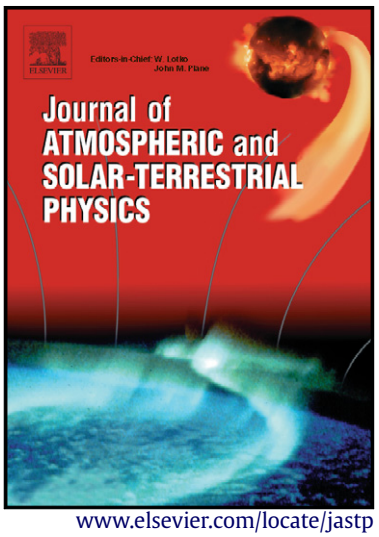

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ESTIMATING THE LOWER IONOSPHERE HEIGHT AND LIGHTNING LOCATION USING MULTIMODE "TWEEK" ATMOSPHERICS

A.V. Shvets[1], T.M. Serdiuk[2], Y.V. Gorishnyaya[1],Y. Hobara[3], and M. Hayakawa[3]

[1] O. Ya. Usikov Institute for Radiophysics and Electronics, of the National Academy of Sciences of Ukraine, 12 Akademika Proskury St., Kharkov 61085, Ukraine

[2] Lazaryan's National University of Railway Transport, 2a Akademika Lazaryana St., Dnepropetrovsk 49010, Ukraine

[3] The University of Electro-Communications, 1-5-1 Chofugaoka, Chofu Tokyo, 182-8585, Japan

Abstract There is proposed a new method of estimating the effective ionospheric height of the Earth-ionosphere waveguide and the propagation distance of tweek-atmospherics. It is based on the compensation of waveguide frequency dispersion of a tweek signal, which enables us to improve the accuracy of deducing the cutoff frequencies, especially in the presence of noise. An approach to solve the inverse problem is suggested that reduces the task of finding both the source range and the waveguide cutoff frequencies by using the multimode characteristics of tweeks to an issue of one-dimensional optimization. Based on the numerical modeling of multimode tweek-atmospherics in the Earth-ionosphere waveguide with exponential vertical conductivity profile of the lower ionosphere, it was shown that the accuracy of estimating the effective waveguide height by the new method is good as about 100-400 m for the first and higher order modes. It then allows us to estimate the parameters of vertical conductivity profile of the lower ionosphere for a wide range of source distances from a few hundred to a few thousand kilometers, as long as two or more tweek harmonics can be detected. Preliminary analysis of experimental tweek records show a decrease of the effective height with increasing the mode number, and the scale height of the exponential vertical conductivity profile for the isotropic lower ionosphere model is estimated to be in a range of 0.4 – 2.5 km.

Keywords: Earth-ionosphere waveguide, tweek-atmospherics, lightning location, lower ionosphere diagnostics.

1. Introduction

Tweek-atmospherics are electromagnetic pulses with duration of 10 – 100 milliseconds, propagating in the natural Earth-ionosphere waveguide, which represent a response of the natural cavity to lightning discharges. These tweeks are formed due to small losses during night time in the ionosphere at altitudes from about 85 to 95 km, which effectively reflects ELF-VLF electromagnetic waves. A study of this altitude range in the ionosphere meets essential difficulties due to relatively small electron densities (10 – 1000 cm^{-3}), and so these tweeks are considered to be a useful natural means for radiosounding the lower ionosphere in a wide frequency band.

Approximation of a flat infinite waveguide with perfectly conducting walls is known to quite well describe the waveguide dispersion effects observed in the experiment, especially near the cutoff frequencies [*Outsu*, 1960]. Based on the waveguide propagation theory, different methods of analysis of tweeks were elaborated to estimate the lower ionosphere effective height, together with the propagation distance. Analyses of instant frequency variations of tweek signals [*Outsu*, 1960; *Yano et al*., 1989; *Shimakura et al*., 1992; *Hayakawa et al*., 1994, 1995; *Ohya et al*., 2008], phase spectrum of the longitudinal magnetic field component [*Rafalsky et al*., 1995], interference between the zeroth and the first order modes in the tweek amplitude spectrum [*Shvets and Gorishnyaya*, 2010] were employed in the frequency domain around the first waveguide cutoff frequency to find both the effective waveguide height and source-to-observer distance. Further spectral peculiarities of long-range tweek atmospherics were studied theoretically and experimentally in *Mikhailova and Kapustina* (1988), and *Yedemsky et al.* (1992). But scare attention was devoted experimentally to the use of multimode tweeks that represent the higher order modes of the Earth-ionosphere waveguide, except

some numerical studies [*Cummer et al*., 1998; *Cummer*, 2000] in which they extended the frequency range of analysis and obtained more detailed information on the lower ionosphere.

The cutoff frequency defined for a particular waveguide mode can be related to a certain altitude in the ionosphere which can be considered as an effective waveguide height. Because the depth of penetration of electromagnetic waves into the ionospheric plasma depends on wave frequency, the effective waveguide height is known to change for different modes. So, determination of the cutoff frequencies for higher order modes of the Earth-ionosphere waveguide by using tweek atmospherics, will provide us with the detailed parameters of the lower ionosphere.

An approximate theory has been developed which describes the tweek formation under an anisotropic, both homogeneous [*Yedemsky et al*., 1992; *Ryabov*, 1992; *Sukhorukov et al*., 1992a] and inhomogeneous [*Sukhorukov et al*., 1992b], ionosphere near the cutoff frequencies. These theories predict the deeper penetration of the lower frequency waves into the ionosphere that leads to a decrease in the effective reflection height for higher order modes. The similar behavior for the higher order modes is also obtained from the simplified theory [*Porrat et al*., 2001] based on the model of an exponential vertical conductivity profile of the isotropic lower ionosphere, which describes satisfactorily spectral peculiarities of nighttime atmospherics in the frequency range up to 5 kHz. *Shvets and Hayakawa* (1998) considered the altitude where the full reflection condition is fulfilled in the nighttime ionosphere for the extraordinary left-hand polarized waves, which form tweeks near the cutoff frequencies [Hayakawa et al., 1994]. It implies that the effective height increases with an increase in the mode number. This was confirmed by our initial estimations [*Shvets and Hayakawa*, 1998] and by recent experimental studies of the higher harmonic tweeks (see e.g. *Maurya et al*., [2012] and references therein), but it contradicts to the theoretical consequence by *Ryabov* (1992) and *Sukhorukov et al*. [1992a]. Thus the evaluation of accuracy of existing techniques and development of new techniques to determine the cutoff frequencies of higher harmonic tweeks is of great interest in interpreting the experimental data.

The waveform of tweeks in the range of waveguide cutoff frequencies at ELF-VLF is resulted mainly from the dispersion properties of the Earth-ionosphere waveguide and the propagation distance. While the amplitude spectrum of a tweek exhibits a complicated structure due to the result of interference between different waveguide modes and depends on the distance, its dynamic spectrum enhances the temporal evolution of instantaneous frequencies of the signal so that we are able to identify one to up to ten tweek harmonics corresponding to the first and higher order waveguide modes.

The inverse problem to find the cutoff frequencies and the source range by using higher tweek harmonics can be solved separately for each mode. Sometimes in the experiment it leads to confusing results when estimating the source range made by different tweek harmonics, which are found to be quite different even for the same tweek. We here propose an approach to solve this problem simultaneously for all harmonics of a detected tweek that reduces the solution to an issue of one-dimensional optimization. To improve the accuracy of deducing the cutoff frequencies with higher tweek harmonics we propose a transformation that nonlinearly "stretches" a tweek waveform along the time axis to compensate the waveguide dispersion for a given source range. This allows us to enhance the frequency resolution of spectral analysis and hence to obtain more accurate estimations of the cutoff frequencies. The techniques proposed were realized programmatically and tested by the numerical simulation. Also they were applied to some examples of experimental records of tweek-atmospherics.

2. Earth curvature and dispersion relations

We consider the formation of tweek atmospherics with taking account of the Earth's curvature. The scheme of propagating rays from a source (S) to the receiver (R) between the curved Earth and ionosphere boundaries is shown in Fig.1, which is based on the ray approach (Yano et al., 1989). The path element of a ray propagating between the Earth surface and the ionosphere is given by,

$$l_n^{sphere} = \left(a^2 + (a+h)^2 - 2a(a+h)\cos\theta_n\right)^{1/2}, \quad \theta_n = \frac{\rho}{2na}, \tag{1}$$

where n is the ray number, a (=6371 km) is the Earth's radius, h is the waveguide height, and ρ is the source-to-observer distance along the Earth surface.

Using an approximation for small angular distances $\cos\theta_n \approx 1 - \frac{\theta_n^2}{2}$ and letting the Earth radius a to infinity, we obtain from Eq. (1) the ray path element for the flat waveguide:

$$l_n^{flat} = \sqrt{h^2 + \left(\frac{\rho}{2n}\right)^2} \tag{2}$$

The total path is multiplied by $2n$ of the path element, and then the propagation time of the n^{th} ray is given by:

$$t_n = \frac{2nl_n}{c}, \quad n = 1,2,3,... \tag{3}$$

For the flat waveguide model an analytical expression for the instant frequency can be obtained from Eqs.(2,3). The time of arrival of the n^{th} ray to the observer is given by:

$$t_n = \frac{1}{c}\sqrt{(2nh)^2 + \rho^2}, \tag{4}$$

and its derivative by n defines the instant frequency:

$$\frac{1}{f(t_n)} = \frac{dt_n}{dn} = \frac{4nh^2}{c\sqrt{(2nh)^2 + \rho^2}}. \tag{5}$$

Substituting $n = \frac{1}{2h}\sqrt{(ct_n)^2 - \rho^2}$ of Eq.(5) to this equation and changing t_n to the continuous time, we obtain the known dependence for the instant frequency (see e.g. *Yano et al.*, 1989):

$$f(t) = \frac{c}{2h}\frac{1}{\sqrt{1 - \left(\frac{\rho}{ct}\right)^2}} \tag{6}$$

This expression is obtained also from the frequency dispersion relation for the group velocity of the first waveguide mode [*Hayakawa et al.*, 1985]. The waveguide mode theory yields that the

123 value $f_c = c/(2h)$ in Eq.(6) is the cutoff frequency of the first mode. The cutoff frequency for higher
124 order modes can be simply obtained by dividing the waveguide height by the mode number m:

125 $$f_{cm} = \frac{mc}{2h} \tag{7}$$

126 Let $\tau = t - \frac{\rho}{c}$ be the time from the moment of arrival of the direct ray to the observer, then
127 the instant frequency of Eq.(6) for the m^{th} order mode is given by (*Yano et al*., 1989):

128 $$f_m(\tau) = \frac{f_{cm}}{\sqrt{1-\left(\frac{\rho}{\rho+c\tau}\right)^2}} \tag{8}$$

129 To compare the frequency dispersion in the spherical and flat models of the waveguide we
130 use Eqs.(1-3). We define the instant frequency related to the middle of the interval between two
131 successive rays: $T_n = (t_{n+1}+t_n)/2$ as a reciprocal of the delay between their arrivals:

132 $$f(T_n) = \frac{1}{t_{n+1} - t_n}. \tag{9}$$

133 Fig.2(a) illustrates the calculation results of Eq.(9) for the first three modes, as plotted
134 versus time $\tau_n = T_n - \rho/c$. The cutoff frequencies are shown by the horizontal dotted lines. The
135 curves calculated for the spherical and flat waveguide models are combined, but the difference
136 between them is hardly recognized in the graph. Fig.2(b) shows that the curves are the differences
137 of instant frequencies $\Delta f = f_{\text{sphere}}(\tau_n) - f_{\text{flat}}(\tau_n)$. All the curves are calculated for distances ρ =
138 1,2,3,4,5 Mm with the same waveguide height h = 88 km.

139 To use analytical representation of the instant frequency by Eq.(8) for the flat waveguide in
140 the analysis of experimental data, we consider the "tail" part of a tweek waveform starting with delay
141 τ_0 from the beginning of the tweek. By limiting the maximal deviation between the instantaneous
142 frequencies by 10 Hz as seen in Fig.2(b) for practical use, we infer the following dependence of τ_0 on
143 the source range:

144 $$\tau_0 \text{ [ms]} \cong 2\rho_0 \text{ [Mm]}. \tag{10}$$

The initial estimation of the source range ρ_0 can be obtained by any appropriate known method. Exclusion of the "head" part of the tweek with fast varying instant frequency as is seen in Fig.2(a), will also facilitate the spectral-time analysis of experimental data.

3. Principles of estimation of ionospheric height and propagation distance of multimode tweek-atmospherics

Let $f_m(\tau)$ be the instant frequency of the m^{th} tweek harmonic extracted from the tweek trace in the spectrogram. The cutoff frequency can be estimated from Eq.(8):

$$F_{cm}(\rho_1, \tau) = f_m(\tau)\sqrt{1-\left(\frac{\rho_1}{\rho_1 + c\tau}\right)^2}\,. \tag{11}$$

Approaching ρ_1 in Eq.(11) to ρ, the propagation distance in the waveguide, will converge this dependence to a horizontal straight line at the cutoff frequency of $F_{cm}(\rho, \tau) = f_{cm}$. In order to find this solution we use a procedure of minimization of the absolute value of the slope b_m of the linear regression line ($r_m = a_m + b_m\tau$) for the dependence $F_{cm}(\rho_1, \tau)$ or minimization of the standard deviation σ_m from the mean value of $<F_{cm}(\rho_1, \tau)>$.

When analyzing a multimode tweek we calculate $|b_m|$ for each of higher order tweek harmonics (Eq.(11)) separately and minimize the mean of the absolute values of the slope coefficients:

$$\Phi(\mathbf{b}) = \frac{1}{M}\sum_{m=1}^{M}|b_m|\,. \tag{12}$$

The similar functional can be constructed to minimize the sum of the standard deviations of estimating the cutoff frequency in each mode.

In such a way that the inverse problem to find the two or more unknown parameters ρ and f_{cm} is reduced to the issue of one-dimensional optimization that gives us simultaneously all the cutoff frequencies for higher order modes and the source range ρ. Once we find the cutoff frequencies, the effective waveguide height for each mode is found from Eq.(7):

$$h_m = \frac{mc}{2f_{cm}} \tag{13}$$

As was mentioned above there are some problems for the accurate measurement of the instant frequency of tweek-atmospherics, and one of them is the fast changing frequency in the beginning of a tweek signal. First, a method based on the compensation of the frequency dispersion in a tweek waveform was proposed by *Shimakura et al*. (1992) and *Hayakawa et al*. (1995). These authors constructed a "pseudo-spheric" which is a complex signal with the frequency modulation determined by the waveguide frequency dispersion defined by Eq.(6), and they multiplied it with the experimental record of the tweek. The amplitude spectrum of the resulting complex signal contains peaks at combination frequencies: the difference frequency and the summation frequency between the two multiplied signals. The difference frequency becomes equal zero and a straight line appears at zero frequency in the spectrogram after low pass filtration when the distance and waveguide height are found correctly and the instant frequencies of a tweek and the corresponding pseudo-spheric coincide with each other.

In this paper we consider this kind of transformation of the waveform of a tweek to compensate the waveguide frequency dispersion simultaneously for all harmonics by nonlinearly scaling a tweek along the time axis.

The arrival time of the reflected n^{th} ray with respect to the direct ray in the flat infinite waveguide, is given from Eq.(4):

$$\tau_n = \sqrt{(2nh/c)^2 + (\rho/c)^2} - \rho/c\,. \tag{14}$$

Let $t_n = 2nh/c$ be the arrival time of n^{th} ray if the propagation were exactly perpendicular to the waveguide boundaries. Substituting t_n in Eq.(14) we can express t_n as a function of τ_n:

$$t_n = \sqrt{\tau_n^2 + 2\tau_n(\rho/c)} \tag{15}$$

The moments t_n are proportional to n and it is obvious that they are equidistantly distributed along the time axis. By changing discrete quantities t_n and τ_n to continuous times t and τ in Eq.(15) we obtain the following continuous transformation of time:

$$t_s = \sqrt{\tau^2 + 2\tau(\rho/c)} \tag{16}$$

The resultant transformation stretches the initial tweek waveform $w(\tau)$ and it is calculated from the following expression:

$$w_s\left[t_s(\tau,\rho)\right] = w(\tau)\,. \tag{17}$$

After the transformation we expect maximization of peak amplitudes at the cutoff frequencies in the amplitude spectrum of the stretched tweek w_s resulted from the equalization of delays between successive rays.

4. Inverse problem solution for model atmospherics

To evaluate the effectiveness of our methods of estimation of the cutoff frequencies of the Earth-ionosphere waveguide and the distance to a lightning discharge, tweek waveforms have been synthesized using a simplified model of ELF-VLF propagation proposed by *Porrat et al*. (2001). The horizontal magnetic component excited by a vertical electric dipole on the perfectly conducting Earth's surface is expressed as the following modal sum [*Wait*, 1962]:

$$H_\phi = j\frac{kIds}{2h}\sum_{m=0}^{\infty}\delta_m S_m H_1^{(2)}(kS_m\rho)\,, \tag{18}$$

where *Ids* is the dipole current moment, h is the ionospheric height, k is the wavenumber in free space, $H_1^{(2)}$ is the Hankel function of the second kind of the 1st order, S_m are the modal eigenvalues, and δ_m, excitation factors. The source current moment is adopted as a single exponential pulse with the spectrum $Ids(\omega) = \dfrac{I_0\tau_1 ds}{1+j\omega\tau_1}$ with appropriate parameters: ds = 4km, I_0 = 20kA, and τ_1 = 40μs. The SI units are used in the formulas throughout the paper unless explicitly mentioned.

The upper boundary, the ionosphere is described by an exponential conductivity profile with a single scale height [*Wait and Spies*, 1964]:

$$\sigma(z) = 2.5\times10^5\,\varepsilon_0 e^{(z-H)/\zeta_0} \quad \text{[S/m]} \tag{19}$$

where ε_0 is the dielectric permittivity of vacuum, ζ_0 is the local scale height, z is the altitude above the ground, and H is the characteristic height of the profile.

For such a conductivity profile *Greifinger and Greifinger* [1978] have shown that the propagation in ELF range is determined by the lower "electrical" and higher "magnetic" heights. The lower altitude h_0 is the height at which the conduction current parallel to the magnetic field becomes equal to the displacement current, $\sigma(h_0) = \omega\varepsilon_0$. The upper altitude h_1 is the height at which the wave-like propagation turns into the diffusive penetration. It is determined by the condition equality of the local wave number to the reciprocal of the local scale height ζ_0 of the refractive index, $4\omega\mu_0\sigma(h_1)\zeta_0^2 = 1$, which is considered as a "reflection altitude" [*Sentman*, 1990]. The frequency dependences for the altitudes h_0 and h_1 are obtained from the above conditions [*Nickolaenko and Rabinowicz*, 1982; Nickolaenko and Hayakawa, 2002]:

$$h_0 = H - \zeta_0 \ln\left(\frac{2.5\times10^5}{2\pi f}\right)$$
$$h_1 = h_0 + 2\zeta_0 \ln\left(\frac{2.39\times10^7}{f\zeta_0}\right) = H + \zeta_0 \ln\left(\frac{1.44\times10^{10}}{f\zeta_0^2}\right). \quad (20)$$

For the TEM mode in Eq.(18) $h = h_0(f)$, $\delta_0 = 1$, and the eigenvalue S_0 is given by [*Greifinger and Greifinger*, 1978]:

$$S_0^2 = \frac{h_1 + j\frac{\pi}{2}\zeta_0}{h_0 - j\frac{\pi}{2}\zeta_0} \quad (21)$$

The higher order modes of tweeks are represented mainly by quasi-TE modes resulted from the coupling of TM waves excited by a vertical dipole into TE waves due to the reflection from the anisotropic ionosphere during night time [*Yamashita*, 1978; *Sukhorukov* et al., 1992; *Sukhorukov*, 1996]. For the higher order modes in Eq.(18) $h = h_1(f)$, the eigenvalues and the excitation factors are given by [*Porrat et al*., 2001]:

$$S_m \approx s_m + j\frac{\pi\zeta_0}{2h_1}\frac{c_m^2}{s_m}, \quad \delta_m \approx 2\frac{c_m^2}{s_m}, \quad f > \sqrt{2}f_{cm}$$
$$S_m \approx s_m + j\frac{\pi\zeta_0}{2h_1}s_m, \quad \delta_m \approx 2s_m, \quad f < \sqrt{2}f_{cm} \tag{22}$$

where f_{cm} is the cutoff frequency of m^{th} mode, $c_m = m\pi / kh_1$ and $s_m = \sqrt{1-c_m^2}$ are the modal cosine and sine.

The waveforms of tweeks were obtained by applying the inverse FFT to the modeled spectra. In the further modeling we calculated the spectra of magnetic field component consisting of nine modes including the zero order one.

An example of such a synthesized waveform and its amplitude and dynamic spectrum are shown in Fig.3(a-c). The model parameters are adopted as follow: H = 88 km, ζ_0 = 1.67 km, ρ = 1600 km.

In order to retrieve the time dependences of the instant frequencies of tweek harmonics we calculated a moving-window spectrum of the tweek waveform with 5.12 ms Hamming window length with a step of 0.5 ms starting from the moment delayed by τ_0 from the pulse onset calculated by Eq.(10). The starting point τ_0 is marked by a triangle on the lower time axis. The peak frequencies were defined more accurately by means of interpolation with a parabola inscribed to the point of a spectral maximum and two adjacent points. The inferred peaks in the dynamic spectrum corresponding to the first five higher order modes are shown in Fig.3(c) by small circles.

The stretched waveform transformed with Eqs. (16) and (17) and its amplitude spectrum are illustrated in Fig.4. To apply the FFT to the data we uniformly resampled the stretched tweek waveform using linear interpolation. We can observe that the intermodal interference practically disappears and the spectrum of the transformed tweek waveform consists of the peaks at the cutoff frequencies corresponding to separate modes.

The pairs of instant frequencies $f_m(\tau)$ and corresponding delays τ retrieved from a dynamic spectrum are used to estimate cutoff frequencies in the first method described in Section 3. Fig. 5 demonstrates the fitting procedure to obtain the effective waveguide from higher order tweek

harmonics of the synthesized tweek waveform shown in Fig.3. Instead of estimating cutoff frequencies from Eq.(12) in this figure we plot the effective waveguide heights given by: $h_m(\rho, \tau) = cm/2F_m(\rho, \tau)$. The model effective heights $h_1(f_{cm})$ theoretically calculated, are shown by the horizontal dotted lines labeled with the corresponding mode numbers. Cases of an underestimated propagation distance, ρ = 1500 km, the exact propagation distance, ρ = 1600 km, and un overestimated propagation distance, ρ = 1700 km are treated in Fig.4(a,b,c) respectively.

We can observe that the points are to be grouped horizontally around certain altitudes when the source-to-observer distance is set correctly. The parameter h_1 is logarithmically decreasing with an increase with frequency as it follows from Eq.(20) and then the effective waveguide height $h_1(f_{cm})$ decreases with increasing the mode number in the adopted model of propagation for the synthesized tweek. Note that underestimation of the source range leads to underestimation of the waveguide height and vice versa.

For the stretched tweek shown in Fig.4, we demonstrate the second method to determine the waveguide height and the propagation distance. With the help of the proposed transformation we compensate the frequency dispersion in the individual modes, when the parameter ρ equals to the factual value of the source range. For the stretched waveform, we can extend the length of a window to construct a moving-window spectrum and hence we can improve the estimation of the cutoff frequency and of the source range and the waveguide height.

We demonstrate the fitting procedure for the stretched tweek waveform so as to obtain the effective waveguide height for higher order modes by varying the source range ρ in Fig.6 just like that shown in Fig.5 in the case of the first method. The cases of an underestimated range, ρ = 1500 km, the exact range, ρ = 1600 km, and un overestimated range, ρ = 1700 km are shown in Fig.6(a,b,c) respectively. We can observe a smaller deviation in the estimations of effective heights in comparison with the nonstretched one (see Fig.5(a-c)) due to a larger moving window length, of about 15 ms used for constructing a dynamic spectrum.

5. Estimation of the accuracy and application to experimental records

In order to evaluate the accuracy of our proposed methods we synthesized atmospherics waveforms of 20 ms length for source range from 500 to 3500 km. For these distances we expect to observe two or more harmonics in the experimental records of tweeks [*Outsu*, 1960;*Gorishnya and Shvets*, 2010] that can be used for estimating the cutoff frequencies of the Earth-ionosphere waveguide. Together with the first and second methods described earlier for comparison we also apply the third "sonogram" method (see e.g. *Ohya et al*., 2008) based on the optimization of two parameters in such a way that the theoretical dependence of the instantaneous frequency Eq.(18) fits to the trace in the spectrogram of a tweek. In the last case we determine a pair of parameters, the source range and the waveguide height, separately for each tweek of harmonics.

In the simulation a white noise was generated by random values following a normal distribution with zero mean and standard deviation 0.2 of the standard deviation σ of the analyzed part of a tweek signal. For each distance of 500, 1500, 2500 and 3500 km we synthesized tweek waveforms with the following parameters of the ionosphere conductivity profile: $H = 88$ km, $\zeta_0 = 1.67$ km. Then noise realizations generated independently were added to them and the required parameters were calculated. This process was repeated 100 times for each distance. Tables 1 and 2 present the overall systematic errors calculated as a difference between the mean value of 100 tests and the model parameter together with the standard deviations for the found distances and heights for different modes.

We can see from the tables that the errors mainly decrease with an increase in the mode number for all methods, which can be connected with an increase in the relative accuracy of the frequency estimation for the higher order modes. As it follows from Tables 1 and 2, while the first and third methods exhibit the concurring accuracy, the second method is found to demonstrate about two times better results in the estimation of the source range and waveguide height.

Examples of estimation of the effective height and the propagation path length by using experimental records of tweeks are demonstrated in Fig.7. Two horizontal magnetic and a vertical

313 electric components, received in the frequency range 0.3 – 13 kHz, were recorded digitally with the
314 sampling frequency of 100 kHz. The records were made at different places onboard the scientific
315 vessel "Academician Vernadsky" in 1991 (more experimental details are given in *Shvets and*
316 *Hayakawa* [1998]).

317 Analysis results of the experimental records are presented with the use of the second technique
318 described above. Tweek waveforms of magnetic components were decomposed into transversal and
319 longitudinal components with reference to the source direction (see *Rafalsky et al*. 1995) and we
320 analyzed the longitudinal component which is composed only by higher-order modes. Fig.7 illustrates
321 the waveforms, amplitude spectra and estimated effective heights for different waveguide modes of
322 the tweeks recorded at 5.55º E, 16.7º S on January 21, 1991 (a, b) and at 3.6º W, 8º S on April 10,
323 1991(c, d). We also indicate the parameters of the exponential ionospheric conductivity profile H and
324 ζ_0, the distance to the source ρ and the source azimuth α counted clockwise from North direction are
325 shown below the corresponding graphs. The circle points with the standard deviation bars denote
326 experimentally found effective heights on the right-most graphs and the solid line is the fitted
327 theoretical dependence of h_1 Eq.(20) which yields the estimated parameters H and ζ_0. We have chosen
328 two tweeks arrived from approximately the same direction but from different distances on each day.
329 We should note that in the examples presented of tweek analysis the waveguide height decreases with
330 increasing the mode number, which is basically in agreement with the models by Sukhorukov et al.
331 [1992a,b] and Porrat et al. [2001]. It is also found that estimations of the waveguide height and the
332 reconstructed conductivity profile parameters are quite close for each pair of tweeks that can indicate
333 quiet ionospheric conditions along the corresponding propagation paths.

334

335 6. Summary and conclusion

336 Both of the proposed techniques for estimating the waveguide cutoff frequencies described in
337 the previous two sections are based on linearizing transformations that compensate the waveguide
338 frequency dispersion of tweek harmonics.

Algorithms of the estimations of Earth-ionosphere waveguide parameters for the above-described techniques are formulated as follows:

First technique:

1) calculation of dynamic spectrum;

2) selection of tweek harmonics;

3) minimization of the functional (12) by seeking an optimal value of the propagation distance, that includes the calculation of the slope of the regression lines of the cutoff frequency estimations in selected tweek harmonics Eq.(11).

Second technique:

1) minimization of functional (12) by seeking an optimal value of the propagation distance, that includes:

i) rescaling the tweek waveform with a new value of ρ by transformations of Eqs. (16, 17);

ii) calculation of dynamic spectrum;

iii) selection of tweek harmonics;

iv) calculation of the slope of the regression lines of the cutoff frequency estimations in selected tweek harmonics.

As we can see, the second algorithm requires much more computer resource and so is time-consuming in comparison to the first method. It includes rescaling a tweek, calculating its dynamic spectrum and selecting tweek harmonics in each cycle of seeking an optimal value of ρ, whereas the calculation of the dynamic spectrum and selecting tweek harmonics are performed only once in the first algorirhm. However, the second algorithm has essential advantage in the estimation accuracy of waveguide parameters due to an increased frequency resolution of the moving-window spectral processing.

As a conclusion of this study we note the following.

An approach has been proposed that reduces the problem of finding two or more unknown parameters, source range and cutoff frequencies, by using higher harmonic tweeks to an issue of one-dimensional optimization.

A new technique has been proposed, based on the nonlinear scaling of a tweek waveform along the time axis that compensates the waveguide frequency dispersion in the signal and allows us to improve the accuracy of estimation of the waveguide cutoff frequencies, especially in the presence of noises.

Limitations of the use of dispersion relation for a flat waveguide model to the spherical waveguide have been elaborated.

Based on the numerical modeling of multimode tweek-atmospherics in the nocturnal Earth-ionosphere waveguide with exponential vertical conductivity profile of the lower ionosphere (Porrat et al., 2001), it was shown that the accuracy of estimating the effective ionosphere height is good enough, about 100-400 m for the first and higher order modes in a case of signal-to-noise ratio of being 5 in the modeled tweeks. It enables us to deduce the parameters of vertical conductivity profile of the lower ionosphere for a wide range of source-to-observer distance from a few hundred to a few thousand kilometers, as long as two or more tweek harmonics can be detected.

Examples of analyses of experimental records of tweeks are found to indicate a decrease in the effective height of the waveguide with increasing the mode number. Estimated scale height of the exponential conductivity profile of the isotropic lower ionosphere model is found to be in a range of 0.4 – 2.5 km.

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454 Figure and Table captions

455 Fig.1. Ray paths in the spherical waveguide (Adapted from Yano et al., 1989).

456 Fig.2. (a) Calculated traces of a tweek and (b) difference of instant frequencies $\Delta f = f_{\text{sphere}}(\tau) - f_{\text{flat}}(\tau)$
457 between the spherical and flat waveguide models .

458 Fig. 3. Waveform (a), amplitude (b), and dynamic spectrum (c) of a modeled tweek-atmospheric
459 calculated for the following parameters: H = 88 km, ζ_0 = 1.67 km, ρ = 1600 km. The vertical dotted
460 lines in the amplitude spectrum (b) indicate the cutoff frequencies.

461 Fig. 4. (a) The stretched waveform, and (b) its amplitude spectrum of the modeled tweek-atmospheric
462 shown in Fig.3. The vertical dotted lines on the amplitude spectrum (b) indicate the cutoff
463 frequencies.

464 Fig. 5. Estimation of the effective waveguide heights for higher order modes, m=1…5 from (Eq.(6))
465 of a modeled tweek-atmospheric shown in Fig 3(a). Horizontal dotted lines show the effective heights
466 calculated from the model. Three cases are dealt with; (a) underestimated range, ρ = 1500 km, (b)
467 exact range, ρ = 1600 km, (c) overestimated range, ρ = 1700 km.

468 Fig. 6. Estimation of the effective heights of the Earth-ionosphere waveguide obtained from the
469 stretched tweek waveform shown in Fig.4(a). Horizontal dashed lines show the effective heights for
470 the corresponding modes. a) underestimated range, ρ = 1500 km, b) exact range, ρ = 1600 km, c)
471 overestimated range, ρ = 1700 km.

472 Fig.7. Waveforms, amplitude spectra and estimated effective heights for different waveguide modes
473 for the tweeks recorded at 5.55º E, 16.7º S on January 21, 1991 (a, b) and at 3.6º W, 8º S on April 10,
474 1991(c, d). Estimated parameters of the exponential ionospheric conductivity profile H and ζ_0, the
475 distance to the source ρ and the source azimuth α counted clockwise from North direction are shown
476 below the corresponding graphs. The circle points with the standard deviation bars denote
477 experimentally found effective heights on the right-most graphs and the solid line is the fitted
478 theoretical dependence of h_1 Eq.(20) which yields the estimated parameters H and ζ_0.

479 Table 1 Errors of estimations of the source distance.

480 Table 2 Errors of estimations of the waveguide effective height.

481

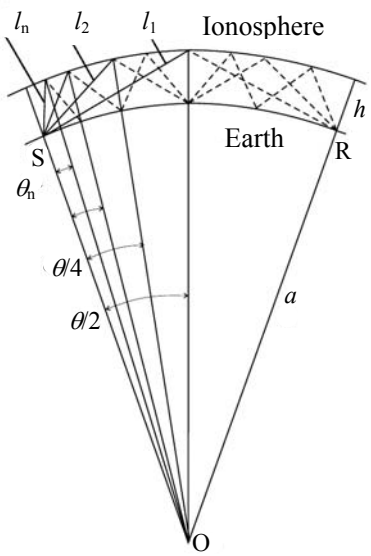
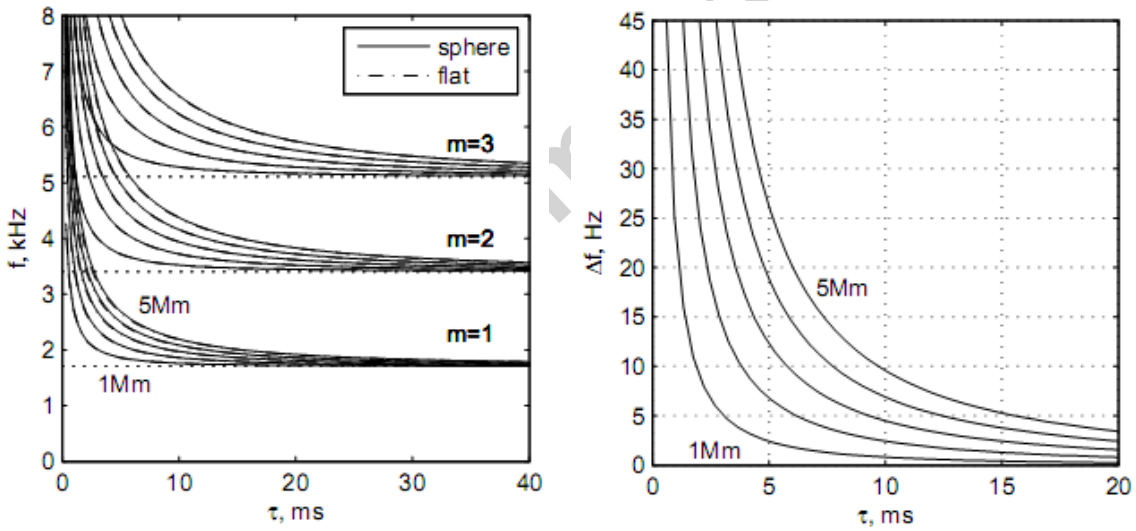
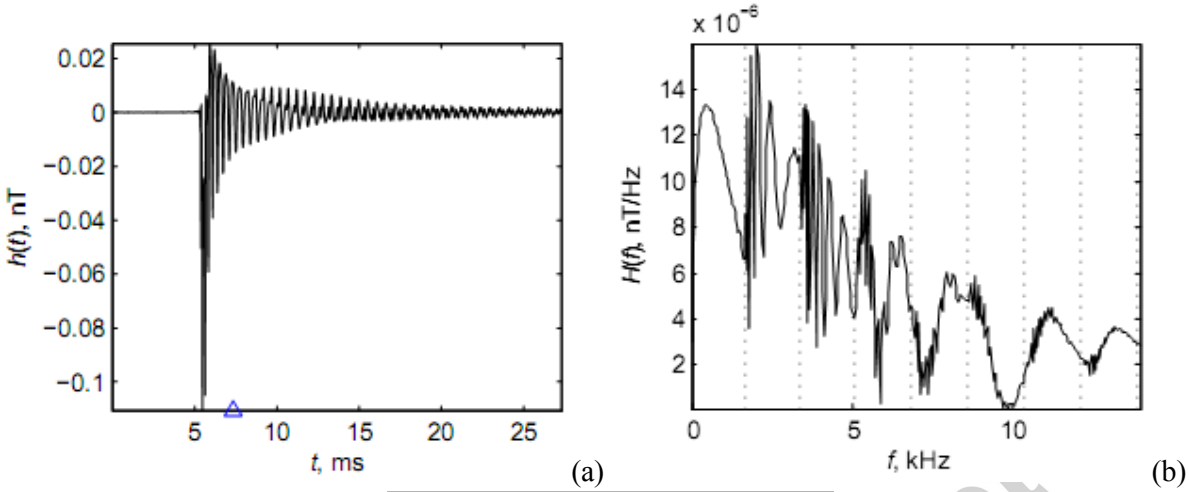


Fig.1.



(a) (b)

Fig.2.



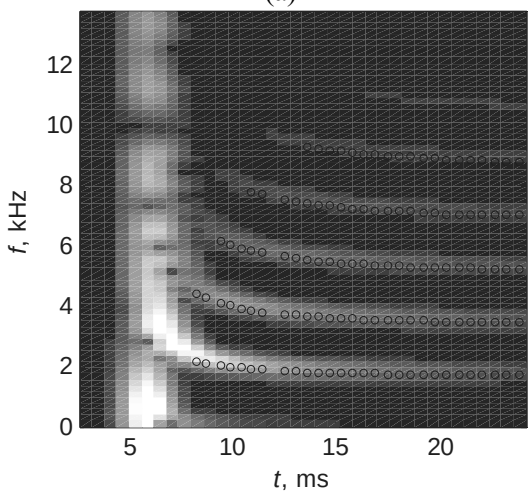


Fig. 3.

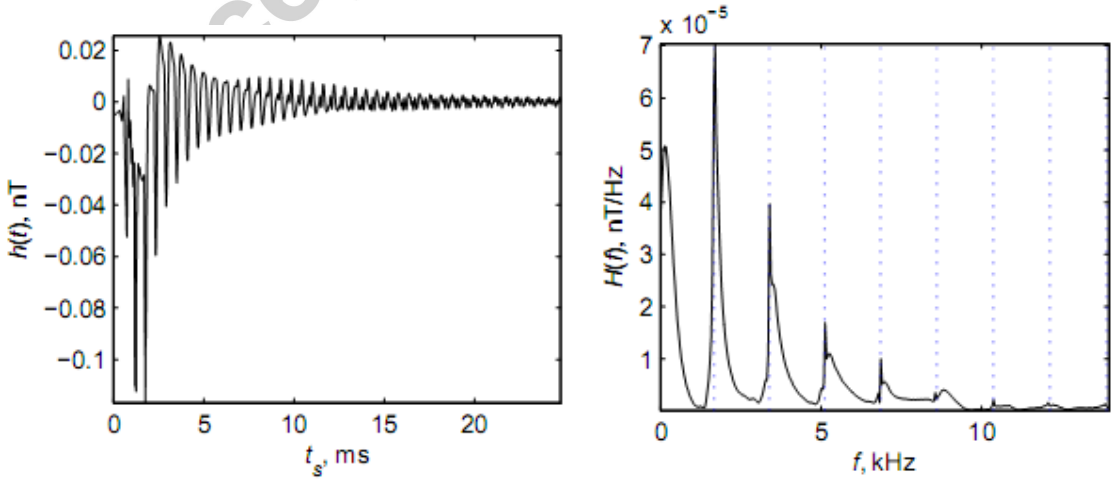
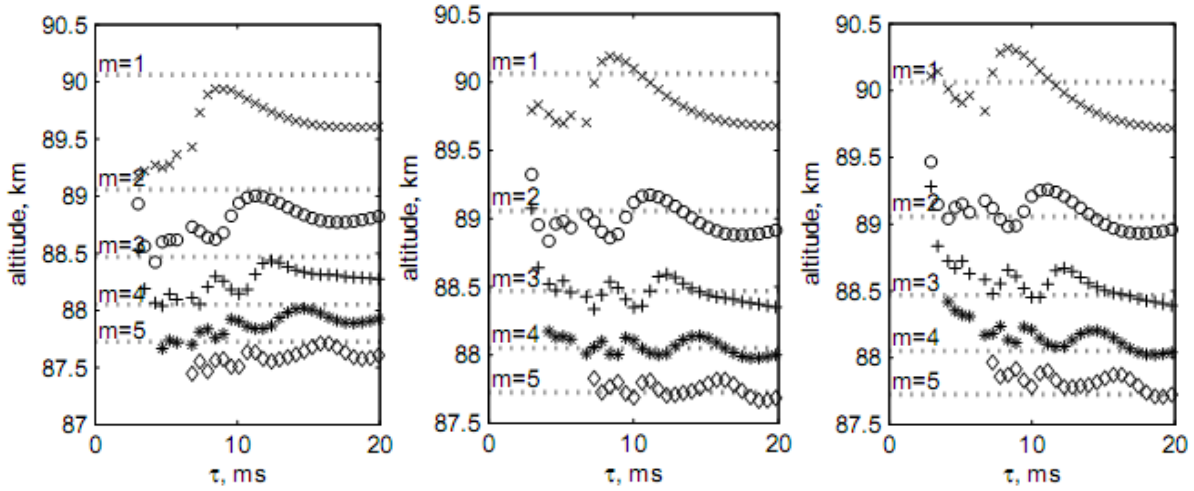
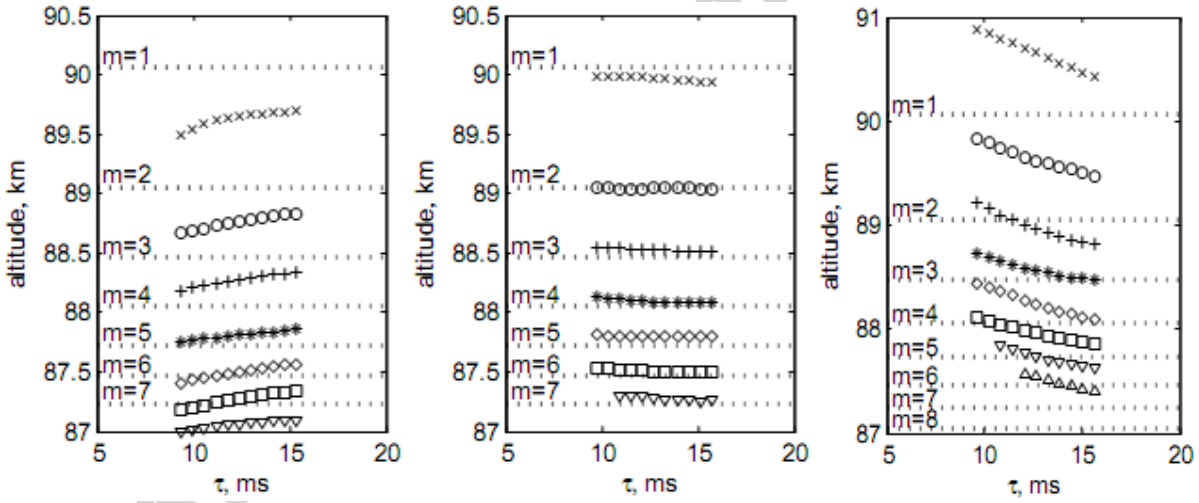


Fig. 4.



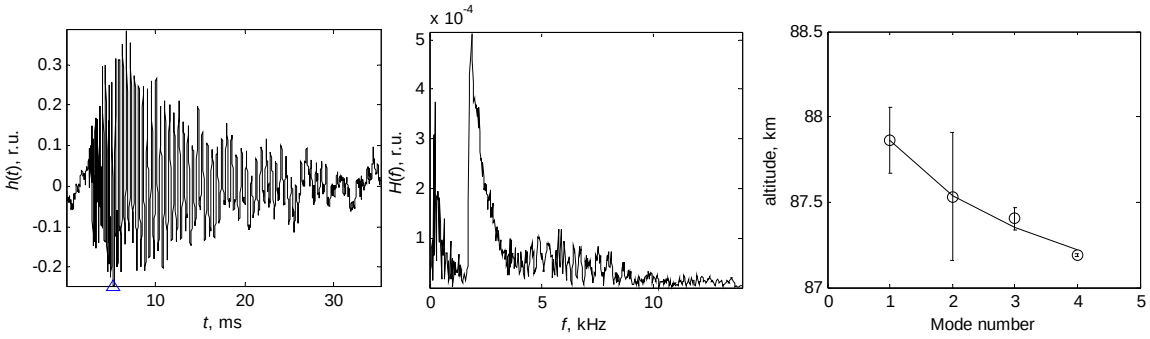
(a) (b) (c)

Fig. 5.

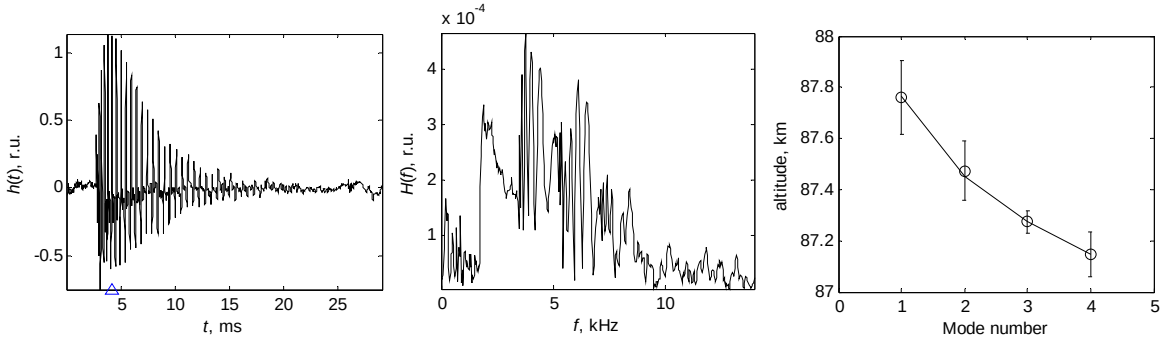


(a) (b) (c)

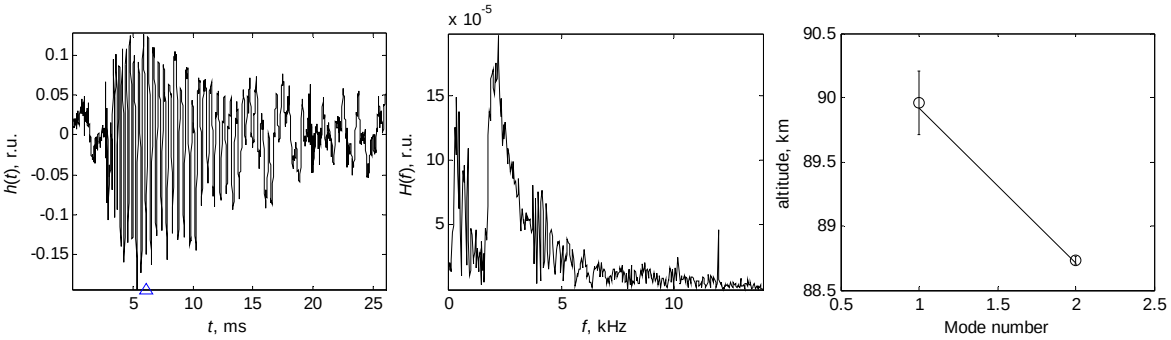
Fig. 6.



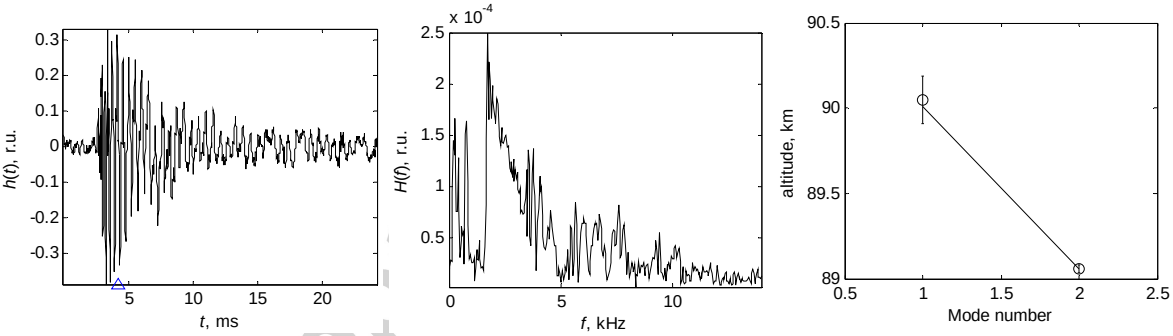
a) 21-Jan-1991, 20:30:47 UT, ρ =2.55 α =62.9°, H=86.2 km, ζ_0 = 0.46 km.



b) 21-Jan-1991, 20:29:22 UT, ρ =1.19 Mm, α=78.6°, H=86.1 km, ζ_0 = 0.44 km.



c) 10-Apr-1991, 01:30:20 UT, ρ =2.68 Mm, α=279.3°, H=88.1 km, ζ_0 = 1.72 km.



d) 10-Apr-1991, 01:24:39 UT, ρ =1.09 Mm, α=276.7°, H=88.0 km, ζ_0 = 1.41 km.

Fig.7.

514

515 Table 1. Errors of estimations of the source distance.

ρ, km	Method 1		Method 2		Method 3	
	Syst. error, km	Std. dev., km	Syst. error, km	Std. dev., km	Syst. error, km	Std. dev., km
500	-18	22	5	9	-30	44
750	10	37	11	10	2	45
1500	18	18	13	26	15	56
2500	55	27	42	49	65	56
3500	207	59	45	42	148	112

516

517 Table 2. Errors of estimations of the waveguide effective height.

Mode #	Method 1		Method 2		Method 3	
	Syst. error, m	Std. dev., m	Syst. error, m	Std. dev., m	Syst. error, m	Std. dev., m
ρ = 500 km						
1	-225	757	-375	175	-24	1371
2	-95	315	-27	64	76	437
3	-2	207	-74	99	152	231
4	-49	232	-11	54	-17	262
5	-10	179	-37	40	-47	148
ρ = 1500 km						
1	-335	151	-241	169	-272	244
2	-80	108	-44	112	-110	69
3	-84	93	39	167	-72	90
4	-52	76	51	115	-55	135
5	-62	137	46	114	-43	476
ρ = 2500 km						
1	-219	131	-267	270	13	52
2	-108	125	-77	181	-319	89
3	-100	93	-32	237	-220	197
ρ = 3500 km						
1	-657	247	-204	190	-5	58
2	-565	191	-50	130	-713	244

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Highlight

*Three techniques are evaluated to find source range and ionosphere height by multimode tweeks.

*Curve-fitting problem for two or more parameters is simplified to one parameter optimization.

*A new technique is proposed based on a nonlinear transformation of tweek waveform.

*Analysis of tweek records shows decreasing ionosphere height with increase of mode number.

*Parameters of the exponential conductivity profile of the ionosphere are estimated by tweeks.