MATHEMATICAL MODELS OF EFFECTIVE TOPOLOGY OF COMPUTER NETWORKS FOR ELECTRIC POWER SUPPLY CONTROL ON RAILWAY TRANSPORT

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Resume

The paper is devoted to analysis of modern directions of innovation-investment formation of intelligent computer networks that control the fast-moving technological processes of electricity supply. It is based on the conclusion that the problem of increasing the productivity of information exchange between information resources and consumers is dominant. A method for increasing the efficiency of information exchange is proposed as a search for the rational location of a new node and the organization of such a set of its connections among the whole set of nodes of the computer network, which provides a minimum average topological distance. Mathematical models of effective topological organization of connections in computer network of power consumption control at the level of traction substations, electric power distances and the railway in general are proposed.

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1 Introduction

Research on the problem of innovation-investment transformation of railway power supply systems, in order to optimize electricity consumption and the formation of energy-saving technologies, including the organization of trouble free transportation, contributed to creating and developing of a new generation of intelligent computer networks that control fast-moving technological processes of electricity supply [1-6]. The concept of intelligent energetics has opened wide opportunities to conduct real-time synchronous vector measurements of the whole set of mode parameters in different points and segments of the distributed power system with rigid association to astronomical time and accuracy to several hundred nanoseconds, to implement direct observation of the dynamic characteristics of the power system, to calculate the coefficients of low-frequency oscillations, as well as to assess the level of instability risk. Monitoring of operating conditions of traction power systems may be achieved by combining controlled reactive power sources, energy storage systems, active harmonic conditioners and distributed generators [4-9]. The work [2] indicates to importance to conduct the dependability analysis for contemporary complex systems. The intelligent power supply network could be presented as a multidimensional object [3]. Modern intelligent means of computer analysis of primary acquired information provide an opportunity to investigate the fast-moving technological processes occurring in the power supply network, to determine in the rate of electric power supply properties and dynamic characteristics at a totally new level, to identify sources of low-frequency oscillations, types and features of their shape and places of occurring, characteristics of abnormal modes, as well as to determine the limit of the current power system stability [7-12]. European Commission has published a nearly 50-page-long document on cybersecurity in transport called the Transport cybersecurity toolkit [12]. Considering the widespread of digital solutions, these guidelines should be taken into account in mathematical models of computer networks. The analysis results of intellectualization procedures of the fast-moving technological process control in energetics allows to conclude that the key feature of intelligent technologies is formation and accumulation of a new knowledge in the subject area. The acquired new knowledge in railway energetics allows to solve the problem of virtual decomposition of the power system into asynchronous operating segments to optimize power supply, to minimize power losses associated with the power flow, to conduct real-time registration of complex types of perturbations in the power supply system and damping of the low-frequency oscillations that destabilize the power system [9-16].

2 Formulation of a problem

Analysis of scientific research conducted by native and foreign scientists has shown that solving a complex problem of fast-moving technological process optimization for traction power supply is associated strictly with solving the problem of forming new knowledge about the physical nature of processes in power systems or complex energy facilities, which, in many cases, cannot be determined by existing methods [4-5]. Acquiring of a new knowledge in the relevant subject area is possible by synthesizing a new class of mathematical models and methods of higher intellectual complexity and dimension, as well as solving a task complex on their basis, which belongs traditionally to the class of creative. Based on the results of basic and applied research in mathematical and computer modelling for the control of complex energy objects, processes and phenomena, it can be confirmed that the organization of intelligent computer networks for the control of complex energy processes is based conceptually on the use of necessary or extra productivity of calculations at any point or segment of the topology of the power supply system [2, 9]. At the same time, a key problem that arises in the real-time control of geographically distributed dynamic power supply system is to increase the productivity of information exchange in multi-hierarchical computer networks that control the fast-moving energy processes. In the scientific literature attention is paid to the fact that to increase the productivity of information exchange in computer networks between information resources and consumers, it is necessary to reduce the transit traffic as a key way to influence the load of each route to balance channel traffic [6, 10]. This approach is usually based on changing the network topology by constructing parity channels. The criterion for their practicability is the expected amount of traffic in the parity channel, as well as the difference between the costs of maintaining this channel compared to the existing network channels through which the traffic is directed. At the same time, when constructing parity channels, little attention is paid to changing the topological distances in the computer network, i.e. minimizing the length of routes. The most important transit attribute of a route in computer networks of power system control is actually the number of transit nodes in the process of information transfer to the final node. This point is proved by the fact that not the geographical location of the components, but their mutual location in the topological space, has the dominant influence on the productivity of information exchange in a computer network. In fact, a decrease in the distance between nodes is associated with a decrease in the topological distance. The topological distance is the minimum amount of message retransmitting between the transit nodes. Whereas the efficiency of computer networks for power system control depends largely on their topological characteristics, in the presented research a fundamentally different approach is proposed to solve the problem of increasing their productivity; this approach is based on local changes in network topology as a basis for reducing the transit traffic and reduction of topological distances. The task of improving the productivity of information exchange in computer networks can be formulated as the search for a rational location of a new node and the organization of such a set of its connections among the whole set of network nodes that provides a minimum average topological distance.

The set of coordinates ξ_l of a new node γ should be determined, which are components of the corresponding vector $\xi = (\xi_1, \xi_2, ..\xi_l, ..\xi_n)^t$ of the contiguity matrix. In the case when $\xi_l = 1$ between the node γ and node l^m, there is a connection, and if $\xi_l = 0$ between nodes γ and l^m, there is no connection.

The aim of this work was to develop mathematical models and methods of increasing the productivity of information exchange in distributed intelligent computer networks for controlling the fast-moving technological processes of power supply at the level of traction substations, power supply distances and railways by organizing effective topological organization of connections between computer networks nodes.

3 Mathematical models

The study of the evolutionary development of power supply systems in railway transport revealed a number of differences from the power utility systems. That is, a number of features are peculiar only to these systems. Railway networks belong to the multi-hierarchical distributed systems. The first main level of the power supply network is represented by traction substations; the second level is power system or power supply distances, each of which includes n traction substations; the third level is represented by the power supply system of a separate railway. Existing topological organization of power supply of the rolling stock, which is realized by supplying electricity between phases $u_a(t) - u_c(t)$ or $u_b(t) - u_c(t)$, is manifested in a significant negative impact in the form of asymmetry and non-sinusoidal voltage and the large irregularity of the moving loads during the transportation further stimulates a significant increase in system-wide electric power consumption. In this connection, the real-time power supply optimization procedures are possible by creating a new control model based on an intelligent distributed computer network, the architecture of which reflects adequately the topology of the railway power supply system. Moreover, the architectural organization of an intelligent computer network should provide the necessary productivity of information exchange at the pace of power supply at all the levels of the hierarchy. Due to this approach, the distributed computing environment for the power control and optimization, as well as energy saving procedures, are presented at three levels. The first level is represented in the form of a local computer network, in the basis of the logical structure of which any physical architecture can be used, which is adequate to the topology of the electrical system of the traction substation. At this level, the organization of local computer networks is dominant in the power supply system, because in the electricity market it is necessary to transmit, in real time, large amounts of initial information related to commercial electricity metering. Procedures for identifying the technical condition of the reliability of the power supply system and forecasting possible normal and abnormal modes, with internal or external adverse effects, are also associated with the real-time formation and transmission of a large amount of rapid emergency information data. The second level of the distributed computer environment is represented in the form of a regional computer network; its nodes or segments may be m local computer networks of the respective traction substations. The third level is represented in the form of a corporate computer network that includes the appropriate number of nodes and segments formed from regional and local computer networks for the power control. The peculiarity of the corporate computer network is in necessity to determine and take into account the relative balance of traffic and transmission capacity at all the levels of the hierarchy, which is very important in formation of a real-time set of control effects to ensure efficient power supply. Based on the above mentioned, the organization of mathematical models is considered for formation of effective topology of computer networks for the power supply control at the level of traction substations, power supply distances and railways.

The local computer network of the traction substation can be represented as a graph $G(V, E)$, where V is a set of nodes representing autonomous computer tools and E is the connections between them represented by the edges of the graph. The technology of increasing the productivity of information exchange in computer networks by analysing and reducing topological distances to reduce the transit traffic is developing for the duration of their existence. Therefore, the transmission capacity of connections between network nodes is not constant, but tends to increase permanently. Accordingly, the graph $G(V, E)$ is unweighted and undirected due to the fact that the information is transmitted in two directions. For further analysis, the metric space of the graph $G(V, E)$ should be determined, which displays the local computer network of the traction substation and what function the topological distance $P(\gamma, u)$ is specified between set objects V, i.e. nodes γ and u of the graph. Topological distance function $P(\gamma, u)$ must satisfy three axioms of the metric space such as the axiom of identity $P(V,u) = 0 <=> \gamma = u$; the axiom of symmetry $P(V,u) = P(u,\gamma)$ and the axiom of triangle inequality $P(\gamma,u) \le P(\gamma,\gamma') + P(V',u)$. Based on the graph theory, the function of topological distance $P(\gamma,u)$ between the two nodes γ and u as the length of the shortest route of the graph $G(V, E)$, $(\gamma,u) \in V$ can be represented as follows [10, 16]

$$P(\gamma,u) = \min_i \left(P(\gamma,i) + P(i,u) \right). \tag{1}$$

Thus defined topological distance $P(\gamma,u)$ satisfies the axioms of the metric given on the set $V \times V$ and a pair (V,P) represents the metric space of the graph $G(V, E)$, where the search problem should be solved on a local computer network of rational location of the new node f, represented by vector $\xi = (\xi_1, \xi_2, ..\xi_l, ..\xi_n)^t$ [10]. In this case, the set of coordinates ξ_l of a new node f, i.e. its connections among multiple nodes V of the network, should be formed to provide a minimum average topological distance as a basis for improving the productivity of information exchange. In view of this, for each γ^{20} node of the graph $G(V, E)$, representing the local network, based on the topological distance function $P(\gamma,u)$, represented by the Equation (1), the sum of the distances P_γ is determined, which can be written as:

$$P_\gamma = \sum_{l=1}^{n} P(\gamma,l). \tag{2}$$

The average sum of distances P_S for the whole set of nodes V of the network, represented by the graph $G(V, E)$, according to Equation (2) can be written as the following mathematical relation:

$$P_S = \frac{1}{n} \sum_{l=1}^{n} \sum_{r>l}^{n} P(l,r). \tag{3}$$

By analogy with Equations (2), (3) can be represented by mathematical dependences to determine the average topological distance $P_{\gamma t}$ for each γ^{20} node of the graph $G(V, E)$ as

$$P_{\gamma t} = \frac{1}{n} \sum_{l=1}^{n} P(\gamma,l) \tag{4}$$

and, respectively, average topological distance P_{mt} among the whole set of nodes V of the network, represented by the graph $G(V, E)$, as

$$P_{mt} = \frac{1}{n^2} \sum_{\gamma=1}^{n} \sum_{l=1}^{n} P(\gamma,l). \tag{5}$$

The results of modelling in the field of improving the productivity of information exchange of computer networks to determine their effective topological organization in the process of a power supply control, shows that in the process of exploration of node sets in different segments of the network, based only on the parameters $P_v, P_S, P_{\gamma,t}, P_{mt}$, represented by Equations (2)-(5), it is impossible to perform a set of procedures for comparing the position of two nodes in different

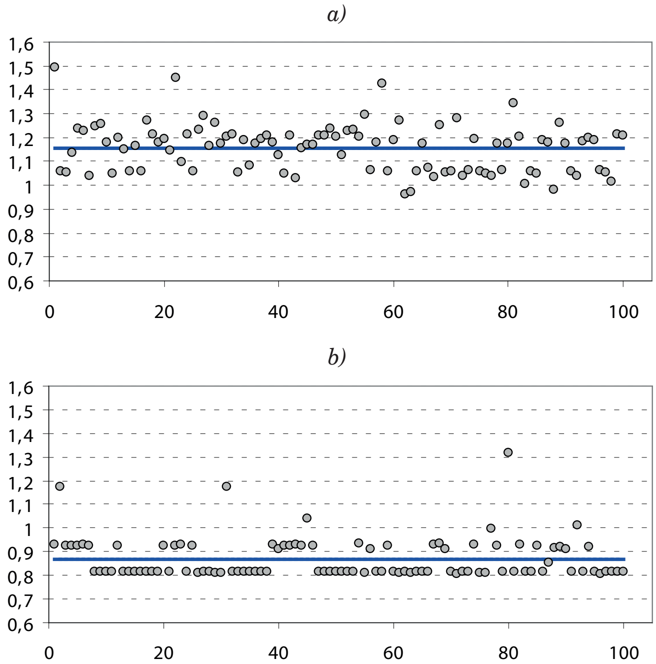

Figure 1 *Value of the distance factor F in a new node as a result of connecting the two random nodes (a) and to two nodes, selected by Barabasi-Albert model;, the abscissa axis is the number of attempts, the ordinate axis is the value of F*

segments of a computer network [16]. Therefore, it is necessary to enter a new parameter F_γ, the absolute value of which characterizes the degree of topological proximity of each γ^{20} node in relation to the set of nodes *V* of the network, presented in the form of a graph *G*(*V, E*). This parameter $F_{\gamma'}$ is the ratio of the sum of distances P_γ of nodes to the average sum of distances P_S among all the nodes *V* of the network, which is called the distance factor and can be written as

$$F_{\gamma'} = \frac{n\sum_{l=1}^{n} P(l,r)}{\sum_{l}^{n}\sum_{r>l}^{n} P(l,r)}. \qquad (6)$$

Thus, the synthesis of the mathematical model for the effective topology organization of the local computer network of the power supply control is based on the solution of a problem of finding the layout of the rational location of a new node γ' among other nodes *V* of initial network *G* := (*V, E*), consisting of nodes *V* and connections *E* between them. Added new node γ' actually forms a new network $G' := (V', E')$ from the set of nodes $V' = V \cup \gamma'$ and new connections E', as long as the sum of the distances between it and other network nodes must be minimal: $(V,E) \rightarrow (V',E');[(V' \cup V'),(F_{\gamma'} \rightarrow \min)]$, i.e. $F_{\gamma' \rightarrow \min}$.

With this approach to organization of the computational process to solve the problem of increasing the efficiency of information data exchange for the computer power supply control network at the traction substation level requires the search for such a contiguity vector $X_{\gamma'} = (x_1, x_2, ..x_l, ..x_n)^t$ of a new node γ', connected to the initial network *G* := (*V, E*), which minimizes the distance factor and is represented by Equation (6), i.e.

$$\begin{cases} F_{\gamma'} \rightarrow \min \\ \sum_{l=1}^{n} x_l = q \end{cases}. \qquad (7)$$

The binary vector $X_{\gamma'} = (x_1, x_2, ..x_l, ..x_n)^t$ of a new node γ' in the mathematical model in Equation (7) is formed in such a way that each *l* coordinate of it takes values $x_l = 1$, if between the nodes γ' and *l* there is the connection and $x_l = 0$ if there is no connection, respectively, with a limited number $q = n - 1$ of the non-zero vector elements $X_{\gamma'}$. A combination of connections that provides the least value for the distance factor $F_{\gamma'}$ of a new node γ' in the mathematical model in Equation (7), namely the minimum average topological distance in the local computer network of the traction substation, is optimal.

The formulated problem belongs to the class of distribution problems and can be considered as a median topic, or its generalization - Weber's problem or Steiner's combined problem [16]. Finding the exact solution to such problems is associated with enumeration methods and, hence, there are limitations on the dimension. To reduce the problem dimension and reduce significantly the number of enumerations, the analysis of the topology of existing connections can be used to select obviously inefficient solutions. In Figure 1 comparison of the experiment result with the random connection of the node to the existing network and the connection using Barabasi-Albert model are showed.

Considering the dimension of each of the graphs

$G(V, E)$, reflecting computer power supply control networks of traction substations, power supply distances and individual railways, is relatively small; the use of enumeration methods to solve problems of the form in Equation (7) can be quite effective.

By analogy with the above mentioned, a mathematical model of effective topological organization of computer control network connections at the level of power supply distances is synthesized. Mathematical dependences to determine the sum of the distances of each node of the graph and the average sum of the distances for the whole set of nodes V of the graph $G(V, E)$, reflecting the computer network power supply distances, are presented as follows

$$P_{\gamma}^{DP} = \sum_{j=1}^{m} P_{\gamma}^{j} \sum_{l=1}^{Q} P^{DP}(\gamma, l)$$
$$P^{DP} = \frac{1}{mQ} \sum_{j=1}^{m} P_{\gamma}^{j} \sum_{\gamma=1}^{Q} \sum_{l>\gamma}^{Q} P^{DP}(\gamma, l), \tag{8}$$

and the distance factor, respectively

$$F_{\gamma}^{DP} = \frac{mQ \sum_{j=1}^{m} P_{\gamma}^{j} \sum_{l=1}^{Q} P^{DP}(\gamma, l)}{\sum_{j=1}^{m} P_{\gamma}^{j} \sum_{l=1}^{Q} \sum_{l>\gamma}^{Q} P^{DP}(\gamma, l)}, \tag{9}$$

where P_{γ}^{DP} is the sum of distances of each node ξ^{DP} of the graph $G(V, E)$ of the power supply distances; P^{DP} is the average sum of distances over the whole set of nodes V of the graph $G(V, E)$ of power supply distances.

The mathematical model of effective organization of the network topology is presented in the form

$$\begin{cases} F_{\gamma'}^{DP} \to \min \\ \sum_{l=1}^{Q} \xi_{l}^{DP} = Q - 1 \end{cases}, \tag{10}$$

where ξ^{DP} is the binary vector $\xi^{DP} = (\xi_1^{DP}, \xi_2^{DP}, ..\xi_l^{DP}, ..\xi_Q^{DP})^t$ of a new node γ', connected to the initial network $G := (V, E)$ of the power supply distance, which minimizes the distance factor in the mathematical model in Equation (10).

Formation model of effective topology of connections of the control computer network for railway power supply by analogy with Equations (8) - (10) can be written as follows

$$P_{\gamma}^{ZL} = \sum_{j=1}^{Q} P_{\gamma j}^{DP} \sum_{l=1}^{G} P^{ZL}(\gamma, l),$$
$$P^{ZL} = \frac{1}{QG} \sum_{j=1}^{Q} P_{\gamma j}^{DP} \sum_{y=1}^{G} \sum_{l>\gamma}^{G} P^{ZL}(\gamma, l),$$
$$F_{\gamma}^{ZL} = \frac{QG \sum_{j=1}^{Q} P_{\gamma j}^{DP} \sum_{l=1}^{G} P^{ZL}(\gamma, l)}{\sum_{j=1}^{Q} P_{\gamma j}^{DP} \sum_{\gamma=1}^{G} \sum_{l>\gamma}^{G} P^{ZL}(\gamma, l)}, \tag{11}$$
$$\begin{cases} F_{\gamma'}^{ZL} \to \min \\ \sum_{l=1}^{G} V_{l}^{ZL} = G - 1 \end{cases},$$

where P_{γ}^{ZL} is the sum of the distances of each node V^{ZL} of the graph$G(V, E)$, reflecting the computer network of railway power supply control; P^{ZL} - the average sum of distances over the whole set of nodes V of the graph $G(V, E)$ of railway; $F_{\gamma'}^{ZL}$ - distance factor of the railway computer network; $V^{ZL} = (V_1^{ZL}, V_2^{ZL}, ..V_l^{ZL}, ..V_G^{ZL})^t$ - binary vector of the connected node γ' to initial network $G := (V, E)$ of power supply control of a separate railway, which minimizes the distance factor $F_{\gamma'}^{ZL}$ in the mathematical model in Equation (11).

4 Conclusion

1. The analysis of modern ideologies of innovation-investment formation of intelligent computer networks and control technologies of the fast-moving technological processes of railway transport power supply by intellectualization of administrative procedures spectrum of power consumption optimization, formation of energy saving technologies and organization of trouble free transportations is carried out.
2. It is shown that the key problem in the real-time control of geographically distributed dynamic power supply systems is the need to increase the productivity of information exchange in computer networks between information resources and consumers by forming control effects to minimize the transit traffic as a dominant way to influence the load of a route and balance channel traffic.
3. A method for increasing the productivity of information exchange in distributed intelligent computer networks for the power supply control is formulated as a search for a rational location of a new node and organization of such a set of its connections among the whole set of computer network nodes which provides the minimum average topological distance by determining the set of coordinates ξ_l of a new node γ, which are components of the corresponding vector $\xi = (\xi_1, \xi_2, ..\xi_l, ..\xi_n)^t$ of the contiguity matrix.
4. Mathematical models and organization methods of the computational process are formulated for solving the problem of increasing the efficiency of information data exchange in the computer network of power supply control at the level of the railway traction substation by organizing a new node γ' connected to the initial network $G := (V, E)$ and its contiguity vector $X_{\gamma'} = (x_1, x_2, ..x_l, ..x_n)^t$, which minimizes the distance factor.
5. A number of mathematical dependences are developed for determining a set of indicators that reflect a number of characteristics of the computer control network such as the sum of distances and the average sum of distances of each node of the graph $G(V, E)$ for all the sets of nodes V; on their basis, mathematical models for the effective topological

organization of connections in the computer network control of power consumption at the level of power supply distance and the railway as a whole are offered.

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