DYNAMIC DESIGN OF GROUND TRANSPORT WITH THE HELP OF COMPUTATIONAL EXPERIMENT

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Abstract

Objectives of ground transport (motor transport vehicle) have been considered. Mathematical model of nonlinear dynamics in spatial motion of asymmetric carriage in the form of Euler-Lagrange equations represented as symmetrical block structure in quaternion matrices has been developed. Kinematic equations and partition matrices of external action in which Rodrigues-Hamilton parameters have been applied describe quaternionic matrices.

Key-words

Dynamic design, computational experiment, mathematical model, Euler-Lagrange equations, quaternion matrices, quasivelocities, Rodrigues-Hamilton parameters.

1. **Introduction. Objectives of dynamic design** are to determine rational values of varied parameters of structural, layout, and construction schemes [1]. Geometrical dimensions and forms, such inertial characteristics as mass, centers of masses, inertia as well as stiffening, dissipative, power and aerodynamic characteristics are varied parameters taking into account effect of surface as screen and its contact interaction with elastic tire [2].

Process of dynamic design solves problems of controllability, stabilization, dynamic load and others [3].

The problems are formulated as nonlinear problems of car performance within spatial motion in terms of kinematic, contact, dynamic, and aerodynamic actions. In this context nonlinear problems of dynamic design can be solved with the help of computational experiment [4,5].

2**. Dynamic scheme**. Computational experiment is carried out according to mathematical models being adequate to problem formulation. Mathematical models depend on dynamic schemes being adequate to problem formulation. Carrying solid body (vehicle body and carried solid bodies) is considered as basic dynamic scheme of motor vehicle. Diversity of dynamic schemes depends on a character of carried bodies relative to carrying ones (internal relations), dynamic interference with external environment for example, wind, road surface and local varieties (kinematic relations). Mathematical model of road surface is developed in the form of ruled surface (Shukhov surface) which guide is determined with the help of spiral line corresponding to program trajectory of a vehicle movement [6]. Dynamic effect on a vehicle is determined with the help of gravity force distributed in terms of volume, inertial forces (centrifugal, gyroscopic, Coriolis, and tangential) aerodynamic forces as well as moments being a result of distributed surface pressure and friction forces specified by a vehicle airflowing based upon wind blasts, turbulent boundary layer between bottom of a vehicle and screening road surface, contact dynamic forces and moment being results of distributed surface forces of pressure and friction stipulated by interaction between elastic tire and road surface within destination, variable field for leading wheels and follower ones.

3. **Coordinate systems.** Vector of force and moment, spatial vector of translation, and vector of linear velocity and angular velocity are represented by their components within following Cartesian coordinate systems:

- Earth-based coordinate system (accepted as inertial one) where direction of one of axes is collinear to a direction of plumb line characterizing gravity force and centrifugal force resulting from Earth's rotation;

- Bound coordinate system (basic trihedron of a vehicle), which pole and axes orientation depend on design characteristics or other specifications (comfortable mounting, balance between geometry and weight etc.);

- Aerodynamic axes are connected with geometric axes of a vehicle's external shape symmetry; reduction centre of aerodynamic forces and moments is assumed as a pole;

- Trihedron of contact forces and moments is guided by a normal to road surface and symmetry axes of elastic tire and road surface plane contact area;

- Coordinate system and pole connected with axes and symmetry center of a wheel determine orientation and displacement of a wheel as regard to datum axes of a vehicle as well as other coordinate systems associated with carried bodies.

Spatial rotations of principal body and carried ones are determined by quaternionic matrices which components are Rodrigues-Hamilton parameters expressed by physical angles (for example, Euler-Krylov's) [7].

4. Mathematical model of nonlinear dynamics of a vehicle while 3D turning and displacing.

4.1. **Dynamic equations by Euler-Lagrange.** Mathematical model of principal body nonlinear dynamics within spatial motion is based on differential equations in the form of Euler-Lagrange ones represented with the help of quaternionic matrices [8]. Principal body angular velocity vector projection and projection of vector of linear velocity of body pole on its basic trihedron are taken as variables of integration-quasivelocities. Dynamic equations involve inertia matrix which components are axial and centrifugal inertial moments reduced to principal body weight and quaternionic matrices which components are coordinates of mass center as well as quaternionic matrices of quasivelocities.

4.2. **Kinematic equations in Rodrigues-Hamilton parameters.** Kinematic equations determine specified quasivelocities through Rodrigues-Hamilton parameters and their time derivatives and components of linear velocities of a pole as for inertial reference system. Principal body turning and displacing within the Earth's coordinate system are determined by means of reverse transformation and numerical integration of kinematic dependences where quasivelocities are assumed as those identified as a result of numerical integration of dynamic equations.

4.3. **Mathematical model structure.** The matrix model of nonlinear dynamics of a vehicle as asymmetrical rigid solid body within spatial motion contains closed system of standard nonlinear heterogeneous differential equations of 1^{st} order relative to six quasivelocities; three pole coordinates; four Rodrigues-Hamilton parameters; and resulting spatial vehicle turning connected by means of identical condition of standardization. Matrix dynamic equations are represented by eight differential equations of 1^{st} order of which 1^{st} and 5^{th} are trivial; 2^{nd}, 3^{rd}, and 4^{th} are equations of moments; and 6^{th}, 7^{th}, and 8^{th} are equations of forces. Matrix kinematic equations are represented by eight differential equations of 1^{st} order of which one to four are linear equations relative to Rodrigues-Hamilton parameters and their solutions are connected with known integral in the form of identical condition and five to eight equations are nonlinear as for basic trihedron pole velocity containing the first of the equations as trivial one. Neglecting trivial equations demonstrate expanded record in a matrix form by Euler operator describing inertial forces and moments including centrifugal, gyroscopic, and Coriolis ones:

-inertial matrix block:

$$
\begin{Vmatrix}
\dfrac{\begin{Vmatrix} \bar{I}_{11}^{y} & -\bar{I}_{12}^{y} & -\bar{I}_{13}^{y} \\ -\bar{I}_{21}^{y} & \bar{I}_{22}^{y} & -\bar{I}_{23}^{y} \\ -\bar{I}_{31}^{y} & -\bar{I}_{32}^{y} & \bar{I}_{33}^{y} \end{Vmatrix}}{\begin{Vmatrix} 0 & y_3^c & -y_2^c \\ -y_3^c & 0 & y_1^c \\ y_2^c & -y_1^c & 0 \end{Vmatrix}} &
\dfrac{\begin{Vmatrix} 0 & -y_3^c & y_2^c \\ y_3^c & 0 & -y_1^c \\ -y_2^c & y_1^c & 0 \end{Vmatrix}}{\begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}}
\end{Vmatrix}
\bullet
\begin{Vmatrix} \dot{\omega}_1^y \\ \dot{\omega}_2^y \\ \dot{\omega}_3^y \\ \hline \dot{V}_{10}^y \\ \dot{V}_{20}^y \\ \dot{V}_{30}^y \end{Vmatrix}
+
\begin{Vmatrix}
\dfrac{\begin{Vmatrix} 0 & -\omega_3^y & \omega_2^y \\ \omega_3^y & 0 & -\omega_1^y \\ -\omega_2^y & \omega_1^y & 0 \end{Vmatrix}}{\begin{Vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{Vmatrix}} &
\dfrac{\begin{Vmatrix} 0 & -V_{30}^y & V_{20}^y \\ V_{30}^y & 0 & -V_{10}^y \\ -V_{20}^y & V_{10}^y & 0 \end{Vmatrix}}{\begin{Vmatrix} 0 & -\omega_3^y & \omega_2^y \\ \omega_3^y & 0 & -\omega_1^y \\ -\omega_2^y & \omega_1^y & 0 \end{Vmatrix}}
\end{Vmatrix}
\bullet
$$

$$
\bullet
\begin{Vmatrix}
\dfrac{\begin{Vmatrix} \bar{I}_{11}^{y} & -\bar{I}_{12}^{y} & -\bar{I}_{13}^{y} \\ -\bar{I}_{21}^{y} & \bar{I}_{22}^{y} & -\bar{I}_{23}^{y} \\ -\bar{I}_{31}^{y} & -\bar{I}_{32}^{y} & \bar{I}_{33}^{y} \end{Vmatrix}}{\begin{Vmatrix} 0 & y_3^c & -y_2^c \\ -y_3^c & 0 & y_1^c \\ y_2^c & -y_1^c & 0 \end{Vmatrix}} &
\dfrac{\begin{Vmatrix} 0 & -y_3^c & y_2^c \\ y_3^c & 0 & -y_1^c \\ -y_2^c & y_1^c & 0 \end{Vmatrix}}{\begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}}
\end{Vmatrix}
\bullet
\begin{Vmatrix} \omega_1^y \\ \omega_2^y \\ \omega_3^y \\ \hline V_{10}^y \\ V_{20}^y \\ V_{30}^y \end{Vmatrix}.
$$

Kinematic equations in an expanded matrix form are:

- spatial turning (orientation):

$$
\begin{Vmatrix} \dot{r}_0(t) \\ \dot{r}_1(t) \\ \dot{r}_2(t) \\ \dot{r}_3(t) \end{Vmatrix}
= \frac{1}{2}
\begin{Vmatrix}
0 & -\omega_{1y}(t) & -\omega_{2y}(t) & -\omega_{3y}(t) \\
\omega_{1y}(t) & 0 & \omega_{3y}(t) & -\omega_{2y}(t) \\
\omega_{2y}(t) & -\omega_{3y}(t) & 0 & \omega_{1y}(t) \\
\omega_{3y}(t) & \omega_{2y}(t) & -\omega_{1y}(t) & 0
\end{Vmatrix}
\cdot
\begin{Vmatrix} r_0(t) \\ r_1(t) \\ r_2(t) \\ r_3(t) \end{Vmatrix}
$$

- spatial displacement:

$$\begin{Vmatrix} \dot{z}_{10}(t) \\ \dot{z}_{20}(t) \\ \dot{z}_{30}(t) \end{Vmatrix} = \begin{Vmatrix} r_0^2+r_1^2-r_2^2-r_3^2 & 2(r_1r_2-r_0r_3) & 2(r_0r_2+r_1r_3) \\ 2(r_0r_3+r_1r_2) & r_0^2-r_1^2+r_2^2-r_3^2 & 2(r_2r_3-r_0r_1) \\ 2(r_1r_3-r_0r_2) & 2(r_0r_1+r_2r_3) & r_0^2-r_1^2-r_2^2+r_3^2 \end{Vmatrix} \cdot \begin{Vmatrix} V_{10}^y \\ V_{20}^y \\ V_{30}^y \end{Vmatrix}$$

Note that Rodrigues-Hamilton parameters are convenient to be explained, for example in Euler-Krylov angles with the help of following simple dependences:

- explanation of Rodrigues-Hamilton parameters in Euler-Krylov angles:

$$2(r_1r_3+r_0r_2)=\sin\beta$$

$$2(r_2r_3-r_0r_1)=-\sin\alpha\ \cos\beta$$

$$r_0^2-r_1^2-r_2^2+r_3^2=\cos\alpha\ \ \cos\beta$$

$$2(r_1r_2-r_0r_3)=-\sin\gamma\ \cos\beta$$

$$r_0^2+r_1^2-r_3^2-r_2^2=\cos\gamma\ \cos\beta$$

4.4**. External effects.** Right part of dynamic equations is a sum of block matrices which structure is determined by a nature of effects on a principal body. Module of considered force and moment vectors are reduces to a principal body weight; for inertial forces (gravity force and weight) it is determined by a value of local acceleration of free fall; in terms of aerodynamic forces and moments it depends on dynamic pressure-weight ratio; for contact forces and moments it depends on the ratio between specific load within contact area and weight. A structure of block matrices of external effects is conservative containing square matrices of 4th order – zero, identity, quaternion – compiled according to the coordinates of application points of the forces under consideration: they are principal body mass centre coordinates within reference trihedron; they are coordinates of a reference point within reference trihedron; and they are coordinates of contact area symmetry center within reference trihedron. Directions of the forces and moments are described in block matrices with the help of quaternion matrices compiled on Rodrigues-Hamilton parameters determining reference trihedron orientation within inertial access for gravity force; orientation of geometrical axes of vehicle symmetry within reference trihedron for aerodynamic forces and moments; and orientation of contact force and moment trihedron within inertial axes and then within reference trihedron axes.

4.4.1. **Inertial forces.** Volumetric gravitational forces and inertial ones depending upon Earth rotation are reduced to resulting force applied within the center of vehicle mass. Unit director vector of the force is collinear to the lead line within reference system connected with principal body. Value of the inertial force is determined as a product of vehicle mass by a module of free fall resulting acceleration within conserved point of the Earth's surface. Expanded record of gravity force matrix block is:

$$-g\cdot\left\|\frac{\left\|\begin{matrix}0 & -y_3^c & y_2^c\\ y_3^c & 0 & -y_1^c\\ -y_2^c & y_1^c & 0\end{matrix}\right\|}{\left\|\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right\|}\right\|\left\|\begin{matrix}r_0^2+r_1^2-r_2^2-r_3^2\\ 2\left(r_1\ r_2-r_0\ r_3\right)\\ 2\left(r_0\ r_2-r_1\ r_3\right)\end{matrix}\right\|.$$

4.4.2. **Aerodynamic forces.** Surface forces of pressure and friction of ram air are reduced to resulting aerodynamic forces applied within related aerodynamic axes through dynamic pressure , midsection area (or plan area), coefficient of longitudinal, standard, and cross-sectional forces or transformed to specific reference point with the help of typical linear dimension and coefficients of drifting, wandering, pitching (differenting, galloping), and rolling. Expanded record of aerodynamic force matrix block is:

$$\frac{qS}{m}\cdot\left\|\frac{\left\|\begin{matrix}0 & -y_3^d & y_2^d\\ y_3^d & 0 & -y_1^d\\ -y_2^d & y_1^d & 0\end{matrix}\right\|}{\left\|\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}\right\|}\right\|\left\|\begin{matrix}r_{0d}^2+r_{1d}^2-r_{2d}^2-r_{3d}^2 & 2\left(r_{1d}r_{2d}-r_{0d}r_{3d}\right) & 2\left(r_{0d}r_{2d}+r_{1d}r_{3d}\right)\\ 2\left(r_{0d}r_{3d}+r_{1d}r_{2d}\right) & r_{0d}^2-r_{1d}^2+r_{2d}^2-r_{3d}^2 & 2\left(r_{2d}r_{3d}+r_{0d}r_{1d}\right)\\ 2\left(r_{1d}r_{3d}-r_{0d}r_{2d}\right) & 2\left(r_{0d}r_{1d}-r_{2d}r_{3d}\right) & r_{0d}^2-r_{1d}^2-r_{2d}^2+r_{3d}^2\end{matrix}\right\|\left\|\begin{matrix}c_{1d}\\ c_{2d}\\ c_{3d}\end{matrix}\right\|$$

where c_{1d}, c_{2d} , c_{3d} are coefficients of standard, longitudinal, and cross-sectional forces;

r_{0d}, r_{1d}, r_{2d}, r_{3d} are Rodrigues-Hamilton parameters determining aerodynamic axes orientation in terms of related ones;

y_1^d, y_2^d, y_3^d – are coordinates of aerodynamic forces reference point within related axes;

q is dynamic pressure;

S is midsection area;

m is mass.

Expanded record of aerodynamic moments matrix block is:

$$\frac{qSL}{m}\cdot\left\|\begin{array}{c}\left\|\begin{array}{ccc}1&0&0\\0&1&0\\0&0&1\end{array}\right\|\\ \hline \left\|\begin{array}{ccc}0&0&0\\0&0&0\\0&0&0\end{array}\right\|\end{array}\right\|\left\|\begin{array}{ccc}r_{0d}^2+r_{1d}^2-r_{2d}^2-r_{3d}^2 & 2\left(r_{1d}r_{2d}-r_{0d}r_{3d}\right) & 2\left(r_{0d}r_{2d}+r_{1d}r_{3d}\right)\\ 2\left(r_{0d}r_{3d}+r_{1d}r_{2d}\right) & r_{0d}^2-r_{1d}^2+r_{2d}^2-r_{3d}^2 & 2\left(r_{2d}r_{3d}+r_{0d}r_{1d}\right)\\ 2\left(r_{1d}r_{3d}-r_{0d}r_{2d}\right) & 2\left(r_{0d}r_{1d}-r_{2d}r_{3d}\right) & r_{0d}^2-r_{1d}^2-r_{2d}^2+r_{3d}^2\end{array}\right\|\left\|\begin{array}{c}m_{1d}\\m_{2d}\\m_{3d}\end{array}\right\|$$

where m_{1d}, m_{2d}, m_{3d} are coefficients of moments of drifting, rolling, and pitching;

L is specific linear dimension.

4.4.3. **Contact forces.** Surface forces of pressure and friction within contact area of wheel and road surface are reduced to resulting contact forces and moments related to characteristics point of contact area (for example, geometrical center of contact area symmetry) with coordinate axes coinciding with symmetry axes of plane area of wheel-road surface contact (tangential plane to road surface in the center of contact area symmetry) depending upon specific load (being normal to road surface of dynamic load distributed over variable zone of contact area), coefficients of wheel contact moments in terms of turning, sloping, rotating, and coefficients of longitudinal and cross-sectional contact forces.

Note. It should be emphasized that coefficients of aerodynamic forces and moments depending upon external shape of a vehicle and involving stream turbulence, screening effect as well as coefficients of contact forces and moments depending upon characteristics of elastic tire and road surface are identified with the help of experimental techniques due to complexity of physical processes in terms of vehicle airflow in the neighbourhood of a screen (road surface) and in terms of wheel-road contact interaction.

5. **Initial conditions.**

5.1. **Physical conditions.**

$z_0(0)$ is a position of hybrid vehicle pole within inertial coordinate system

$$\left[z_{10}(0), z_{20}(0),\ z_{30}(0)\right]$$

$\dot{z}_0(0)$ is components of linear velocity of hybrid vehicle pole within inertial coordinate system

$$\left[\dot{z}_{10}(0), \dot{z}_{20}(0),\ \dot{z}_{30}(0)\right]$$

$\alpha(0),\beta(0),\gamma(0)$ are Euler-Krylov's angles determining hybrid vehicle orientation within inertial space;

$\dot{\alpha}(0),\dot{\beta}(0),\dot{\gamma}(0)$ are time derivatives (angular velocities) of Euler-Krylov's angles determining hybrid vehicle orientation within inertial space.

5.2. **Initial conditions for introduced variables.**

Rodrigues-Hamilton parameters determining hybrid vehicle orientation within inertial space are:

$$r_0(0)=\cos\frac{\gamma(0)}{2}\cos\frac{\beta(0)}{2}\cos\frac{\alpha(0)}{2}-\sin\frac{\gamma(0)}{2}\sin\frac{\beta(0)}{2}\sin\frac{\alpha(0)}{2},$$

$$r_1(0)=\cos\frac{\gamma(0)}{2}\cos\frac{\beta(0)}{2}\sin\frac{\alpha(0)}{2}+\sin\frac{\gamma(0)}{2}\sin\frac{\beta(0)}{2}\cos\frac{\alpha(0)}{2},$$

$$r_2(0)=\cos\frac{\gamma(0)}{2}\sin\frac{\beta(0)}{2}\cos\frac{\alpha(0)}{2}-\sin\frac{\gamma(0)}{2}\cos\frac{\beta(0)}{2}\sin\frac{\alpha(0)}{2},$$

$$r_3(0)=\sin\frac{\gamma(0)}{2}\cos\frac{\beta(0)}{2}\cos\frac{\alpha(0)}{2}+\cos\frac{\gamma(0)}{2}\sin\frac{\beta(0)}{2}\sin\frac{\alpha(0)}{2}.$$

Components of hybrid vehicle angular velocity within related coordinate system are:

$$\omega_{1y}(0)=\dot{\alpha}(0)\cos\beta(0)\cos\gamma(0)+\dot{\beta}(0)\sin\gamma(0),$$

$$\omega_{2y}(0)=-\dot{\alpha}(0)\cos\beta(0)\sin\gamma(0)+\dot{\beta}(0)\cos\gamma(0),$$

$$\omega_{3y}(0)=\dot{\alpha}(0)\sin\beta(0)+\dot{\gamma}(0).$$

Components of hybrid vehicle pole linear velocity within related coordinate system are :

$$V_{1oy}(0)=\dot{z}_{10}(0)\cos\beta(0)\cos\gamma(0)+\dot{z}_{20}(0)\left[\sin\alpha(0)\sin\beta(0)\cos\gamma(0)+\cos\alpha(0)\sin\gamma(0)\right]+$$
$$+\dot{z}_{30}(0)\left[\sin\alpha(0)\sin\gamma(0)-\cos\alpha(0)\sin\beta(0)\cos\gamma(0)\right],$$
$$V_{2oy}(0)=\dot{z}_{10}(0)\left[-\cos\beta(0)\sin\gamma(0)\right]+\dot{z}_{20}(0)\left[\cos\alpha(0)\cos\gamma(0)-\sin\alpha(0)\sin\beta(0)\sin\gamma(0)\right]+$$
$$+\dot{z}_{30}(0)\left[\cos\alpha(0)\sin\beta(0)\sin\gamma(0)+\sin\alpha(0)\cos\gamma(0)\right],$$
$$V_{3oy}(0)=\dot{z}_{10}(0)\sin\beta(0)+\dot{z}_{20}(0)\left[-\sin\alpha(0)\cos\beta(0)\right]+\dot{z}_{30}(0)\cos\alpha(0)\cos\beta(0).$$

Conclusions.

Properties of mathematical model. The matrix differential equations of 1st order are reduced directly to Cauchy form for which effective numerical integration techniques have been developed. Excess trivial equations – 1st, 5th, 13th, as well as identical condition of standardization are required to control (verify) accuracy of numerical integration. Following invariants also verify the objectives:

$$\omega_{1y}^2+\omega_{2y}^2+\omega_{3y}^2=4\left(\dot r_0^2+\dot r_1^2+\dot r_2^2+r_3^2\right)$$

$$V_{10y}^2+V_{20y}^2+V_{30y}^2=\dot z_{10}^2+\dot z_{20}^2+\dot z_{30}^2$$

$$r_0\dot r_0+r_1\dot r_1+r_2\dot r_2+r_3\dot r_3=0$$

$$r_0^2+r_1^2+r_2^2+r_3^2=1.$$

Nonavailability of trigonometric functions in dynamic and kinematic equations allows excluding mathematical features typical for the functions in the process of numerical integration. That favours PC functioning reducing calculation period. The structure of proposed matrix equations of vehicle motion and symmetry properties of matrices being applied provide clearness of mathematical model, programmability, and efficient use of mathematical PC support improving mental activities on the whole.

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