

# Calculation of the traction power supply systems using the functions of resistance

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## Summary

The article is devoted to the development of a new method of calculating the instantaneous traction power supply circuits of the system. On the basis of the design scheme of generalized traction substations zone provided analytical functions of the resistance. Using the known functions of the current distribution of functions putted into the concept of resistance feeders traction substations. The functions of resistance and current distribution used in a more straightforward method of calculating the instant schemes that allow to formalize electrical calculations of electrical traction power supply systems.

**Key words:** electric traction, electric power supply, instant scheme, calculation, resistance function, feeder currents, voltage losses, power losses.

## Introduction

Calculation of traction power supply systems is an important step in the design of electrified railways. Having an impact on the capital costs, which are determined by the cost of the necessary equipment. The equipment, in turn, is determined basing on the calculated parameters of the traction power supply.

In addition to the general industrial power supply systems, traction power supply system have characteristic differences, which preclude the use of conventional techniques for calculation of the design parameters. Due to the lack of modern computing capabilities in the development of methods for calculating the traction power supply systems, they have been greatly simplified, which entails being left assume factor to avoid the influence of unaccounted factors.

Modern trends in the development process require more accurate methods of calculating the parameters of the technical systems that primarily determine the need of usage, the parameters of the power equipment, and ultimately having a significant impact on the cost of project implementation.

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### **The analysis of publications**

The fundamentals theory of parameter calculation of the traction power supply systems on the territory of the post-Soviet space were laid by prof. Marquardt K. G., who are summarized and categorized the possible methods of calculation for traction power supply [1, 2]. The methods of calculation in these works are divided into probabilistic method and the method of using train schedule. The method of calculating the instant schemes play the key role. Prof. Arzhannikov and prof. Marikin made a great contribution in developing the theory of the controlled traction power supply [3, 4].

Any of the researchers are based on the well-known postulate of the current distribution of various schemes of power, without having to summarize and review methodology for calculating direct Instant schemes, which is in modern textbooks on electric power supply [5].

The attempts to summarize and review some aspects, which impede understanding of the essence processes occurring in the traction power supply, were made by the authors of the article. The works concerned the application of space-time formulation of electrical quantities in the traction power supply systems. As well as more accurate accounting of mode voltage consumption of electric rolling constant power and the development of controlled elements in the traction power supply [6-8].

This analysis showed that the problematic part of the calculation of traction power supply systems have difficulty in calculating the instantaneous schemes, methods that are too difficult to be summarized and complicated algorithmization the development of modeling aids, as far as its used cumbersome methods of the theory of graphs and matrices, multipoles or phase coordinates.

### **Problem formulation**

According to the authors of this article referred to the complexity of the traction power supply systems calculations provide the view at the complex systems and processes occurring in it without analysis and generalization of reasons. Thus, the widespread belief that the specificity of the traction load is continuously changing it in time and space, merely reflects the view from the top of the hierarchical system. It is proposed researchers to consider from the bottom to up and speculate regarding the traction load on the assumption that the parameters of the traction power supply vary depending on the location of the load.

Thus, it is proposed to investigate the change in the resistance of traction network, describe the patterns of analytic functions, and develop a fundamentally new method of calculation.

### Enunciate the fundamentals of the method

Suppose that the traction load moving at a constant speed, constant input current along a straight section of the electrified railway. It is well known, that the voltage on the electric current collector is equal to the voltage on the tires traction substation at its location near the substations and will be parabolically decline in its adherence to the middle section. In this case we define the variation of the resistance, designating it  $f(x)$ , the moving traction load.

In the context of two-way separate power scheme (fig. 1, all the nodes are broken), the variation of the resistance will be parabolically, namely

$$f(x) = \frac{r_0 x \cdot r_0 (L - x)}{r_0 x + r_0 (L - x)} = r_0 \left( x - \frac{x^2}{L} \right),$$

where  $r_0$  – is the resistance of 1 km of traction network, Ohm / km;

$L$  – is the length of the traction substations zone km;

$x$  – is the coordinate of the load location, km.

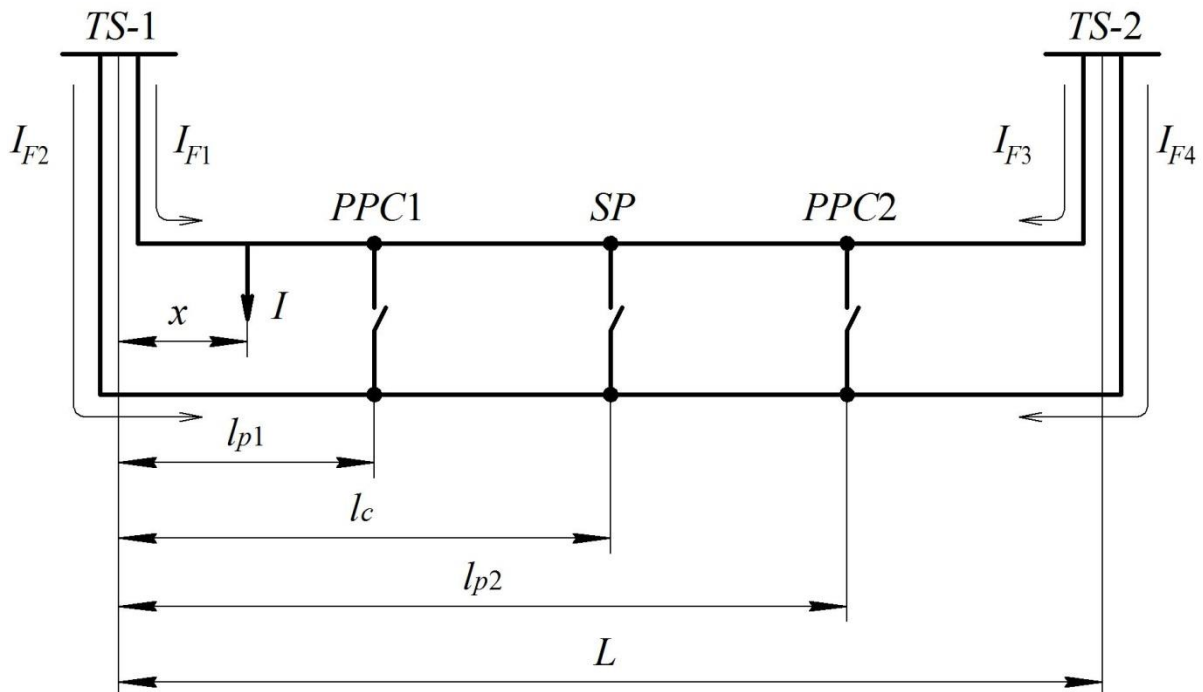


Fig. 1. Generalized computational scheme of traction substations zone

The functions of resistance for the rest scheme of power supply received in [8], providing them without additional mathematical calculations.

The formula for the nodal power scheme (fig. 1., SP node is on)

$$f(x) = r_0 \times \begin{cases} x - \left( \frac{L + l_c}{2Ll_c} \right) x^2, 0 \leq x \leq l_c; \\ \frac{L \left( x - \frac{l_c}{2} \right)}{L - l_c} - \frac{\left( L - \frac{l_c}{2} \right) x^2}{L(L - l_c)}, l_c \leq x \leq L, \end{cases}$$

where  $l_c$  – is the coordinate of the node connection for the contact networks, km.

The parallel scheme of the power supply (fig. 1, PPC1, PPC2, SP nodes are on) will be characterized by a function of the resistance:

$$f(x) = r_0 \times \begin{cases} x - \frac{x^2}{2L} - \frac{x^2}{2l_{p1}}, 0 \leq x \leq l_{p1}; \\ -\frac{x^2 - 2l_c x + l_c l_{p1}}{2l_c - 2l_{p1}} - \frac{l_c x^2 - l_{p1} x^2}{2L(l_c - l_{p1})}, l_{p1} \leq x \leq l_c; \\ \frac{x^2 - 2l_{p2} x + l_c l_{p2}}{2l_c - 2l_{p2}} - \frac{l_c x^2 - l_{p2} x^2}{2L(l_c - l_{p2})}, l_c \leq x \leq l_{p2}; \\ -\frac{(L - x) \cdot (Ll_{p2} - 2Lx + l_{p2} x)}{2L(L - l_{p2})}, l_{p2} \leq x \leq L, \end{cases}$$

where  $l_{p1}, l_{p2}$  – is the coordinates of the first and second points of connection contact networks, km.

Graphically derived resistance functions of the traction network are parabolic dependence (fig. 2), which by means of correction of input data  $l_c, l_{p1}, l_{p2}$  can be adapted to any real-world conditions. In particular, the calculation of asymmetric power scheme, when the distance between the nodes of the parallel connection of contact networks are not the same.

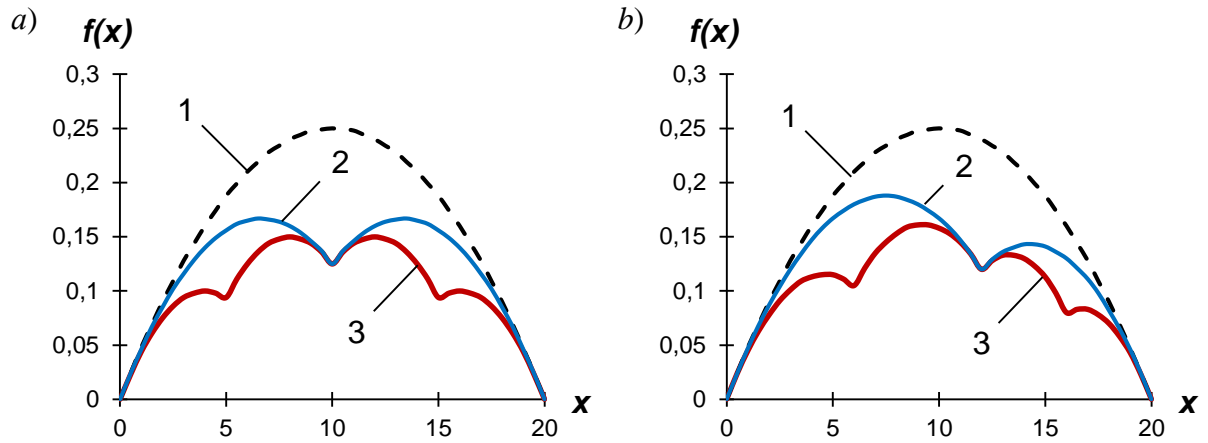


Fig. 2. Graphical representation of functions of resistance for symmetric (a) and asymmetric (b) power schemes: 1 – two-way power scheme; 2 – nodal; 3 – parallel

The theory of calculation the instant schemes of electric power supply specifies the usage of functions for power distribution feeders of traction substations. The physical meaning of these functions is to find the current of each feeder, which is expressed by a part of the load current, depending on its location, for any power supply scheme. Generally, the analytical expression of current distribution functions are determined by the formula:

$$\varphi_i(x) = \frac{I_{Fi}(x)}{I(x)},$$

where  $I_{Fi}(x)$  – is the current of feeder with number  $i$  of power supply scheme, A;

$I(x)$  – is the current of the load, A;

$x$  – is the coordinate of the load location, km.

The common use of these functions are only for calculation of nodal power scheme, but the definition can be easily applied to other schemes.

The analytical expressions for the generalized calculation scheme given below (fig. 1). Two-way power scheme with separate supply tracks will have the following functions of the current distribution for each feeder (fig. 3)

$$\varphi_1(x) = 1 - \frac{x}{L}; \varphi_2(x) = 0; \varphi_3(x) = \frac{x}{L}; \varphi_4(x) = 0.$$

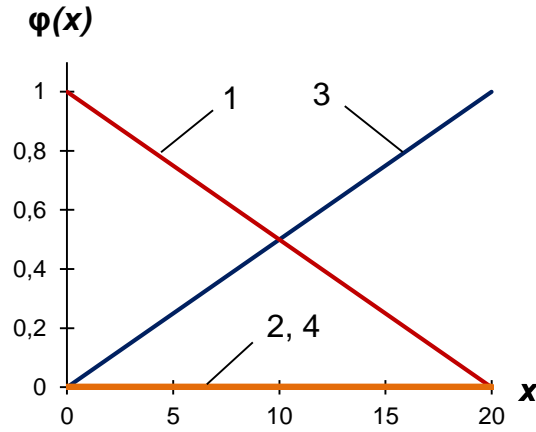


Fig. 3. Current distribution functions for the feeders of traction substation  
in case of two-way power scheme

For the nodal power scheme the following current distribution functions is valid. They graphic representation is shown at the fig. 4.

$$\varphi_1(x) = \begin{cases} 1 - \frac{L+l_c}{2Ll_c} \cdot x, & 0 \leq x \leq l_c; \\ \frac{1}{2} \left( 1 - \frac{x}{L} \right), & l_c < x \leq L; \end{cases} \quad \varphi_2(x) = \begin{cases} \frac{L-l_c}{2Ll_c} \cdot x, & 0 \leq x \leq l_c; \\ \frac{1}{2} \left( 1 - \frac{x}{L} \right), & l_c < x \leq L; \end{cases}$$

$$\varphi_3(x) = \begin{cases} \frac{x}{2L}, & 0 \leq x \leq l_c; \\ 1 - \frac{(2L-l_c)(L-x)}{2L(L-l_c)}, & l_c < x \leq L; \end{cases} \quad \varphi_4(x) = \begin{cases} \frac{x}{2L}, & 0 \leq x \leq l_c; \\ \frac{l_c(L-x)}{2L(L-l_c)}, & l_c < x \leq L. \end{cases}$$

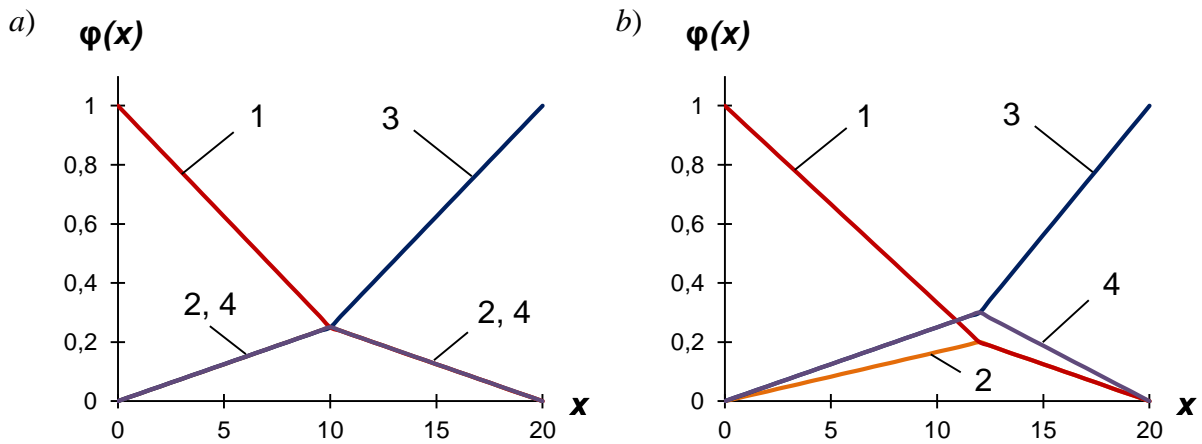


Fig. 4. Current distribution functions for the feeders of the nodal power scheme:  
a – symmetric; b – asymmetric

Finally, a power scheme of full parallel overhead lines connection will be described by the following functions. Distribution functions for the parallel power scheme (fig. 5) is obtained by replacing the coordinates of the sectioning post (SP) on the coordinates of points of the parallel connection (PPC1, PPC2). Notable for this scheme is that the presence of middle-node connections and its location does not affect the current distribution between the feeders of the traction substations. Middle node affects the distribution of currents inside the power scheme and therefore the voltage distribution will be change.

$$\varphi_1(x) = \begin{cases} 1 - \frac{L + l_{p1}}{2Ll_{p1}} \cdot x, & 0 \leq x \leq l_{p1}; \\ \frac{1}{2} \left( 1 - \frac{x}{L} \right), & l_{p1} < x \leq L; \end{cases} \quad \varphi_2(x) = \begin{cases} \frac{L - l_{p1}}{2Ll_{p1}} \cdot x, & 0 \leq x \leq l_{p1}; \\ \frac{1}{2} \left( 1 - \frac{x}{L} \right), & l_{p1} < x \leq L; \end{cases}$$

$$\varphi_3(x) = \begin{cases} \frac{x}{2L}, & 0 \leq x \leq l_{p2}; \\ 1 - \frac{(2L - l_{p2})(L - x)}{2L(L - l_{p2})}, & l_{p2} < x \leq L; \end{cases} \quad \varphi_4(x) = \begin{cases} \frac{x}{2L}, & 0 \leq x \leq l_{p2}; \\ \frac{l_{p2}(L - x)}{2L(L - l_{p2})}, & l_{p2} < x \leq L. \end{cases}$$

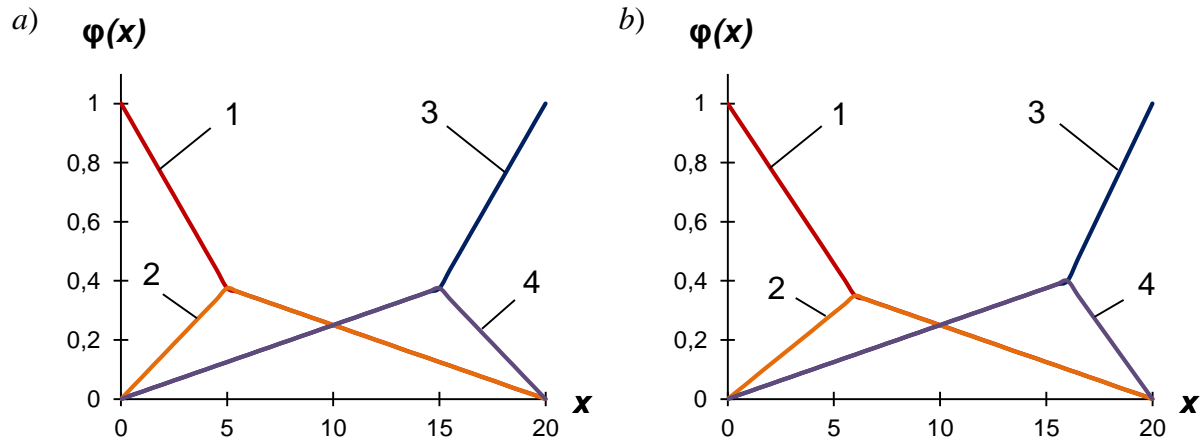


Fig. 5. Current distribution functions for the symmetric (a) and asymmetric (b) parallel power scheme

To complete the theory of calculation need to introducing the definition of resistance function for the feeder, which is the resistance that creates a voltage drop to anywhere in the power scheme for the current of the feeder. Thus, in general form

$$f_F(x) = \frac{\Delta U^{(F)}(x)}{I_F(x)},$$

where  $\Delta U^{(F)}(x)$  – is the voltage drop from the buses of traction substation to the  $x$  coordinate caused by the current of this feeder, V.

If the section of power supply scheme has only one considered feeder current flows, feeder resistance function may be expressed in a particular case by a common function of the resistance and the current distribution function, namely

$$f_{Fi}(x) = \frac{f(x)}{\Phi_i(x)}.$$

Graphical representation the functions of feeder's resistance are shown at the fig. 6-8 in the case of symmetric and asymmetric power supply schemes.

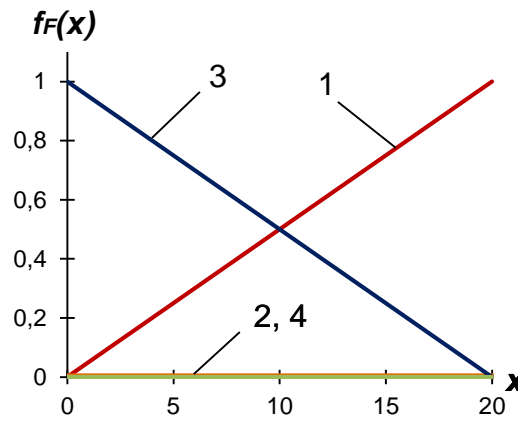


Fig. 6. Resistance functions for the feeders of two-way power supply scheme

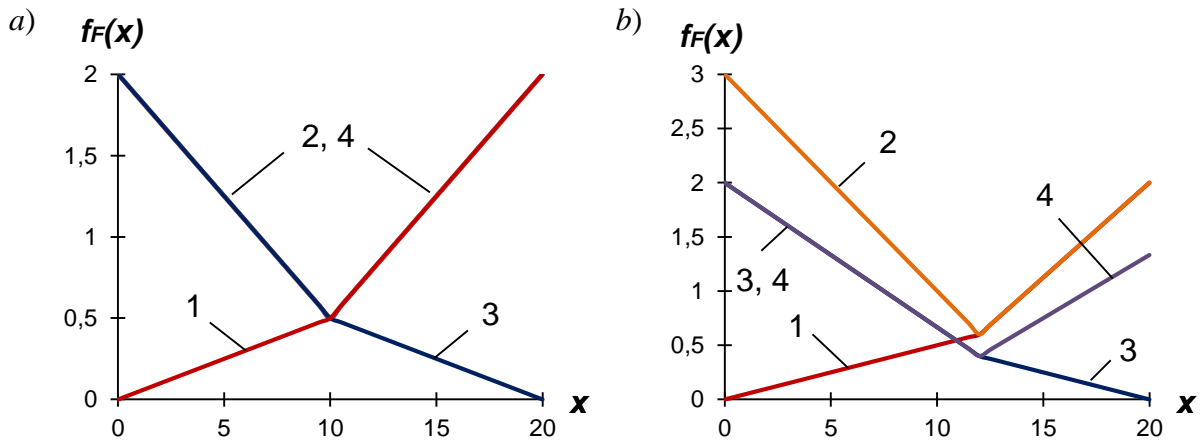


Fig. 7. Resistance functions for the feeders of symmetric (a) and asymmetric (b) nodal power scheme



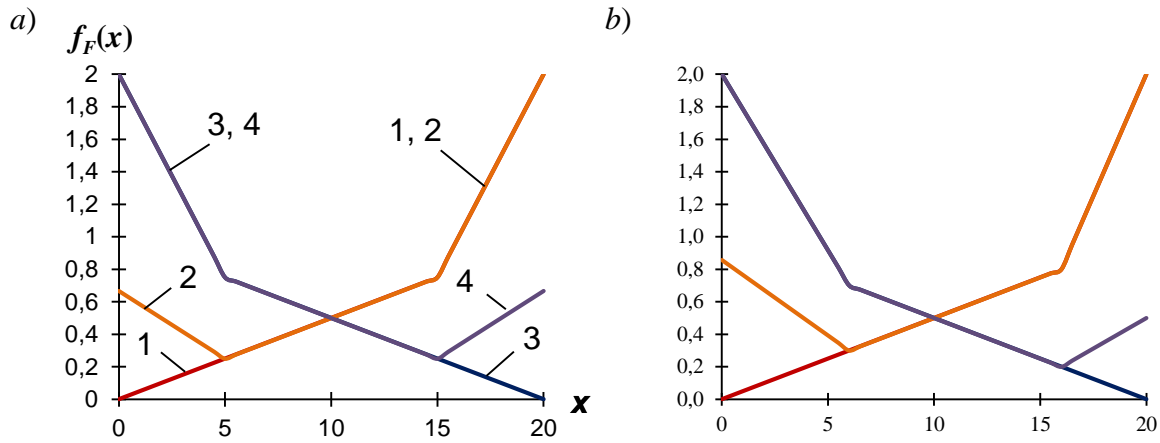


Fig. 8. Resistance functions for the feeders of symmetric (a) and asymmetric (b) parallel power scheme

### Using in calculation

The basic values that required for the calculation of traction power supply systems are feeder's currents of the traction substations, power losses in a power supply scheme and voltages on the pantographs of electric locomotives. Here are the basic expressions that allow numeric calculation these values using the developed method. For simplicity, we will not take into account the internal resistance of the traction substations since electrical calculations for typical electric power devices don't constitute a special difficulty.

Feeder's currents of traction substations when load is moving on the 1-st track determined by multiplying the current traction load on the proper function of the current distribution, as follows from its definition.

$$I_{Fi}(x) = I(x) \cdot \varphi_i(x).$$

For multiple loads in the zone between substations is able to apply superposition the current components for the each load on the feeder.

$$I_{Fi}(x) = \sum_{j=1}^n I_j(x) \cdot \varphi_{ij}(x),$$

where  $j$  – is the number of traction load in the zone between substations;

$n$  – is the quantity of loads in the power scheme.

The voltage at the pantograph of an electric locomotive can be expressed through a common function of resistance

$$U(x) = U_{TS} - I(x) \cdot f(x).$$

This formula is valid only for a single traction load. If there are multiple loads must take into account their mutual influence on passing and depending on the power supply scheme and on the adjacent track. In general, we have

$$U_j(x) = U_{TS} - I_j(x) \cdot f(x) - \sum_{\substack{k=1 \\ k \neq j}}^{n_1} \Delta U'_k(x, x_k) - \sum_{k=n_1+1}^{n_1+n_2} \Delta U''_k(x, x_k),$$

where  $U_{TS}$  – is the voltage at the buses of traction substation, V;

$k$  – is the number of traction load on its track;

$n_1, n_2$  – is the quantities of loads on the 1-st and 2-nd track respectively;

$\Delta U'_k(x, x_k)$  – is the distribution function of the voltage drop for the load with number  $k$  on passing track, V;

$\Delta U''_k(x, x_k)$  – is the same on the adjacent track, V.

The distribution function of the voltage drop (fig. 9) is enough easily formalized with the resistance functions of the feeder. For example, a nodal power supply scheme when the load is on the 1st track will have the following expressions

$$\Delta U'_k(x, x_k) = \begin{cases} I_{F1}(x_k) \cdot f_{F1}(x), & 0 \leq x \leq x_k; \\ I_{F3}(x_k) \cdot f_{F3}(x), & x_k < x \leq L; \end{cases}$$

$$\Delta U''_k(x, x_k) = \begin{cases} I_{F2}(x_k) \cdot f_{F1}(x), & 0 \leq x \leq l_c; \\ I_{F4}(x_k) \cdot f_{F3}(x), & l_c < x \leq L. \end{cases}$$

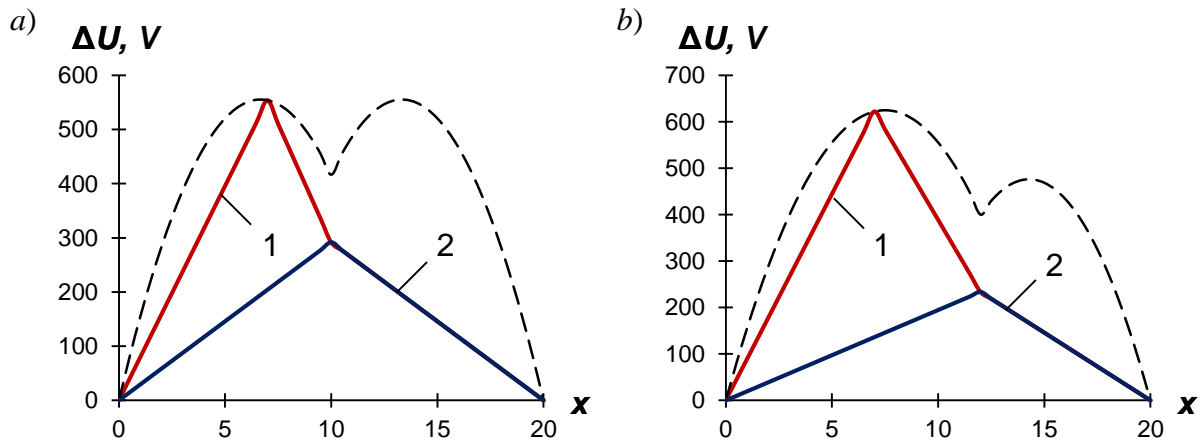


Fig. 9. Voltage distribution functions for the symmetric (a) and the asymmetric (b) nodal power supply scheme: 1 – on the passing track; 2 – on the adjacent track

The power losses in the traction network using presented method may be defined in two ways. Using the current feeders and feeder's resistance function the components of power losses (fig. 10) may be determined, namely

$$\Delta P(x) = \sum_{i=1}^{N_F} I_{F_i}^2(x) \cdot f_{F_i}(x),$$

where  $N_F$  – is the quantity of the feeders, for the considered power schemes  $N_F = 4$ .

This formula will be more convenient if the power losses in a traction network need to be shared between the traction power substations or the feeders. The same result in total may be obtained by the general formula which using the load current and the total resistance function, i.e.

$$\Delta P(x) = I^2(x) \cdot f(x).$$

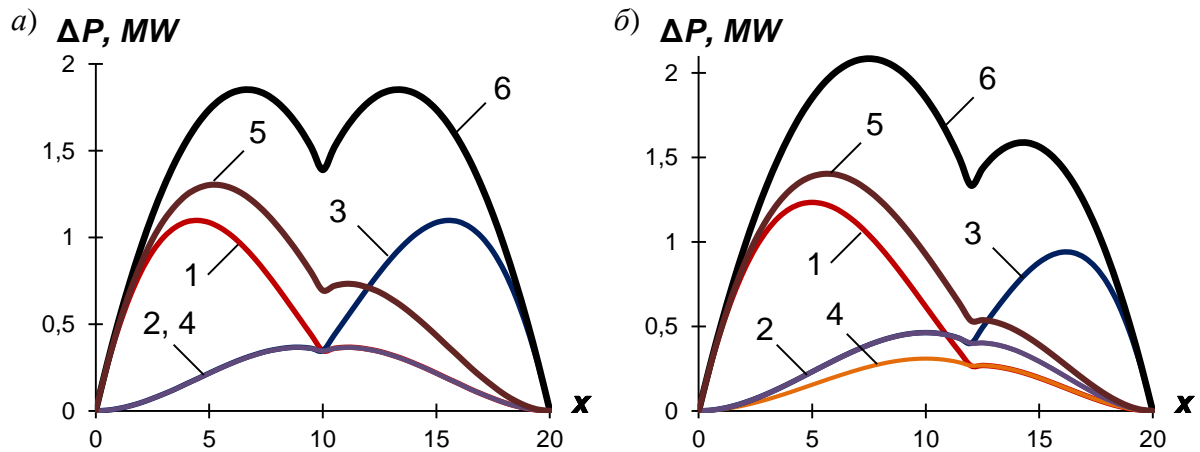


Fig. 10. Power losses for the symmetric (a) and asymmetric (b) nodal power supply scheme: 1-4 – the components appropriate to the feeders of traction substations; 5 – losses that account for a substation; 6 – total power losses in the traction network

## Conclusions

1. Difficulties in calculation of traction power supply systems are caused by the prevailing view on the complex systems and processes occurring in it after the fact without analysis and generalization of reasons. To simplify the calculations necessary to revise the understanding the traction load specifics and reasoning with respect to the traction load on the assumption that the parameters of the traction power supply vary depending on the location of the load.

2. The proposed method of traction power supply calculation lies in describing the resistance of traction network with analytic functions, which together with the functions of the current distribution and feeder's resistance allow formalizing the calculations in electric traction systems.

3. The using of developed calculation method is possible in research projects aimed to optimizing the operating modes of the traction power supply systems using controlled elements and working in a global intelligent power system.

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## **Obliczanie systemów zasilania trakcji przy użyciu funkcji odporności**

### **Streszczenie**

Artykuł poświęcony jest opracowanie nowej metody obliczania chwilowe układów zasilania trakcji systemu. Na podstawie wyników programu projektowania uogólnionej strefie podstacji trakcyjnych pod warunkiem funkcji analitycznych oporu. Korzystanie znane funkcje obecnego podziału funkcji putted koncepcji oporu podstacji trakcyjnych karmniki. Funkcje oporu i bieżącej dystrybucji wykorzystywane w bardziej prosty sposób obliczania bieżących schematach, które pozwalają, aby sformalizować obliczeń elektrycznych systemów zasilania trakcji elektrycznej.

**Słowa kluczowe:** trakcja elektryczna, zasilanie elektryczne, czat schematu, kalkulacja, funkcja odporność, prądy zasilające, straty napięcia, straty mocy.

## **Расчет систем тягового электроснабжения с помощью функций сопротивления**

### **Резюме**

Статья посвящена разработке нового метода расчета мгновенных схем системы тягового электроснабжения. На основании обобщенной расчетной схемы межподстанционной зоны приведены аналитические зависимости функций сопротивления. С использованием известных функций токораспределения введено понятие функций сопротивления фидеров тяговых подстанций. Используемые в совокупности функции сопротивления и токораспределения составляют более простую

методику расчета мгновенных схем, которые позволяют формализовать электротехнические расчеты электротяговых систем.

**Ключевые слова:** электрическая тяга, электроснабжение, мгновенная схема, расчет, функция сопротивления, токи фидеров, падение напряжения, потери мощности.