal nondimensional parameters in plane t/b_1, b_2/b_1 for single-criterion and two-criteria optima are shown. The dark points present values of these parameters for the standard profiles.

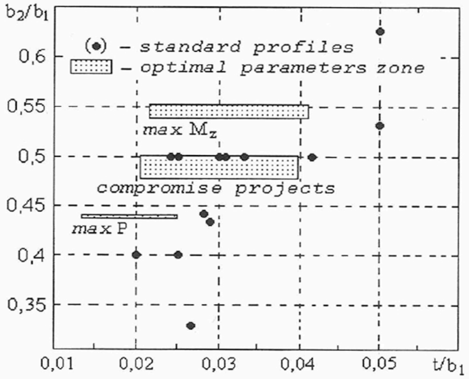


Figure 8: Domains of optimal nondimensional parameters and the standard profiles.

We see that the majority of the profiles fall in (or close to) the optimal zone for compromised projects, but some profiles are rather far from this zone. So we can indicate those profiles which are not optimal at any load considered (any combination of axial force and bending moment in the symmetry plane).

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