Proceedings of 23rd International Scientific Conference. Transport Means 2019.

Features of the Nonlinear Calculation of the Stress-Strain State of the "Rock Massif–Excavation Support" System Taking into Account Destruction

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Abstract

The algorithm for estimating the reliability of the results of the calculation of the stress-strain state of the «rock massif – excavation support» system, taking into account the non-linear relationship between stresses and strains, as well as the destruction of the rock, is presented. In the course of a numerical experiment, it was shown that to determine the reliability of the calculation results, it is enough to find the relative error between the maximum efforts in the lining and then the extremum (more precisely, the maximum) of the "relative error - depth of the excavation center".
KEY WORDS: *extended development, geomechanical model, plane stress state, plane strain state, displacements, stresses, finite element method, physical nonlinearity, successive loading method, rock destruction*

1. Introduction

The aim of this research is disclosing the developed simplified method for assessing the reliability of the calculation of the stress-strain state of the “rock massif – excavation support” system. This technique takes into account non-linear work and rock destruction using the finite element method. Analysis, generalization of the results of theoretical calculations and numerous experiments using the finite element method.

The algorithm that assesses the reliability of the results of the calculation of the stress-strain state of the rock massif containing the extended production and its support is described. This algorithm takes into account non-linear relationships between stresses and strains, as well as rock fracture. In the course of the experiment, to determine the reliability of the calculation results, it is sufficient to find the relative error between the maximum efforts in the lining and then the extremum (maximum) of the "relative error - depth of the excavation center" dependence.

For the first time, a simplified algorithm is proposed that evaluates the reliability of the results of the calculation of the stress-strain state of the “rock massif – excavation support” system while taking into account non-linear work and rock destruction.

The method presented in this paper allows one to reasonably explain the results of the calculation of extended mine workings, taking into account non-linear work and rock destruction.

2. Analysis of Achievements in this Area

At present, the calculation of the stress-strain state of the “rock massif – excavation support” system, taking into account non-linear work and rock destruction, is performed as follows:

1. Full load is divided into several stages (usually from 10 to 1000 and more).
2. First, the first load step is applied. In this case, the calculation is performed according to a linear scheme.
3. Next, an analysis of the acting forces and stresses within each of the finite elements is performed and the modulus of elasticity changes within it.
4. After that, a new load stage is applied and again, as required, the modules of total deformation change within each of the elements.
5. If, within any finite element, the combination of stresses and/or deformations reaches some critical deformation, this element is excluded from the design scheme.

6. After that, a new numbering of nodes and elements areperformed, further calculations are already performed for a new design scheme with a reduced number of finite elements and, possibly, nodes.

7. If such a number of finite elements are destroyed, the system turns into a mechanism, the calculation stops even if not 100% of the load is applied to the calculated system. At the same time, the efforts in the finite elements and the displacements of their nodes are printed [1, 2].

When executing such an algorithm, the problem will be that in this case, the calculated forces and displacements will often correspond to the conditions of operation of the structure that is not fully loaded, i.e. they will be understated.

The research materials presented in this paper are aimed at solving this problem.

Aim of research. When writing this article, explain on a concrete example the methodology developed by the authors for estimating the reliability of the calculation of the stress-strain state of the "rock massif – excavation support" system, which is a physically non-linear system.

Materials and methods of research. Taking into account the nonlinear work and destruction of the rock, the essence of the proposed methodology for checking the adequacy of the results of the calculation of "rock massif – excavation support" is as follows:

1. It is necessary to divide the load from the overlying soil layers q, which is applied to the upper boundary of the computational domain (Fig. 1), into several parts (*n* steps), the value of which is equal to $q^* = \frac{q}{n}$.

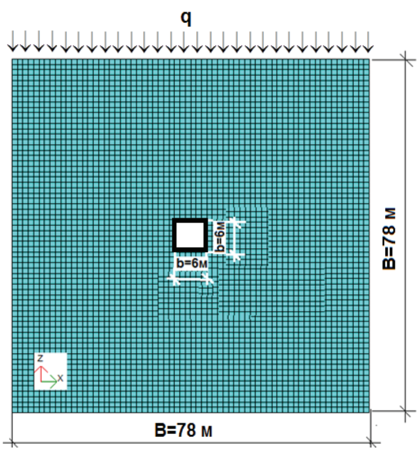

Fig. 1 The calculated area of the rock massif that accommodates mining production (along the perimeter of the excavation, a black (bold) color shows a reinforced concrete lining; B and b - the size of the computational area and excavation)

2. The task should be solved comprehensively in a nonlinear and linear formulation for an external load equal to: $q_1 = q^* = q/n$; $q_2 = 2\cdot q^* = 2\cdot q/n$; $q_3 = 3\cdot q^* = 3\cdot q/n$; . $q_n = n\cdot q^* = q$.

It is necessary to determine the maximum effort in the lining or the support structure of the excavation (for the case of plane deformation with a rectangular section of the tunnel, these are axial forces *N*, shearing forces *Q* and bending moments *M*).

3. For each of the values obtained in the course of calculations by a linear and nonlinear scheme, it is necessary to calculate the relative errors of the form:

$$\left.\begin{aligned} \varepsilon_{N,min} &= \frac{N^{nl}_{min} - N^{l}_{min}}{N^{nl}_{min}}; \\ \varepsilon_{Q,min} &= \frac{Q^{nl}_{min} - Q^{l}_{min}}{Q^{nl}_{min}}; \\ \varepsilon_{M,min} &= \frac{M^{nl}_{min} - M^{l}_{min}}{M^{nl}_{min}}; \\ \varepsilon_{N,max} &= \frac{N^{nl}_{max} - N^{l}_{max}}{N^{nl}_{max}}; \\ \varepsilon_{Q,max} &= \frac{Q^{nl}_{max} - Q^{l}_{max}}{Q^{nl}_{max}}; \\ \varepsilon_{M,max} &= \frac{M^{nl}_{max} - M^{l}_{max}}{M^{nl}_{max}}. \end{aligned}\right\} \quad (1)$$

4. Further, these dependencies should be presented in graphical form.
5. After that, the graphs need to find the first maxima.
6. Next, it is necessary to localize these maxima and approximate them by dependencies in the form:

$$\left.\begin{aligned} \varepsilon_N &= a_{N,0} + a_{N,1} \cdot q^* + a_{N,2} (q^*)^2; \\ \varepsilon_Q &= a_{Q,0} + a_{Q,1} \cdot q^* + a_{Q,2} (q^*)^2; \\ \varepsilon_M &= a_{M,0} + a_{M,1} \cdot q^* + a_{M,2} (q^*)^2. \end{aligned}\right\} \quad (2)$$

7. Next, using the formulas:

$$q_{N,\frac{min}{max}} = -\frac{a_{N,1}}{2 \cdot a_{N,2}}; \quad (3)$$

$$\left.\begin{aligned} q_{Q,\frac{min}{max}} &= -\frac{a_{Q,1}}{2 \cdot a_{Q,2}}; \\ q_{M,\frac{min}{max}} &= -\frac{a_{M,1}}{2 \cdot a_{M,2}}; \\ q_{av} &= \frac{q_N + q_Q + q_M}{3} = -\frac{a_{N,1}}{6 \cdot a_{N,2}} - \frac{a_{Q,1}}{6 \cdot a_{Q,2}} - \frac{a_{M,1}}{6 \cdot a_{M,2}}. \end{aligned}\right\}. \quad (4)$$

from the overlying layers of soil is determined by the maximum value of the load on the upper boundary of the computational region, for which the results of the calculations will be reliable.

7.1. A more reliable result can be obtained using the methods of mathematical statistics [3, 4].

8. If dependences (1) do not have points corresponding to a maximum, the results of calculations are also reliable.

Next, let's illustrate the above algorithm with a specific example.

Let's find the pressure of rocks in the roof of the calculated area of excavation, for which the results of determining its stress-strain state with a physically nonlinear basis, taking into account the destruction of the rock, are reliable.

Table 1

The actual and equivalent (fictitious) properties of sandstone, adopted during the numerical experiment

№	Rock	Kind of environment	Specific weight γ, kN/m^3	Poisson's ratio ν, d.u.	Compressive strength *Rc*, MPa	Tensile strength *Rp*, MPa	Elastic modulus *E*, MPa
1	Sandstone	Actual	23,0	0,300	60	12	40000
2		fictitious	23,0	0,418	60	12	19780

Let's consider a long horizontal production of square shape with dimensions of 6x6 m, located inside the rock massif composed of sandstones [5, 6]. The actual properties of the rock mass are presented in the first row of Table 1.

In this case, let's take into account that the rock that contains the extended production is in a state of plane deformation [7]. In this case, it is necessary to use not the actual, but the reduced material properties (Table 1, line 2), which should be determined by the formulas:

$$\left.\begin{aligned} \nu^* &= \frac{\nu}{1-\nu}; \\ E^* &= \frac{E}{1-\nu^2}. \end{aligned}\right\}. \quad (5)$$

For the convenience of justifying the obtained results and their clarity, the nonlinearity of the properties of sandstone and the nature of its destruction are described in the work by an exponential dependence [1], Fig. 2.

The parameters $\varepsilon(+)$ and $\varepsilon(-)$ are determined by the formulas:

$$\left.\begin{aligned} \varepsilon(+) &= \frac{Rp}{E}; \\ \varepsilon(-) &= -\frac{Rc}{E}. \end{aligned}\right\}. \quad (6)$$

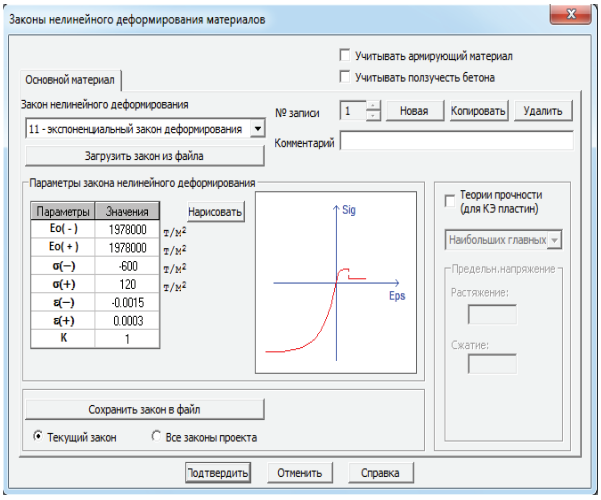

Fig. 2 The "strain - load" diagram adopted to determine the stress-strain state of the rock massif

When determining them, it is taken into account that in the mechanics of a solid deformable body, the stresses corresponding to compression are taken with a "-" sign, and tensile stress - with a "+" sign.

For the sake of simplicity of substantiating the results of calculations, reinforced concrete lining of a mine workout was simulated using a linear isotropic medium. The lining thickness was assumed to be 30 cm, the elastic modulus $E = 20000$ MPa Poisson's ratio $\nu = 0.2$.

The computational domain (Fig. 1) was loaded with its own weight – the weight of the rock γ and the load q applied to its upper boundary. A distributed load of 0, 100, 200, 300, 400 and 500 tons / sq.m was successively applied to the upper boundary of the computational domain.

The calculation results are summarized in Table 2.

In graphical form, the dependences of the maximum and minimum efforts in the tunnel lining are presented in Fig. 3.

The fragments of curves with characteristic extrema taken for approximation in Figure 3 are shown in Fig. 4.

In this case, the curves obtained in the course of approximation presented in Fig. 4, the analytical dependencies are summarized in Table 3, the third column of the table presents the ordinates calculated using formulas (3) and corresponding to the extremes $q_{N,\frac{min}{max}}$, $q_{Q,\frac{min}{max}}$, $q_{M,\frac{min}{max}}$.

Table 2

The results of the calculation of the stress-strain state of the tunnel lining

Excavation depth	Base model	Name of the force					
		Axial forces N, tons		Shear forces Q, tons		Bending moments M, t·m	
		min	max	min	max	min	max
0,0	Linear	-42	2	-10	9	-3	3
	Nonlinear	-62	4	-15	13	-5	5
200	Linear	-156	8	-37	33	-13	13
	Nonlinear	-213	14	-58	48	-19	19
400	Linear	-270	14	-64	56	-22	22
	Nonlinear	-332	27	-118	87	-39	39
600	Linear	-384	20	-91	80	-34	31
	Nonlinear	-361	30	-137	98	-45	45
800	Linear	-499	25	-118	104	-40	40
	Nonlinear	-294	23	-118	90	-38	38

Taking into account the data presented in Table 3, let's find the value of the load on the upper boundary of the computational domain, for which the results of the calculation of its stress-strain state are reliable.

There is:

$$q_{cp} = \frac{1}{6} \cdot \left(q_{N,min} + q_{N,max} + q_{Q,min} + q_{Q,max} + q_{M,min} + q_{M,max} \right) = \frac{250 + 338 + 375 + 370 + 365 + 365}{6} = 344 \ tons \qquad (7)$$

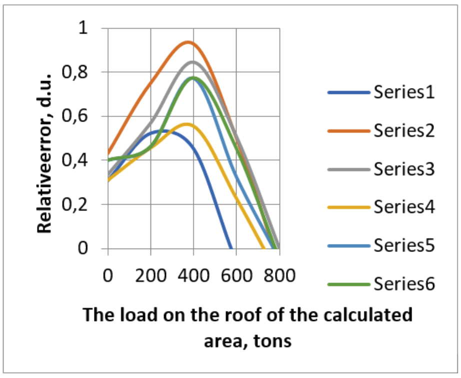

Fig. 3 The dependence of the forces in the lining of the excavation from the load in the calculated area

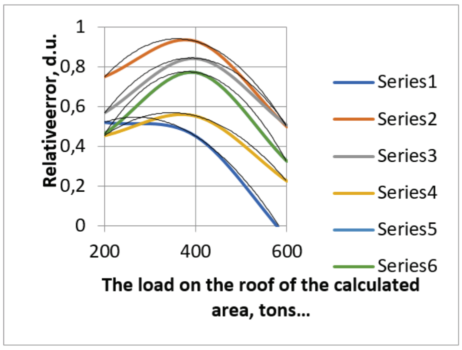

Fig. 4 The dependence of the forces in the lining of the excavation from the load in the vicinity of the first maximum

Table 3

The results of the approximation dependencies in Fig. 4

№	Type of function and its approximation	The value of the variable corresponding to the extremum, tons
1	2	3
1	$\varepsilon_{N,\min} = -6 \cdot 10^{-06} \cdot (q^*)^2 + 0,003 \cdot q^* + 0,1488$	250
2	$\varepsilon_{N,\max} = -8 \cdot 10^{-06} \cdot (q^*)^2 + 0,0054 \cdot q^* + 0,0367$	338
3	$\varepsilon_{Q,\min} = -8 \cdot 10^{-06} \cdot (q^*)^2 + 0,006 \cdot q^* + 0,3231$	375

In order to clarify the result using the methods of mathematical statistics, it is necessary to perform the following steps:

1. Using formula (7) finding the standard:

$$\bar{q} = q_{cp} = 344 \text{ tons}. \tag{8}$$

2. Finding the biased estimate of the standard deviation:

$$S_{dis} = \sqrt{\frac{\sum_{i=1}^{n} (\bar{q} - q_i)^2}{n}} = 43,6 \text{ tons}. \tag{9}$$

3. Then using the formula:

$$|\bar{q} - q_i| - \nu \cdot S_{dis} < 0, \tag{10}$$

where for a sample with the number of terms in the number n = 6 and $\chi = 2,07$ (the value of the statistical Student's criterion) [3, 4]. There are:

$$\left.\begin{array}{l} |344 - 250| - 2,07 \cdot 43,58 = 3,8 > 0; \\ |344 - 337,5| - 2,07 \cdot 43,58 = -83,7 < 0; \\ |344 - 375| - 2,07 \cdot 43,58 = -59,2 < 0; \end{array}\right\} \tag{11}$$

$$\left.\begin{array}{l} |344 - 250| - 2,07 \cdot 43,58 = 3,8 > 0; \\ |344 - 337,5| - 2,07 \cdot 43,58 = -83,7 < 0; \\ |344 - 375| - 2,07 \cdot 43,58 = -59,2 < 0. \end{array}\right\} \tag{12}$$

Since condition (7) is not satisfied for the first equality (8), recalculation is performed. According to the results of the calculation:

$$\bar{q} = q_{av} = \frac{337,5+375+370+365+365}{5} = 362,5 \text{ tons}.$$

Next, perform a check for a "rebound":

$$\left.\begin{aligned}
&|362,5-337,5|-2,07\cdot 22,8=-40,7<0;\\
&|362,5-375|-2,07\cdot 22,8=-16,2<0;\\
&|362,5-370|-2,07\cdot 22,8=-21,2<0;\\
&|362,5-365|-2,07\cdot 22,8=-26,2<0;\\
&|362,5-365|-2,07\cdot 22,8=-26,2<0.
\end{aligned}\right\} \quad (13)$$

Next, using the formulas:

$$\left.\begin{aligned}
\gamma_g &= \frac{1}{1\pm\delta};\\
\delta &= \frac{t_\alpha\cdot V}{\sqrt{n}};\\
V &= \frac{\sigma_A}{\bar{q}};\\
\sigma_A &= \sqrt{\frac{1}{n-1}\cdot\sum_{i=1}^{n}\left(\bar{q}-q_i\right)^2}.
\end{aligned}\right\} \quad (14)$$

The safety factor γ_g.is found. In this case, it is necessary to take into account that with $n = 5$ and the confidence coefficient α = 0.95; $t_\alpha = 2.01$ [4].

$$\left.\begin{aligned}
\sigma_A &= \sqrt{\frac{1}{n-1}\cdot\sum_{i=1}^{n}\left(\bar{q}-q_i\right)^2} = \sqrt{\frac{2608}{4}} = 25,5;\\
V &= \frac{\sigma_A}{\bar{q}} = \frac{25,5}{362,5} = 0,07;\\
\delta &= \frac{t_\alpha\cdot V}{\sqrt{n}} = \frac{2,01\cdot 0,07}{\sqrt{5}} = 0,063;\\
\gamma_g &= \frac{1}{1\pm\delta} = \frac{1}{1\pm 0,063} = \begin{cases}0,94\\1,07\end{cases}.
\end{aligned}\right\} \quad (15)$$

The final required load on the roof of the computational domain is found using the formula:

$$q^p = \frac{q}{\gamma_g} = \frac{262,5}{1\pm 0,063} = \begin{Bmatrix}341 \text{ tons}\\365 \text{ tons}\end{Bmatrix}. \quad (16)$$

Here q^p – the calculated load on the roof from the overlying layers of the rock, to which, with a probability of 95%, the results of calculation by a nonlinear scheme taking into account the destruction of the rock are reliable.

To determine the depth to which the results of calculations are reliable, it is necessary to use formulas of the form:

$$h^p = \frac{q}{\bar{\gamma}};\quad \bar{\gamma} = \frac{\sum_{i=1}^{m}\gamma_i\cdot h_i}{\sum_{i=1}^{m}h_i};\quad \sum_{i=1}^{m}\gamma_i\cdot h_i \le q^p, \quad (17)$$

where h^p – the calculated depth, for which the results of determining the stress-strain state of the "rock massif – excavation – excavation lining" system are reliable. At the same time, γ_i and h_i –the specific gravity and thickness of the *i*-th rock layer located above the computational domain.

To illustrate the above presented results with loads $q^* =$ 0, 200, 400, 600 and 800 kPa, the authors of the work constructed a picture of the destruction of the rock within the computational domain (Figs. 4-8).

It follows from the figures that the greatest destruction occurs at $q^* = 600$ t/m², and at $q^* = 400$ t/m² and $q^* = 800$ t/m², the pattern of destruction is approximately identical. This fact confirms the thesis that in this case it was not possible to visually assess the reliability of the calculation results, since the destruction $q^* = 600$ t/m² more than with $q^* = 800$ t/m², and at $q^* = 400$ t/m² and $q^* = 800$ t/m² are identical. This is contrary to modern ideas about the nature of the destruction of rocks.

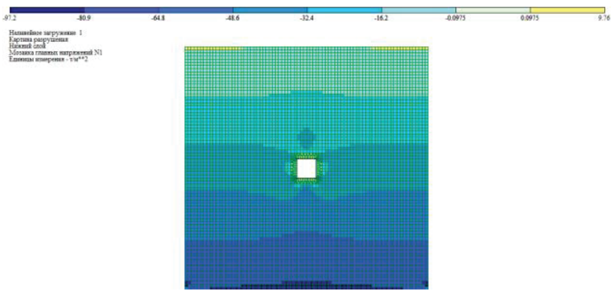

Fig. 5 Picture of destruction. The load on the upper boundary of the computational domain is $q^* = 0$ t/m². No destructions

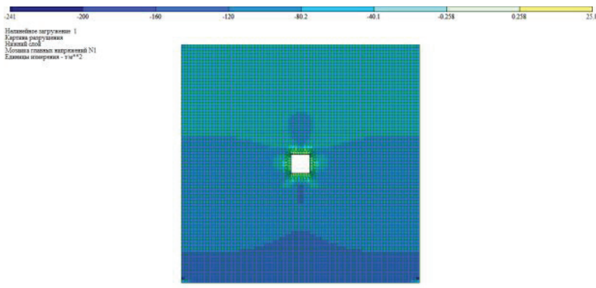

Fig. 6 Picture of destruction. The load on the upper boundary of the computational domain is $q^* = 200$ t/m². No destructions

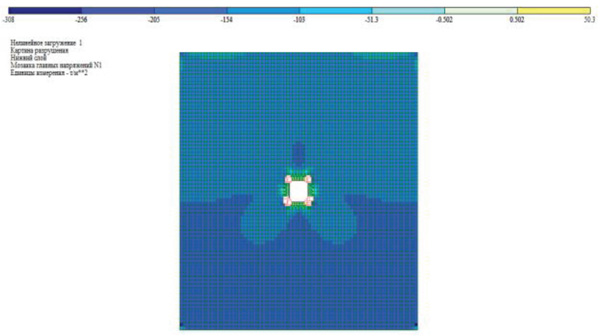

Fig. 7 Picture of destruction. The load on the upper boundary of the computational domain is $q^* = 400$ t/m². Destructions are local

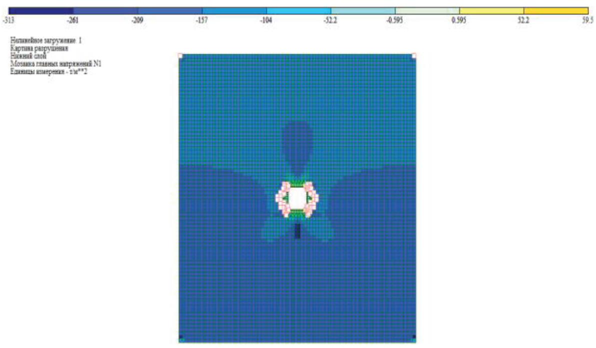

Fig. 8 Picture of destruction. The load on the upper boundary of the computational domain is $q^* = 600$ t/m²

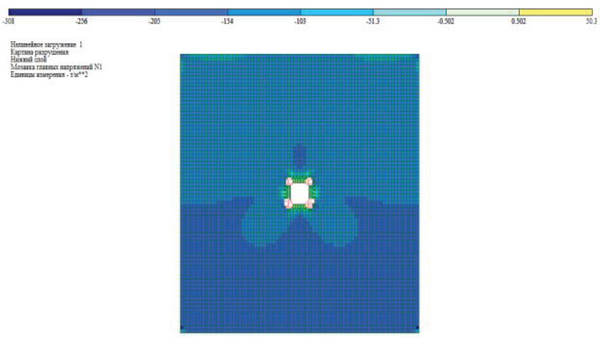

Fig. 9 Picture of destruction. The load on the upper boundary of the computational domain is $q^* = 800$ t/m²

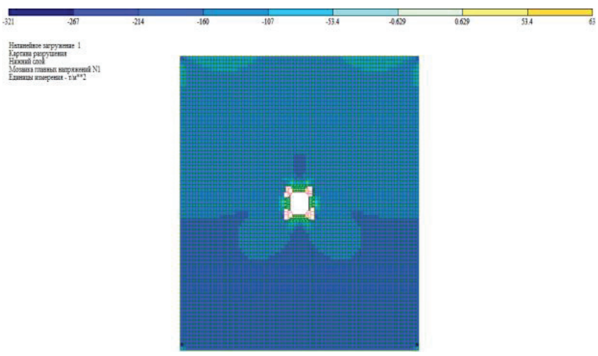

Fig. 10 Picture of destruction. The load on the upper boundary of the computational domain is $q^* = 362,5$ t/m²

Further, the authors of the work performed a calculation for the established load $q^* = 362,5$ t/m² to the upper limit of the computational domain (Fig. 10).

From Fig. 10 it follows that in this case the destruction is much less than at $q^* = 600$ t/m² and close to the picture of destruction at $q^* = 400$ t/m² which corresponds well to modern ideas about the nature of the destruction of rocks.

3. Conclusion

The method of estimating the reliability of the calculation of the stress-strain state of the system "rock massif – concrete lining of the tunnel" system is developed.

With this method it is possible to obtain reliable results, approximate with the exact solution when solving problems by the method of sequential loading.

In this regard, the method presented in this paper can find its place in solving applied problems of geomechanics, soil mechanics and, in general, nonlinear mechanics of continuous deformable media.

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