derailment; forensic science; wheel; sleeper; movement resistance; speed; force; kinetic energy

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MATHEMATICAL MODEL OF THE DERAILED TRAIN WHEEL MOTION

Summary. It is developed a mathematical model of the derailed train wheel motion, which makes it possible to determine a change in its speed when moving on the track panel. When constructing the model the methods of theoretical mechanics and train traction were used. It was found that the highest speed reduction occurs as a consequence of the wheel impact on the sleeper. The model allows creating a generalized mathematical model of the train motion in the emergency mode.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ДВИЖЕНИЯ СОШЕДШЕГО С РЕЛЬСОВ ЖЕЛЕЗНОДОРОЖНОГО КОЛЕСА

Аннотация. Разработана математическая модель движения сошедшего с рельс железнодорожного колеса, позволяющая определить изменение его скорости при движении по рельсошпальной решетке. При построении модели использованы методы теоретической механики и тяги поездов. Установлено, что наибольшее снижение скорости происходит вследствие удара колеса о шпалу. Модель позволяет создать обобщенную математическую модель движения поезда в аварийном режиме.

1. INTRODUCTION

The possibility of preventing railway accidents is one of the main problems during forensic railway transport enquiry. Solution of this problem requires calculation of the braking path of the train, the value of which largely depends on the rolling stock movement resistance.

At the present stage of development of the train traction science the methods for determining the movement resistance of locomotives and cars in normal operating conditions are developed at a sufficient level [1-3]. However, during the railway transport accident there are factors that have a significant influence on the movement resistance value, and the methods of quantitive evaluation of the above mentioned influence are insufficiently developed [4, 5].

A typical example is the case, when in the process of movement, braking or after collision with motor vehicles at the railway crossing, some wheel sets derail and move along the track panels for several hundred meters (Fig. 1).

The fact that there is no methodology for calculating the additional movement resistance, which occurs in this case, greatly complicates the determination of the place of the actual start of braking. So, it is necessary to develop a methodology for calculating the additional movement resistance of the rolling stock from the derailed wheel sets and specify the methodology of calculating the braking path of the train.

To solve this problem, it is necessary to develop a methodology for calculating the movement speed of the rolling stock with derailed wheel sets, as well as to specify the method of calculating the braking path of the train. This is exactly what determines the relevance of this article at the present stage of development of expert studies of railway transport accidents.

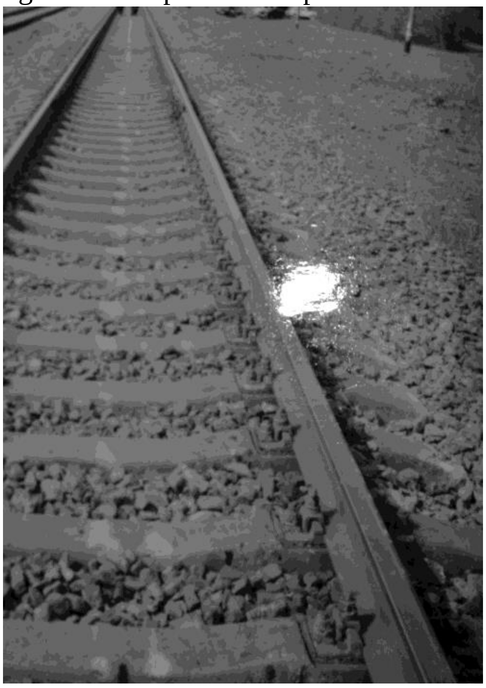

Fig. 1. The traces of the railway wheel motion on the track panel
Рис. 1. Следы движения железнодорожного колеса на рельсошпальной решетке

2. PROBLEM DEFINITION

The article is devoted to solving scientific and technical problem of improving the traction calculations of emergency modes of train motion. This will solve the practical tasks of determining the braking path during forensic enquiries and internal investigation of derailments and collisions of rolling stock, as well as develop proposals to prevent the railway accidents.

3. ANALYSIS OF INVESTIGATIONS AND PUBLICATIONS

Results of investigations devoted to the development of train traction starting from its emergence are systematized in the work [1].

Well-known experts in the field of train traction participated in carrying out the drawbar tests of rolling stock and studied the resistance forces. They are: N. P. Petrov, Yu. V. Lomonosov, A. M. Babichkov, A. I. Dolinzhev, V. F. Yegorchenko, V. M. Kazarinov, O. N. Isaakyan, P. A. Gurskiy, P. N. Astakhov, P. T. Grebenyuk, P. P. Stromskiy, V. G. Inozemtsev, A. N. Dolganov and others.

Among the early works it should be noted the work of N. P. Petrov "Resistance of trains on the railways", which appeared in 1892. The work considered the components of the resistance force to train movement and the influence of various factors on their value, as well as it proposed the calculation formulas for determining the movement resistance of rolling stock.

In 1915 it was published the second edition of the book "Traction calculations" [6] of Yu. V. Lomonosov, in which were summarized the methods of traction calculations developed by this time.

Based on the results of large number of tests, the scientists published generalizing monographs, articles in number of special issues on the train motion mechanics [7,8].

"Rules of traction calculations for train operation", the latest edition of which [9] was published in 1985 were refined and supplemented according to the research results. It is still the guiding document for traction calculations on the railways of the former Soviet Union.

The main theoretical approaches that were the basis of methodologies for railway forensic examinations are given in the works of E.N. Sokol, the Doctor of Technical Sciences [10-15]. They observe the cases of wheel set motion in the emergency mode, but the final point, which is observed in these works, is the moment of wheel flange rolling onto the rail head and movement on it.

Movement of different types of wheels on the different surfaces is considered in fundamental and special works in physics, theoretical mechanics and strength of materials [16-19], which set out the common approaches to the definition of resistance and speed of the wheel motion, however the case of the railway wheel rolling along the track panel is not considered.

4. PURPOSE AND TASKS OF INVESTIGATION

Purpose of the study is to improve the traction calculations of the railway rolling stock during motion in emergency mode. This will make it possible to determine the speed change and additional resistance occurring at the same time.

Task of the study is to develop a method for determining the movement speed of railway wheel moving on the track panel.

5. MAIN PART OF INVESTIGATION

Let us represent railway wheel as a solid non-deformable metal disk with radius R and the weight Q, which is loaded by the force P.

The wheel movement on the track panel will be divided into the following stages: rolling on the surface of ferroconcrete sleeper; falling into the space between sleepers; elastic impact on the next sleeper; climbing on the surface of the following sleeper.

Rolling of the wheel flange along the surface of the concrete sleeper is accompanied by destruction of its surface. The flange destroys the sleeper and deepens into it by the value h'; at this a force of rolling resistance W arises.

Let us define the linear speed of the wheel at the final point of motion along the sleeper v_1. The diagram of the wheel motion on the surface of the concrete sleeper is shown in the Fig. 2.

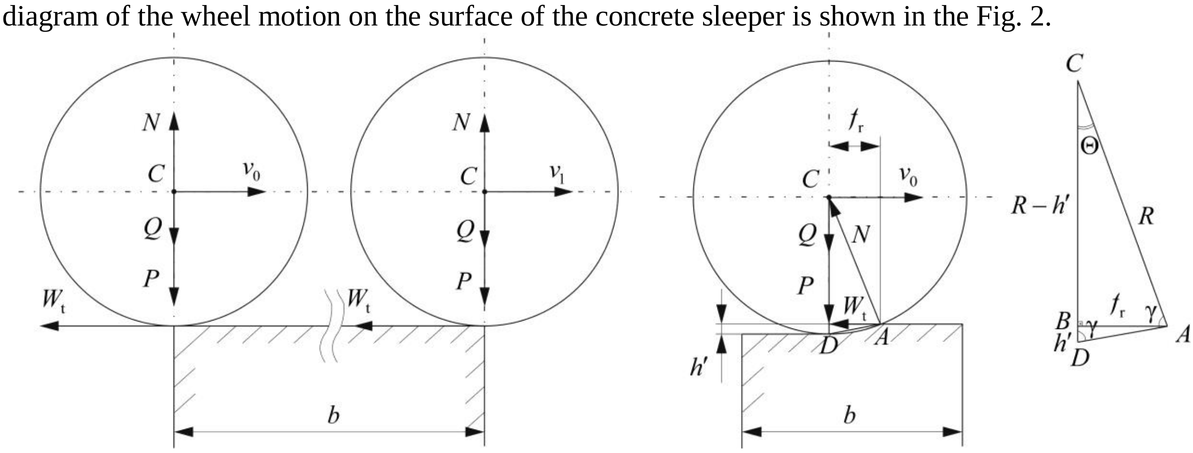

Fig. 2. The diagram of the wheel motion on the surface of the concrete sleeper
Рис. 2. Схема движения колеса по поверхности железобетонной шпалы

The resistance force of the wheel motion on the surface of the concrete sleeper W_t will be:

$$W_{\mathrm{t}} = \frac{f_{\mathrm{r}}}{R}(P+Q)$$

where f_{r} - is the coefficient of rolling friction.

The dependence of the value f_{r} on the value h' can be found using the geometric relationships of the triangles CAD, CAB, DAB represented in the Fig. 2, b:

$$\frac{f_{\mathrm{r}}}{h'} = \mathrm{tg}\gamma = \mathrm{tg}\left(90 - \frac{\Theta}{2}\right);$$

$$\frac{R-h'}{f_{\mathrm{r}}} = \mathrm{ctg}\Theta = \frac{\mathrm{ctg}^2\frac{\Theta}{2} - 1}{2\mathrm{ctg}\frac{\Theta}{2}};$$

$$\mathrm{ctg}^2\frac{\Theta}{2} - 1 = 2\mathrm{ctg}\frac{\Theta}{2}\left(\frac{R-h'}{f_{\mathrm{r}}}\right);$$

$$\frac{f_{\mathrm{r}}^2}{h'^2} - 1 = 2\frac{f_{\mathrm{r}}}{h'}\left(\frac{R-h'}{f_{\mathrm{r}}}\right);$$

$$f_{\mathrm{r}} = \sqrt{2h'R - h'^2}\ .$$

To determine the speed of the wheel movement v_1 at the end of the sleeper the theorem on change of kinetic energy of the system is used [16].

Kinetic energy of the wheel at the initial moment of motion on the surface of sleeper may be defined by the formula

$$T_0 = \frac{(P+Q)}{g}\frac{v_0^2}{2} + \frac{J_{c_z}\omega_0^2}{2},$$

where P – is a wheel load; Q – is the weight of the wheel, J_{c_z} - is the axial moment of inertia of the wheel, v_0 - is the initial linear speed of the wheel; ω_0 - is the initial angular speed of the wheel.

Axial moment of the wheel inertia under the made assumptions is determined by the formula

$$J_{c_z} = \frac{Q}{g}\cdot\frac{R^2}{2}.$$

Then the kinetic energy of the wheel in the initial moment of movement on the sleeper will be equal to

$$T_0 = \frac{P}{g}\cdot\frac{v_0^2}{2} + \frac{Q}{g}\cdot\frac{v_0^2}{2} + \frac{QR^2\omega_0^2}{4g}$$

Taking into account the relationship between angular and linear speeds $v = \omega R$:

$$T_0 = \frac{P}{g}\cdot\frac{v_0^2}{2} + \frac{3}{4}\frac{Q}{g}v_0^2 = \frac{v_0^2}{2g}\left(P + \frac{3}{2}Q\right) \quad (1)$$

Similarly, the kinetic energy of the wheel at the final moment of motion along the sleeper is:

$$T_1 = \frac{v_1^2}{2g}\left(P + \frac{3}{2}Q\right) \quad (2)$$

where v_1 − is a linear speed of the wheel at the final moment of motion along the sleeper.

The work of external forces (in this case – the movement resistance force when moving the wheel for the distance *b*).

$$\sum A^e = -W_{\mathrm{t}}\cdot b,$$

where b − is a sleeper width.

According to the theorem of the kinetic energy change of the system

$$T_1 - T_0 = \sum A^e \,, \tag{3}$$

where T_1 and T_0 − is the kinetic energy of the system at the final and initial moment of movement on the sleeper correspondingly.

Let us substitute (1) and (2) in (3) and after transformations we obtain

$$\left(v_1^2 - v_0^2\right)\frac{P + \frac{3}{2}Q}{2g} = -W_t b \,,$$

$$v_1 = \sqrt{v_0^2 - \frac{2W_t bg}{P + \frac{3}{2}Q}}$$

We define the linear speed of the wheel in the final moment of rolling into the space between sleepers v_2. The diagram of the wheel motion in this case is shown in the Fig. 3.

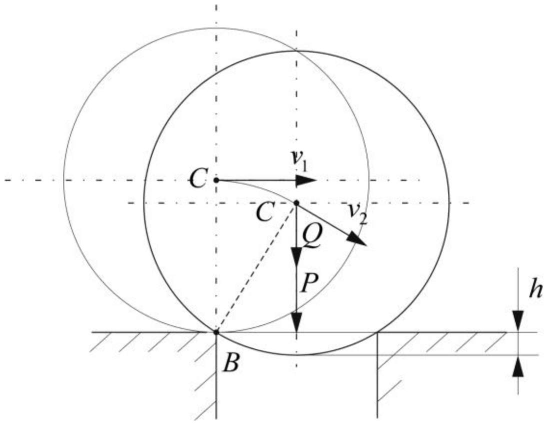

Fig. 3. The diagram of the wheel motion between sleepers
Рис. 3. Схема движения колеса между шпалами

According to the theorem on the change of kinetic energy for this section

$$T_2 - T_1 = \sum A^e \,.$$

where T_2 - is a kinetic energy of the wheel at the moment of impact on the next sleeper (end point of the falling into the space between sleepers).

The work of external forces in this case will be the sum of the work of forces P and Q at the moving h

$$\sum A^e = A(P) + A(Q) = (P + Q)h \tag{4}$$

The kinetic energy of the wheel at the start of falling into the space between sleepers

$$T_1 = \frac{1}{2} J_1 \omega_1^2 \,.$$

where ω_1 - is the angular speed of the wheel at the initial moment; J_1 - is the moment of inertia of the wheel relative to the point B.

$$J_1 = J_c + \frac{P + Q}{g} R^2 \,,$$

where J_c - is the moment of inertia of the wheel during the rotational movement relative to center of masses of the wheel - the point *C*.

$$J_\mathrm{c} = \frac{Q}{2g} R^2 .$$

Then

$$J_1 = \frac{R^2}{g}\left(P + \frac{3}{2}Q\right). \tag{5}$$

Thus,

$$T_1 = \frac{R^2}{2g}\left(P + \frac{3}{2}Q\right)\omega_1^2 . \tag{6}$$

The kinetic energy in the final moment of falling into the space between sleepers (before the impact on the next sleeper) is determined similar to the expression (6):

$$T_2 = \frac{R^2}{2g}\left(P + \frac{3}{2}Q\right)\omega_2^2 . \tag{7}$$

where ω_2 - is the angular speed of the wheel at the end of this stage.

After substituting (6) and (7) in (4) we obtain:

$$\frac{R^2}{2g}\left(P + \frac{3}{2}Q\right)\left(\omega_2^2 - \omega_1^2\right) = (P+Q)h ,$$

$$\frac{\left(P + \frac{3}{2}Q\right)}{2g}\left(v_2^2 - v_1^2\right) = (P+Q)h ,$$

$$v_2 = \sqrt{v_1^2 + \frac{2(P+Q)gh}{P + \frac{3}{2}Q}} .$$

The value h by which the wheel center is lowered in the space between the sleepers depends on the distance between the axles of sleepers l_s, the sleeper width b and the radius of the wheel (at the top of the flange) R:

$$h = R - \sqrt{R^2 - \frac{(l_\mathrm{s} - b)^2}{4}} .$$

The next stage of movement of the wheel with the weight *Q* loaded by the force *P* with initial speed of the mass center v_2 is impact on the next sleeper (Fig. 4, a).

Let us define the linear speed of the wheel movement v_3 at the start of climbing on the sleeper (at the end of the impact).

To solve this problem the theorem of the change of angular momentum is used [16]

$$\overline{L}_\mathrm{e} - \overline{L}_\mathrm{b} = \sum \overline{M}_A^e\left(\overline{S}\right),$$

where $\overline{L}_\mathrm{e}$ and $\overline{L}_\mathrm{b}$ - are the angular momenta of the wheel relative to the point *A* at the end and at the beginning of the impact.

Since the wheel is influenced only by the impact impulse $\overline{S}$, whose line of action passes through the point *A* (Fig. 4, a), the geometric sum of the moments of external impact impulses will be equal to zero

$$\sum \overline{M}_A^e\left(\overline{S}\right) = 0 .$$

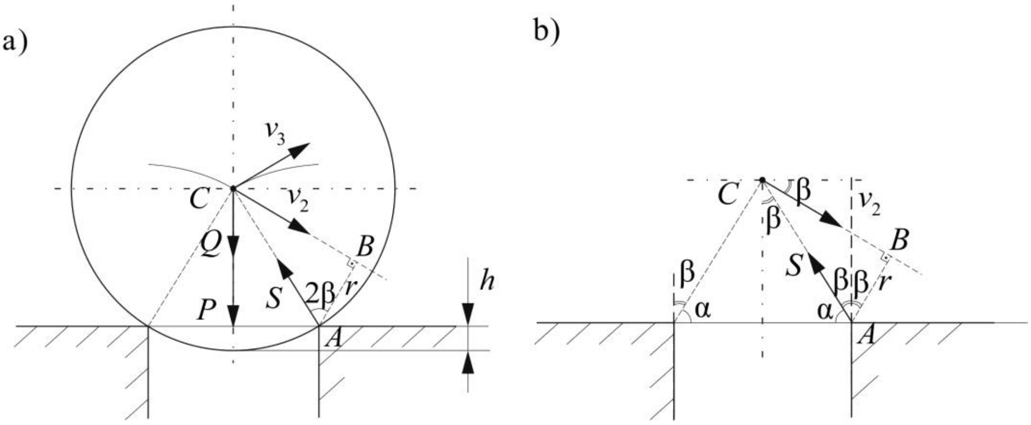

Fig. 4. The diagram of the wheel motion at the moment of impact
Рис. 4. Схема движения колеса в момент удара

Since the wheel is influenced only by the impact impulse $\bar{S}$, whose line of action passes through the point *A* (Fig. 4, a), the geometric sum of the moments of external impact impulses will be equal to zero

$$\sum \overline{M}_A^e(\bar{S}) = 0 .$$

Then

$$\bar{L}_e = \bar{L}_b \tag{8}$$

Angular momentum $\bar{L}_b$ relative to the point *A* at the beginning of impact (Fig. 4, b) is [16]

$$\bar{L}_b = \bar{r} \times m\bar{v}_2 .$$

In our case

$$L_b = \frac{Q+P}{g} v_2 r , \tag{9}$$

where

$$r = R\cos 2\beta . \tag{10}$$

Angular momentum of the wheel at the end of the impact L_e is [17]

$$L_e = J_1 \omega_3 , \tag{11}$$

where J_1 - is the wheel inertia moment relative to the point *A*, which is determined by the expression (5); ω_3 - is the angular speed of the wheel at the end of the impact.

After substituting (9) and (11) in (8) taking into account (10) and (12) we obtain:

$$\frac{Q+P}{g} v_2 R\cos 2\beta = \frac{R^2 \omega_3}{g}\left(P + \frac{3}{2}Q\right),$$

$$\omega_3 = \frac{(Q+P)v_2 \cos 2\beta}{R\left(P + \frac{3}{2}Q\right)},$$

$$v_3 = v_2 \frac{Q+P}{P + \frac{3}{2}Q} \cos 2\beta .$$

Now we define the linear speed of the wheel motion v_4 at the moment of its climbing on the sleeper. Motion diagram of the wheel is shown in the Fig. 5.

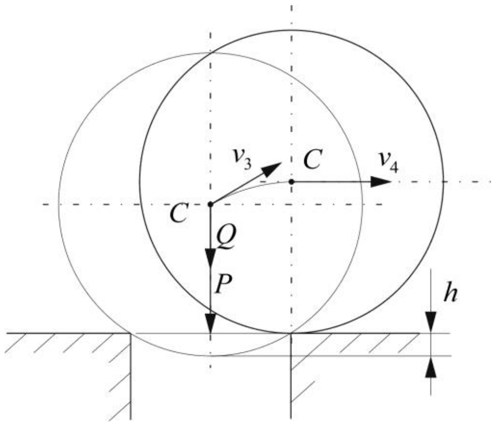

Fig. 5. The moment of the wheels climbing on the sleeper
Рис. 5. Момент подъема колеса на шпалу

To solve this problem we use the theorem on the change of kinetic energy of the system, which in this case will have the following form:

$$T_4 - T_3 = \sum A^e , \tag{13}$$

where T_3 - is the kinetic energy of the wheels at the start of the wheel climbing on the sleeper; T_4 - is the kinetic energy of the wheel at the end of wheel climbing on the sleeper.

$$T_3 = J_1 \frac{\omega_3^2}{2} = \frac{R^2}{2g}\left(P + \frac{3}{2}Q \right)\omega_3^2 . \tag{14}$$

Similarly, taking into account the relationship between the angular and linear velocities

$$T_4 = \frac{v_4^2}{2} = \left(P + \frac{3}{2}Q \right). \tag{15}$$

The work of external forces in this case will be equal to

$$\sum A^e = -(P+Q)h \tag{16}$$

After substituting (14), (15) and (16) in (13) we obtain:

$$\frac{v_4^2}{2g}\left(P + \frac{3}{2}Q \right) - \frac{R^2}{2g}\left(P + \frac{3}{2}Q \right)\omega_3^2 = -(P+Q)h ,$$

$$v_4^2 - v_3^2 = -\frac{2g(P+Q)h}{P + \frac{3}{2}Q} ,$$

$$v_4 = \sqrt{v_3^2 - \frac{2g(P+Q)h}{P + \frac{3}{2}Q}}$$

To derive the calculation formulas for determination of the speeds at the end of each stage (v_1 , v_2 , v_3 , v_4) the following factors are introduced:

$$A = \frac{2gW_t b}{P + \frac{3}{2}Q}, \quad B = \frac{P+Q}{P + \frac{3}{2}Q}, \quad C = \frac{2g(P+Q)h}{P + \frac{3}{2}Q} = 2ghB$$

Then

$$v_1 = \sqrt{v_0^2 - A} ,$$

$$v_2 = \sqrt{v_0^2 - A + C} ,$$

$$v_3 = B \cdot \cos 2\beta \cdot \sqrt{v_0^2 - A + C}\ ,$$
$$v_4 = \sqrt{B^2 \cdot \cos^2 2\beta \cdot \left(v_0^2 - A + C\right) - C}\ .$$

6. EXAMPLE OF CALCULATION.

Using the developed methodology it is established the final speed of the wheel motion at the following initial data: P = 110360 N; Q = 6867 N; β = 0.316 rad; b = 0.174 m; R = 0.503 m; h' = 0.01 m; l_s = 0.625 m; v_0 = 5…20 m/sec.

The calculation results are summarized in the Table 1

Tab. 1

v_0, [m/sec]	v_1, [m/sec]	v_2, [m/sec]	v_3, [m/sec]	v_4, [m/sec]
5	4.93	5.04	3.95	3.82
10	9.97	10.02	7.85	7.79
15	14.98	15.01	11.77	11.72
20	19.98	20.01	15.68	15.65

7. CONCLUSIONS

Thus, as a result of the conducted investigation, there were obtained the calculation formulas to determine the railway wheel motion speed on the track panels for different stages of movement: on the surface of ferro-concrete sleeper, at the moment of falling into the space between sleepers, at the moment of impact, at the moment of climbing on the next sleeper. It should be noted that this sequence of stages is unchangeable on further motion on the track panel.

The study also established the dependency of the movement resistance to the wheel rolling on the surface of ferro-concrete sleeper on the depth of the track, which leaves the wheel on the surface of sleeper.

It was found that the highest speed reduction occurs as a result of the wheel impact on the sleeper.

The proposed method can be used to determine the average weighted resistance to the wheel set movement on the track panel, which is a step towards solving the problem of improvement of traction calculations of emergency modes of the train movement.

For the purpose of further development of this methodology it is reasonable to determine the influence of energy losses for impact on the change of speed, as the wheel impact on the sleeper is not elastic (as it is assumed in the given model), and it is accompanied by the destruction of the sleeper surface.

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