# TWO-CRITERIA OPTIMIZATION OF H-SECTION BARS UNDER BENDING AND COMPRESSION

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The two-criteria optimum design problem for thin-walled members with two axes of symmetry (of H-type) subjected to compression and bending is solved. Components of the objective function vector are the compressive force and the bending moment in the web plane. Pareto-optima and "compromise" optimal projects are obtained and compared with the standard profiles.

## 1. INTRODUCTION

In papers [1, 2] the optimum design problem for thin-walled members subjected to compression and/or bending was considered in the framework of the theory of multi-criteria optimization of structures (vector-valued optimization). The objective function vector components were the axial compressive force and the bending moments in two planes. Such an approach enables us to obtain optimal parameters with account of multi-various character of the loading of thin-walled members, undergoing the action of various loads, separately or in a combination, and, in particular, to obtain "compromise" optimal projects.

In [1, 2] we considered only beams with channel- and lipped channel cross sections. In this paper the consideration is expanded to H-section profiles (as typical profiles with two axes of symmetry). Only requirements of stability (overall and local) are taken into account (the material is suggested to be elastic).

## 2. STATEMENT OF THE PROBLEM AND METHOD OF SOLUTION

We seek for optimal parameters of a simply supported H-cross-section beam under action of compressive force $P$ and/or bending moment $M$ in the web plane (Fig. 1). Two equivalent (dual) approaches for the optimization problem are possible: 1) to minimize the weight of a structure for given loading; 2) to maximize the load for given weight (volume, or cross-section area). As we consider the beam for various loads, it is preferable to employ here the second approach. So the cross-section area $A$ and the length of the beam $L$ are considered given, as well as the material properties (Young's modulus $E$, Poisson's ratio $\nu$). Design variables are dimensions of the cross-section elements – width and thickness of the web $b_w$, $t_w$ and flanges $b_f$, $t_f$.

The optimization problem is stated as a nonlinear programming one. ***Constraints of the problem*** are buckling constraints taking into account the overall buckling (flexural and torsional) and local buckling. Critical forces for the *overall flexural and torsional buckling* $P_f$, $P_t$ and critical moment for flexural-torsional buckling $M_{ft}$ (lateral buckling), as well as their critical combinations, were calculated according to the linear theory of thin-walled bars (Vlasov, [3]).

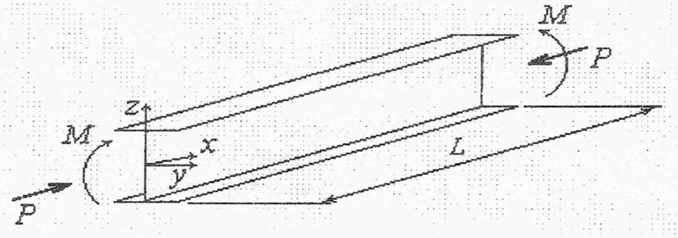



Fig. 1. H-section beam and its loading

*The linear local buckling loads* (compressive force $P_l$, bending moment $M_l$ and their combinations) were obtained from solution of the buckling problem for the beam-column considered as an assemblage of strips-plates [5]. Each plate (web, flange) was divided on several strips, for which the longitudinal stresses might be considered as constant, and the solution of the buckling problem was constructed by conjugation of the solutions for each the strip with exact conjugation conditions at the contact lines. Boundary conditions on the free edges and conjugation conditions on the contact lines result in a characteristic equation, which determines the local critical load.

A certain difficulty exists due to the dense spectrum of critical loads for a cluster of short-wave local modes relating to different numbers of longitudinal halfwaves $m$. Transition from a certain $m$ to another at varying design variables makes the local buckling constraints nonsmooth, and this complicates the optimization procedure. In order to deal only with smooth constraints we considered local buckling constraints separately for each $m$ (in the range $m$=2–35), i.e. a set of constraints was considered instead of one constraint.

Strength constraints were not imposed, i.e. the assumption was adopted that the yield limit is sufficiently high. This assumption is warranted for not too large values of the weight parameter $G^*$ (see below).

All constraints were formulated in the following dimensionless parameters of the load, weight and stress:

$$P^* = \frac{P}{L^2 \cdot E}10^6, \quad M^* = \frac{M}{L^3 \cdot E}10^8, \quad G^* = \frac{A}{L^2}10^3, \quad \sigma^* = \frac{\sigma}{E}10^3 \qquad (1)$$

The cross-section was characterized with dimensionless geometric parameters $b_w / L$, $b_f / b_w$, $t_w / b_w$, $t_f / b_f$. All computations were performed in these dimensionless parameters. After specifying the weight parameter $G^*$ the solution of the optimization problem determines all optimal dimensionless parameters. To obtain values of dimensional optimal parameters one should specify additionally $L$ value.

The nonlinear programming problem was solved by the linearized method of reduced gradient [4]. Usually a few iterations were needed (of order 30–50) to obtain the solution with required accuracy (of order $10^{-3}$).

## 3. RESULTS OF THE SOLUTION

The following optimization problems were considered sequentially.

- Optimization with a single criterion – maximum of the bending moment ($M^*$) or the compressive force ($P^*$) – for given weight parameter $G^*$. The objective functions in these cases are

$$M^* = \max\min(M^*_{ft}, M^*_l), \quad P^* = \max\min(P^*_f, P^*_t, P^*_l). \qquad (2)$$

- Constructing Pareto-optimal projects with $M^*$ and $P^*$ as objective function vector components.
- Constructing a "compromise" optimal project with a global criterion comprising $M^*$ and $P^*$ in a certain combination.

### *3.1. THE SINGLE-CRITERION OPTIMIZATION*

***Beams under the bending moment.***

There were computed optimal dimensionless parameters for the range of parameter $G^*$ (0; 0.5) in which the assumption about elastic deformation of the material is found to be justified. Optimal bars turn out to be equally stable with respect to flexural-torsional and local buckling. It has been revealed that two dimensionless parameters – $b_f/b_w$ and $t_f/t_w$–weakly depend upon $G^*$ and practically are nearly constant: $b_f/b_w = 0.5-0.6$, $t_f/t_w = 2.35-2.55$. However parameters $t_f/b_f$ and $t_w/b_w$ noticeably increase with $G^*$, as is shown in Fig.2.

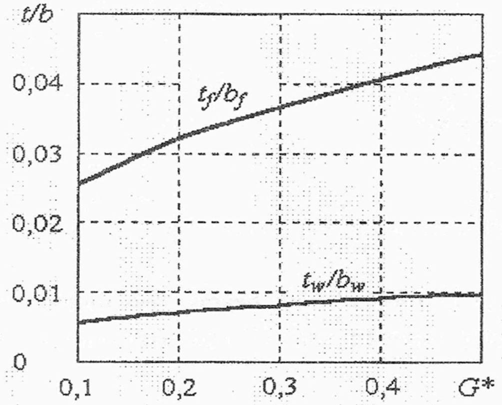
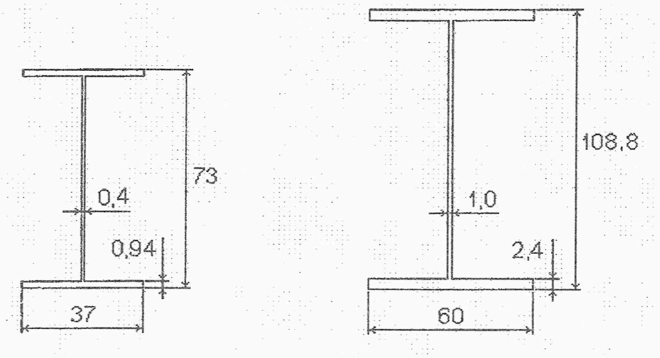



Fig. 2. Optimal thickness to width ratios for web and flange under bending

It is established that the dependence of the dimensionless critical moment $M^*$ on weight parameter $G^*$ can be approximated with the following simple formula (with high accuracy):

$$M^* = 30.865\, G^{*2} \tag{3}$$

Typical optimal cross-sections are illustrated in Fig. 3 for two $G^*$– values (*dimensional* parameters in mm are given for $L = 1$ m). These profiles differ from the standard ones by very thin web.

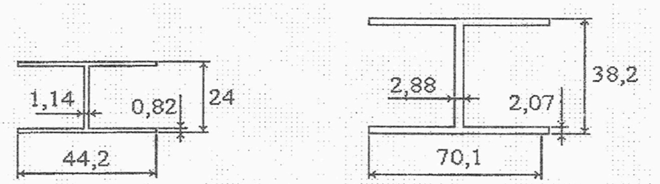



a) $G^* = 0,1$ б) $G^* = 0,4$

Fig. 3. Optimal profiles of beam under bending for $L = 1$ m (dimensions in mm)

***Bars under compression.***

Optimal bars under compression turn out to be equally stable with respect to three modes: flexural, torsional and local buckling. Therefore the optimal profiles noticeably differ from those for bent beams, as shown in Fig.4, where optimal cross-sections of compressed H-sections for two values of $G^*$ at $L = 1$ m are given.




a) $G^* = 0,1$ б) $G^* = 0,4$

Fig. 4. Optimal profiles of compressed H-section bars at $L = 1$ m (in mm)

Similarly to the case of optimal bent beams, two dimensionless parameters – $b_f / b_w$ and $t_f / t_w$ – weakly depend upon $G^*$ and practically are nearly constant, but their values are quite different comparing to the case of bending: $b_f / b_w \approx 1.84$, $t_f / t_w \approx 0.72$. Flanges become thinner, the height of webs smaller.

Parameters $t_f / b_f$ and $t_w / b_w$ increase with $G^*$, as is shown in Fig.5.

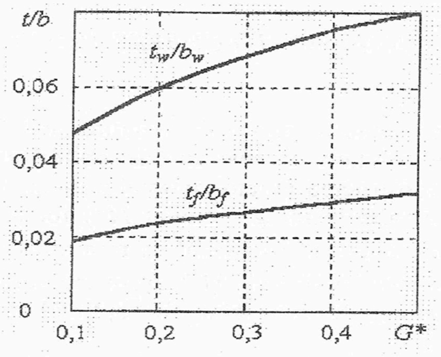



Fig. 5. Thickness to width ratios for web and flange of optimal compressed bars

The dependence of the dimensionless force $P^*$ on weight parameter $G^*$ can be approximated with high accuracy, similarly to (3), with the following power function:

$$P^* = 5.405\, G^{*5/3} \tag{4}$$

The $P^* - G^*$ and $M^* - G^*$ dependencies (3), (4) for optimal H-bars in the single-criterion optimizations are presented in Fig. 6.

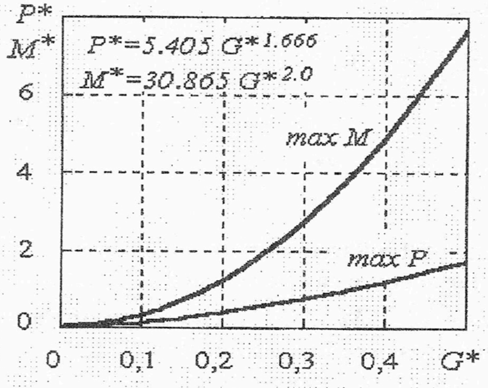



Fig. 6. Dependencies $P^*$ - $G^*$ and $M^*$ - $G^*$ for H-sections in the single-criterion optimizations

It is interesting to compare optimal H-bars under compression with optimal channel sections studied in [1]. For optimal channel sections in [1] the following relationship between $P^*$ and $G^*$ was obtained: $P^* = 2.794\,G^{*1.668}$. Comparing with (4) one can conclude that optimal H–section bar sustains compressive force approximately two times as much than optimal channel section bar.

### *3.2. TWO-CRITERIA OPTIMIZATION. PARETO-CURVES AND COMPROMISE OPTIMAL PROJECTS*

The optimal cross sections obtained above for bending and for compression are rather different, and therefore solution of two-criteria optimization problem is of particular interest. There were constructed Pareto–optimal projects (optimal profiles for any combination of the force $P^*$ and bending moment $M^*$) for several values of the weight parameter $G^*$. They were obtained by means of minimization in $P^*$ with the constraint on $M^*$, which gradually increased $M^*$ from the value, corresponding to optimal bar under compression, up to the value for optimal beam under bending moment.

The Pareto-curves are presented in Fig. 7 in the plane $P^*$ - $M^*$ for several values of $G^*$ (solid lines). Dashed lines relate to single-criterion optimal projects (each point of the dashed line gives the value of critical force or moment for optimal H-section at certain value of $G^*$, for *max P* - or *max M*- projects).

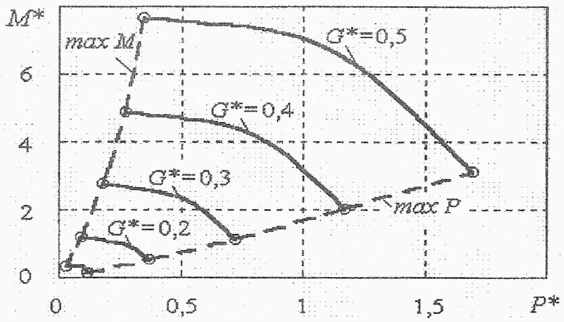



Fig. 7. Pareto-curves on the plane $P^*$ - $M^*$

Note that configurations of the Pareto-curves for different $G^*$ values are similar, so by a certain normalizing these curve can be reduced (approximately) to a single curve in relative parameters $P^*/G^{*5/3}$, $M^*/G^{*2}$. The shape of Pareto-curves noticeably differs from those for channel cross-section, obtained in [1].

In order to obtain "compromise" optimal projects, the optimization problem was solved with a global criterion of the form

$$F = \alpha \frac{P^*}{P^*_{\max}} + (1-\alpha)\frac{M^*}{M^*_{\max}}, \qquad (5)$$

where $P^*_{max}$ and $M^*_{max}$ are the values of $P^*$ and $M^*$, obtained in the single-criterion optimizations in $P^*$ and $M^*$, respectively, $\alpha$ is a "weight" coefficient which is chosen by designer. There were carried out calculations for various $\alpha$ values: 0; 0.25; 0.5; 0.75; 1 (values $\alpha = 0$ and $\alpha = 1$ relate to single-criterion optimizations).

Optimal parameters of H-sections for these $\alpha$ values, as well as values of $P^*$ and $M^*$, are presented in Table 1 for $G^* = 0.5$, $L = 1$ m.

Table 1. Compromise optimal H-sections for different values of coefficient $\alpha$ ($G^*$=0.5, $L$=1 m)

| $\alpha$ | $P^*$ | $M^*$ | Dimensions (mm) | | | | Ratios | |
|---|---|---|---|---|---|---|---|---|
| | | | $b_f$ | $b_w$ | $t_f$ | $t_w$ | $b_f/b_w$ | $t_f/t_w$ |
| 0 | 0.35 | 7.64 | 65 | 115 | 2.8 | 1.1 | 0.56 | 2.53 |
| 0.25 | 0.54 | 7.5 | 66 | 95 | 3.1 | 1.0 | 0.70 | 3.17 |
| 0.5 | 1.58 | 4.1 | 70 | 39 | 2.9 | 2.6 | 1.8 | 1.08 |
| 0.75 | 1.71 | 3.11 | 75 | 41 | 2.4 | 3.3 | 1.84 | 0.732 |
| 1.0 | 1.71 | 3.11 | 75 | 41 | 2.4 | 3.3 | 1.84 | 0.732 |

We see that the variation of $\alpha$ noticeably influences the optimal parameters. At small $\alpha$ optimal cross section parameters are close to those derived in single-criterion optimization in $M^*$. If $\alpha > 0.7$, the optimal project coincides with one obtained in single-criterion optimization in $P^*$, i.e. for $\alpha = 1$.

The obtained optimal dimensionless parameters $b_f / b_w$, $t_f / b_f$, $t_w / b_w$ have been compared with their values for standard H-sections. For some standard profiles (wide flange light profiles) these parameters turn out to be close to the calculated optimal ones in the case of prevailing bending moment (for $\alpha < 0.4$). But the most standard profiles are rather far from the optimal H-sections for any $\alpha$ values, in distinction from the standard channel sections [1]. As a rule, standard profiles have more thick webs and flanges. We may conclude that these standard profiles are not optimal independent of what combinations of compression and bending acts on the member.

Summarizing, we would like to emphasize that the solution of the two-criteria optimization problem allows one to separate those H-profiles, which are not optimal for any loading conditions.

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