Development of Methods for Optimizing Reactive Power Modes Based on Neural Network Technologies

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Abstract. The high cost of electric power, as well as the considerable length and branching of electrical networks, necessitate reduce electric power consumption, and losses in electrical networks. One of solutions of this problem is optimizing the reactive power mode. Reducing the reactive power factor at the point of common coupling (PCC) to the economic level established by the power system is not taking into account that in a complex network, power flows with a non-optimal arrangement of compensating devices and improper determination of their power can reach large values, that resulting in an increase in losses in the network. A program has been developed that implements prediction algorithms using neural networks, as well as optimizing the reactive power mode.

Keywords – forecasting, electrical network, compensating of reactive power, optimization, neural networks, modeling, reactive power.

I. INTRODUCTION

Electric load forecasting, along with the problems of energy efficiency and energy saving, are given serious consideration in the design and operation of power supply systems for industrial enterprises. Electrical loads forecasting makes it possible to efficiently plan and quickly manage the operation of the power supply system (PSS). A reliable forecast allows us to calculate the optimum operating conditions of PSS, improve their efficiency and reliability. With own sources of electric power in industrial enterprises planning of energy consumption, the economy of loading the generators, and, consequently, the cost of the generated electricity depend on the accuracy of the forecast.

The problem of efficient use of electric power is becoming more urgent than ever because of a sharp increase in its value. An important factor that requires a reduction in electric power consumption is the electrical networks that have exhausted their service life, can hardly bear the increased loads and require modernization. Optimization of the energy consumption mode will entail a reduction in the production cost, preservation of the enterprises competitiveness.

II. ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

To solve the problem of optimizing the electrical network mode and reducing losses, it is necessary to know

the distribution of power flows in the branches of the network. On account of a large number of electrical receivers and possible changes in the configuration of the electrical network, power flows are random processes, and as a consequence, the flow distribution is a rather complex task, especially in case of branched, ring networks.

One of the most common methods for calculating losses in an electrical network is the equivalent resistance method [1]. In accordance with this method, it is necessary to find the equivalent resistance of some conditional unbranched circuit, the current in which is equal to the current at the head line of the network, and the losses are equal to the losses in the network. It is assumed that when the current in the head line and the currents in all the other parts of the network change proportionally. However, it should be noted that this method of estimating losses is very approximate, and does not allow to determine with high accuracy the flow distributions in the electrical network of modern industrial enterprises, especially in case of rapid changes in loads and various factors affecting the technological process.

III. OBJECTIVES OF THE STUDY

Development of methods for optimizing the reactive power mode in a complex branch network from the viewpoint of economy.

IV. MAIN RESEARCH MATERIALS

One way out of this situation is to reduce losses and increase the transfer capability of electrical networks. The solution of this problem is primarily related to the optimization of the production process:

- The time separation of the loads of the most powerful electric receivers to reduce peak loads, and as a result, the reduction of current and power losses in the network.
- Restriction of electric receivers idling.
- Reduction of electricity consumption for auxiliary.
- Work in night shifts, when the total load of the network is reduced, etc.

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As a result of the above measures, the electrical load is planed, power losses in electrical networks are reduced. A special place in this problem is occupied by forecasting the consumption of reactive power. Minimizing payment for reactive energy, compensating reactive power is low effective without a correct prediction of reactive loads of industrial enterprises. The problem becomes even more urgent because of the growing electricity tariffs and the overall goal of saving energy resources. The problem of reducing the losses of active power in the elements of electrical networks (cable and air lines, transformers, etc.) is pressing as well. The losses of active power depend both on the value of the active power P and on the reactive power Q, flowing through the element with resistance r:

$$\Delta P = ((P^2 + Q^2)/U^2) \times r, \, kW \tag{1}$$

Therefore, compensation of reactive power is very important for reducing the loss of active power and energy. Qualitative forecasting makes it possible to develop algorithms for adjusting the power of compensating devices and, in some cases, dispense with rather expensive highspeed controls for compensating devices, varying the compensation power in manual mode. Nowadays, one of the most widespread methods of forecasting is predicting the average value over the previous time interval, statistical methods, as well as predicting with the help of neural networks that came into being not so long ago [2]. Let's consider the question of efficiency and expediency of application of each of the methods when choosing an algorithm for controlling the power of compensating devices.

A. Forecasting using a neural network.

The neural network is a powerful method of simulating processes and phenomena, allowing to reproduce extremely complex dependencies. Its main peculiarity is the use of the training process, in which the user is given input data, then the goal values are set up and the training process starts, which automatically adjusts the network parameters. The theory of neural networks arose from studies of artificial intelligence, namely the attempts to artificially reproduce a nervous biological system with the connections between neurons. These peculiarities created the prerequisites for the successful application of neural networks for forecasting. The neural network does not predict the future; on the basis of input parameters, it "tries to assess the state of the predicted value at the moment and reproduce its future behavior as accurately as possible." Theoretical aspects of the creation and operation of neural networks are described in [3]. Fig. 1 shows a general view of an artificial neuron.



Fig.1. Artificial neuron

There are three distinct functional operations that take place in this example neuron. First, the scalar input p is multiplied by the scalar weight w to form the product wp, again a scalar. Second, the weighted input wp is added to the scalar bias b to form the net input n. Finally, the net input is transmitted through the transfer function f, which produces the scalar output a. The names given to these three processes are: the weight function, the net input function and the transfer function. [4]

B. The method of forecasting using statistical methods

It is one of the most widespread and developed for today [5]. One of the most commonly used methods of statistical prediction is the extrapolation of data. It is based on the assumption that the process under consideration has two components - a constant (trend line), which is a function of time, and a random uncorrelated component with zero mean value, the estimation of which is necessary to determine the characteristics of the forecast in terms of accuracy. Extrapolation methods focus on identifying the best description of the trend and determining the predicted values by extrapolating it. The generalization of the extrapolation operation can be written as the value of the function [6]:

$$y_{i+L} = f(y_i^*, L)$$
 (2)

where y_{i+L} - the extrapolated level value,

L – the prediction period,

 y_i^* - the level accepted for the extrapolation base.

Extrapolation of trends in dynamical series is relatively widely used in practice because of its simplicity [7], the possibility of implementing on the basis of a relatively small amount of information. Absence of other information besides the separately considered dynamical series is often decisive in the choice of this forecasting method.

C. The method of forecasting by the average

This method is the simplest one presented in this work. The essence of the method lies in the fact that there is an average of a number of previous values (in the case under consideration, it is 24, that is, for the previous day), and then a vector of forecast values is made up, and in the considered interval they are all equal, and the graph of this forecast represents a straight line. This method of forecasting is the most outdated for today, but due to its simplicity, it has found wide application in the practice of the operation of power supply systems at a number of industrial enterprises. Fig. 2 shows the errors of various methods for forecasting reactive loads, obtained on the basis of earlier studies [8,9].

D. Optimization of the electrical network

As mentioned above, one of the most accurate and costeffective methods for predicting electrical loads is neural networks. A comparison of forecasting methods is presented in the Table I. Despite some shortcomings (complexity of training, large data sampling for network training), neural networks have an undeniable advantage - during training they "learn" to recreate very complex dependencies, taking into account many factors, which in the end becomes advantageous over traditional methods of forecasting [10,11].



Fig.2. Comparison of error values of prediction methods

Concerned network has hidden layer of sigmoid neurons followed by an output layer of linear neurons. Multiple layers of neurons with nonlinear transfer functions allow the network to learn nonlinear relations between input and output vectors. The linear output layer is used for nonlinear regression.

TABLE I. COMPARISON OF FORECASTING METHODS ERRORS

Neural network	Traditional methods			
0.02 %	-3.39 %			

The basic idea of the back propagation error method is propagating the signals from the network outputs to its inputs in the opposite direction to the direct propagation of signals in normal operation. To be able to use the method of back propagation of error, the transfer function of neurons must be differentiable. This method is a modification of the classical gradient descent method [12]. The scheme of the neural network with the back propagation of the error is shown in Fig.3.



Fig.3. General scheme of neural network with back propagation of the error

Optimization of the reactive power mode in the simulated network is carried out by forecasting the reactive loads, then the compensation power is adjusted in accordance with this forecast.

To predict the loads, a neural network with back propagation of the error was developed. The neural network contains the number of neurons equal to three times the number of inputs (determined by selection), the learning function is implemented using the Levenberg-Marquardt algorithm because of its high accuracy and speed of training [13]. The Levenberg-Marquardt (LM) algorithm is an iterative technique that locates the minimum of a multivariate function that is expressed as the sum of squares of non-linear real-valued functions. It has become a standard technique for non-linear least-squares problems, widely adopted in a broad spectrum of disciplines. This algorithm can be thought of as a combination of steepest descent and the Gauss-Newton method. When the current solution is far from the correct one, the algorithm behaves like a steepest descent method: slow, but guaranteed to converge. When the current solution is close to the correct solution, it becomes a Gauss-Newton method [14]. An example of a simplified scheme of the created network is shown in Fig.4, and its hidden layer - in Fig.5.

The consumption data of reactive power for the previous period, equal to 120 hours (determined empirically) are supplied at the input of the neural network. Then the forecast is made for one hour ahead and compared with the real consumption data of reactive power. The prediction error is calculated for each hour and on average for the selected time period. During the preparation the 75% of the data is used to train the neural network and 25% - to assess the accuracy of prediction.

This neural network is effective only if there is a sufficient amount of initial data that can be obtained only at the substation. The loads of each node of the ring network are generally unknown. Designing, creating and setting up networks for transferring data on the load values at each connection of a ring network in real time requires additional financial costs.



Fig.4. Example of neural network structure



Fig.5. Model of the hidden layer of a neural network

A comparison of the simulated the reactive load and the forecast on its basis with the help of neural networks is shown in Fig. 6.



Fig.6. Real parts of the reactive load and prediction with the help of neural networks.

To reduce the costs caused by the flow of reactive power through the network, it is impossible to determine the optimum power at each node, considering it separately [15]. Installing the compensating devices in one node changes the efficiency of the compensating devices installation in other nodes. This is especially true for complex networks, where increasing the power of the compensating devices at one point may lead to an increase in power flows across other parts of the network to such a level that the increased losses can minimize the economic effect of the compensating devices installation.

Let us consider the economic value of reactive power Q_{ec} in terms of minimum costs resulting from the purchase, transmission and compensation of reactive power. The graphs of the given costs dependence on the value of reactive power compensation are shown in Fig. 7.

The expenses resulting from the transfer, compensation and payment for reactive power are calculated as follows:

$$C = C_{trans} + C_{comp} + C_{pay}, \qquad (3)$$

where C_{trans} – costs for the transfer of reactive power,

 C_{comp} – costs of reactive power compensation,

 C_{pay} – costs associated with payment for the flow of reactive power.

To solve such problems, gradient optimization methods are usually used, realizing iterative algorithms of gradual approximation to the optimal solution [16]. To determine the direction of motion to a minimum, calculate the partial derivatives of the total costs [17] (objective function) of the compensating devices (CD) power at each node. Physically, they represent specific partial decreases in total costs, UAH / kVAr per year, when installing a unit power of the CD in various nodes. Further low power of the CD (a portion) is distributed between nodes in proportion to the values of the derivatives. Obviously, with such a distribution, most of the portion of the CD will be installed into the nodes with large values of the derivatives, since the decrease in reactive power in these nodes most significantly reduces the total costs. At new values of reactive power, the nodes again determine the partial derivatives, which will have lower values than in the previous step. The next portion of the total power of the CD is distributed between the nodes in proportion to the new values of the derivatives, etc.



Fig.7. Graphs of the given costs dependence on the value of reactive power compensation.

At each optimization step, the reactive power level of the network is calculated. Transformers of networks communication of different voltage classes are also involved in the calculation, the choice of optimal transformation ratios of which makes it possible to keep the voltage levels in the nodes within the permissible limits.

When the reactive loads of nodes change after several iterations, some derivatives may become negative, which indicates a too high CD power, determined for these nodes at previous iterations. When distributing a new CD portion, a negative portion of the CD power will be added to these nodes and the CD power will decrease in them, while increasing in the nodes with positive derivative values. The calculation is over when the derivatives in the nodes with the accumulated CD power become close to zero, indicating that further increase or decrease in CD power will only increase the total costs. The costs for each of the iterations are calculated as follows:

$$C_{o} = (Q^{2} / U^{2}) \times R \times c_{eq} + C_{comp} (Q_{init} - Q_{fin}), \qquad (4)$$

where Q – reactive power in the node, kVAr, U – voltage, V,

R – matrix of active resistance nodes, Ohm,

 c_{eq} – cost of losses of active energy, UAH / kW* h,

 C_{comp} – compensation costs, UAH,

 Q_{init} – initial value of reactive power, kVAr,

 Q_{fin} – value of reactive power after compensation, kVAr.

To build a matrix of nodal reactive powers, it is necessary to know the values of reactive power in the nodes of the network with a certain time interval (for example, 1 hour). In this case, the rows are the node numbers of the network under investigation, and the columns are the hours. In this way, we get a single data array for the electrical network. 2019 IEEE 6th International Conference on Energy Smart Systems (2019 IEEE ESS)

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1n} \\ Q_{21} & Q_{22} & Q_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ Q_{m1} & Q_{m2} & Q_{mn} \end{bmatrix}, VAr$$
(5)

The matrix of nodal resistance of a radial network is determined directly by the network scheme. Each diagonal element of the matrix represents the sum of the resistances of the sections from the node under consideration to the power center, and the off-diagonal element is the sum of the resistances of the sections common to the pair of nodes under consideration.

$$R = \begin{bmatrix} R_{11} & R_{12} & \dots & R_{1n} \\ R_{21} & R_{22} & \dots & R_{2n} \\ \dots & \dots & \dots & \dots \\ R_{n1} & R_{n2} & \dots & R_{nn} \end{bmatrix}, Ohm$$
(6)

To analyze and further optimize the reactive power mode, let us consider the circuit shown in Fig. 8 as an example. Table II shows the parameters of the substitution circuit for the network elements reduced to the 6 kV side.



Fig.8. Simplified network scheme

TABLE II. PARAMETERS OF THE SUBSTITUTION SCHEME

Number of node	Rtrans. Ohm ^{.3}	Rreactor, Ohm ^{.3}	R cab per lenght, Ohm ⁻³ /meter	Lcab., meters	R cab., Ohm ⁻³	R network to power source R, Ohm ³
2	2.6	4.3	0.040	2100	84	91
3	2.6	4.3	0.160	250	40	47
4	2.6	4.3	0.069	1960	136	143
5	2.6	4.3	0.040	250	10	17
6	2.6	4.3	0.405	500	203	209
7	2.6	4.3	0.104	1960	204	211
8	2.6	4.3	0.405	20	8	15
9	2.6	4.3	0.069	250	17	24
10	2.6	4.3	0.040	300	12	19
11	2.6	4.3	0.040	300	12	19
12	2.6	4.3	0.040	300	12	19

The matrix of reactive loads in the nodes Q, are presented in Table III. The lines are nodes of the network, the columns are the hours (the loads are presented with a discreteness of 1 hour).

 TABLE III.
 The Matrix of Reactive Loads for the Test Circuit for 5 Hours, kVAr

N₂	Hours					
of node	1	2	3	4	5	
1	2734.5	2739.5	2737.0	2732.2	2722.5	
2	2541.9	2536.3	2548.5	2553.7	2548.0	
3	1531.3	1538.8	1549.6	1559.1	1570.3	
4	2486.2	2482.6	2486.5	2476.2	2479.4	
5	2861.6	2861.8	2870.8	2891.5	2904.5	
6	929.5	934.4	944.8	951.8	958.1	
7	2287.6	2298.2	2313.0	2325.4	2342.9	
8	2520.0	2541.3	2551.7	2568.3	2584.0	
9	238.8	238.8	239.5	239.2	239.6	
10	0.0	0.0	0.0	0.0	0.0	
11	0.0	0.0	0.0	0.0	0.0	
12	0.0	0.0	0.0	0.0	0.0	

The vector of partial derivatives is the following:

$$\sigma = (2c_{eq} / U^2) \times R \times Q_t - C_{cost.comp}, \qquad (7)$$

where Q_t – transposed matrix of reactive power values in the nodes, kVAr,

 $C_{cost.comp}$ — column matrix of specific annual costs for compensating devices, USD.

Based on the data from the tables, the node impedance matrix R is compiled for each bus section. To obtain a matrix of nodal resistance of a radial network, each diagonal matrix element is the sum of the resistances of the sections from the node under consideration to the power source, and the off-diagonal is the sum of the resistances of the sections common to the pair of nodes under consideration. This matrix is presented in Table IV for each of the bus sections, respectively.

The obtained values of the reactive loads are again substituted into the formula 4 and the derivatives at the new point are determined. As the derivatives near to zero, the iteration step naturally reduces, so as not to miss the zero point. As in any iterative process, the more iterations are made, the closer to the zero value the derivatives can be brought; as a result of the decrease in the CD power step.

 TABLE IV.
 NODE RESISTANCE MATRIX FOR THE CIRCUIT, OHM-3

2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
2.6	90	6.9	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
2.6	6.9	46.9	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
2.6	2.6	2.6	142	6.9	2.6	2.6	2.6	2.6	2.6	2.6	2.6
2.6	2.6	2.6	6.9	16.9	2.6	2.6	2.6	2.6	2.6	2.6	2.6
2.6	2.6	2.6	2.6	2.6	209	6.9	2.6	2.6	2.6	2.6	2.6
2.6	2.6	2.6	2.6	2.6	6.9	210	2.6	2.6	2.6	2.6	2.6
2.6	2.6	2.6	2.6	2.6	2.6	2.6	15	2.6	2.6	2.6	2.6
2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	24	2.6	2.6	2.6
2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	19	2.6	2.6
2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	19	2.6
2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	19

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In connection with the fact that in the process of successive approximations it is impossible to accurately reduce the derivatives to zero (it is only possible to approach it infinitely), an admissible difference of the derivatives from zero has been established, at which the iteration process terminates.

The values of total monthly costs for 6 months using the presented methods of optimizing reactive power modes for a simulated network is presented in the Table V.

TABLE V. TOTAL MONTHLY COSTS

Total costs, USD						
Month	Neural networks with optimization algorithm	Traditional method				
1	103.35	1174.10				
2	109.38	969.60				
3	109.25	989.97				
4	99.31	1058.55				
5	90.29	1251.01				
6	82.08	1190.05				

V. CONCLUSIONS

Forecasting with the help of neural networks is one of the most effective and progressive methods for forecasting electrical loads nowadays. Its mean error of forecasting is 0.02%, against 3.12% by traditional methods. This is due to the ability of the network to "fully understand" the reactive load curve and try to reproduce it in the future, which is actually an imitation of human brain activity. In addition, the neural network has one very important advantage - the ability to add parameters to the input of the network, which, when forecasted, will allow to take into account such factors as the ambient temperature, the season of the year, etc. Consequently this will further reduce the forecast error.

Neural networks, despite their advantages, have a significant drawback - a complex mechanism for creating and configuring - you must first make up a program to predict the neural network, and then set up its work. In addition, when using a neural network, a fairly large sampling of data is required to set the weighting coefficients and the displacements between network neurons. It is also necessary to use high performance computing equipment for neural networks.

Forecasting with further optimization of reactive power modes by means of gradient methods of successive approximations of the objective function to zero value is effective. As a result, optimized levels of reactive power have been obtained at the network nodes, costs for reactive power, as well as for its overflows and the losses of active power caused by them, were reduced up to 10 times, as indicated in the comparison. It should also take into account the fact that an accurate forecast also allows more effectively planning the network's operation mode in the future.

However, the problem is that most compensating devices have discrete-level power control, and this can negatively affect the accuracy of compensation and optimization of reactive power flows. The more efficient application of the proposed methods for optimizing reactive power modes is using the continuously adjustable sources such as synchronous compensators or thyristor static compensators.

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