Purpose. Designing new models of construction machines is closely related to the development of slewing gear, and that, in turn, has a drive whose power and dimensions depend on the rotational resistance and the reduced friction coefficient in the units. The absence of analytical dependencies for determining the reduced coefficient of friction for the rotation of construction machines, first, restricts the designer's ability to select materials, and secondly, does not allow the adoption of optimal design solutions. Therefore, the purpose of the article is to find analytical solutions to determine the rotational resistance in the slewing gear of construction machines, which allows projecting more advanced gears and machines in general. Existing techniques are based on empirical dependencies and experimental coefficients that reduce the accuracy of calculations, increase the size and cost of work. It is proposed to improve the accuracy and simplify the process of determining the rotational resistance and the magnitude of the reduced rotational resistance coefficient of the building tower cranes.

Methodology. The set objectives can be achieved by means of analytical dependencies for determination of rolling friction coefficients over linear and point contacts. This will enable to find the more accurate value of the resistance coefficient, and the constructor during the calculations to take targeted measures to reduce it, using the mechanical constants of materials of the units and their geometric parameters. The calculation is based on Hertz contact deformation theory and the body point plane motion theory. Findings. The obtained dependencies will allow analytically to find the resistance of rolling resistance of rollers in construction machines with fixed and rotating pillars, with circular rotary devices, as well as in ball and roller slewing rings. The calculated values of the rotational resistance coefficients for some types of mechanisms give similar values with those recommended, while for others they significantly differ and require their refinement in reference values. Originality of the work consists in the use of analytical dependences for determining the reduced coefficient of the rotational resistance over linear and point contacts using Hertz contact deformation theory and Tabor partial analytic dependencies theory. Practical value. The obtained dependencies will allow to design new types of slewing gear units of the construction machines and to reveal the additional rotational resistances.

Keywords: construction machine; resistance; rotation; turn; slewing ring; rail; rolling friction

Introduction

There are the following types of slewing gears (SG) of the construction machines:

a) with a fixed pillar consisting of an upper support with a thrust and radial bearings;

b) with a rotating pillar: consists of a pillar connected to the revolving portion of the construction tower crane;

c) with a circular flat or tapered rail consisting of a series of conical or cylindrical rollers, which come in contact with two rails on the revolving and
non-revolving portions of the construction crane; 
g) with a slewing ring: consists of ball or roller 
single-row or multi-row structures (full-slewing 
and part-slewing excavators, motor graders).

One of the main causes of rotational resistance is rolling resistance [12, 13]. There are many 
studies and suggestions for its definition, but all of 
them are either inaccurate, like Reynolds’s asser-
tion that rolling resistance is the result of sliding 
friction at the contact point, or require an experi-
mental determination of one or more coefficients.

The analytic dependence of Tabor [3] on de-
termining the rolling friction coefficient, which 
is based on Hertz contact deformation theorem [6], is 
quite successful. Tabor obtained the following ana-
lytical dependences for determining the rolling 
friction coefficient, \( k \):

- for a linear contact:

\[
k = \frac{3b}{\alpha} \quad (1)
\]

- for a point contact

\[
k = \frac{3b}{16\alpha} \quad (2)
\]

where \( b \) – half-width of the contact pattern; \( \alpha \) – coefficient of hysteresis losses.

However, the presence in these formulas of the 
coefficient \( \alpha \) nullifies their practical application.

In [5], there are formulas analogous to (1) and 
(2) without coefficient \( \alpha \), namely: \( k = 0.11b \) and 
\( k = 0.1b \), that essentially differ from those offered 
by Tabor, and the absence of their coefficient of 
hysteresis losses testifies to their inaccuracy.

In [4], there are proposed the dependences for 
determining the rolling friction coefficient with the 
use of Tabor analytical dependences and the exper-
imental values of the rolling friction coefficient for 
the wheels of cranes with a flat champignon and 
bull-headed rails [1, 2].

Similarly to formulas (1) and (2) they are ob-
tained in the following form:

- for flat champignon rail:

\[
k = 0.225b e^{-1.2R} \quad (3)
\]

- for bull-headed rail:

\[
k = 0.16b e^{0.2R} \quad (4)
\]

The difference in numerical values from the 
half-width of the contact pattern is obviously due 
to the rounding of the coefficient \( k \) in experiments 
to ten millimeters, as well as to the fact that their 
values are obtained the same for several wheel di-
diameters (400, 500, 560, 630): \( k = 0.5 \) mm in the 
case of a flat champignon rail and \( k = 0.6 \) mm for 
the bull-headed rail).

It should be noted that formulas (3) and (4) are 
obtained independently of (1) and (2), and since 
the coefficients before \( b \) for such a class of prob-
lems can be considered close by value, we will 
assume that the general values of \( k \) in these for-
mulas coincide. Having considered that the coeffi-
cients before \( b \) in Tabor's formulas are obtained 
analytically and are exact, the value of \( \alpha \) can be 
found by changing the coefficients before \( R \) in 
the exponents. This equality can be achieved by taking 
the following values \( \alpha \) in formulas (1) and (2):

\[
\alpha = e^{-1.13R} \text{ and } \alpha = e^{0.23R}.
\]

**Purpose**

Designing new models of construction machines 
is closely related to the development of slewing 
gear, and that, in turn, has a drive whose power and 
dimensions depend on the rotational resistance and 
the reduced friction coefficient in the units [14–16]. 
The absence of analytical dependencies for deter-
mining the reduced coefficient of friction for the 
rotation of construction machines, first, restricts the 
designer’s ability to select materials, and secondly, 
does not allow the adoption of optimal design solu-
tions. Therefore, the purpose of the article is to find 
analytical solutions to determine the rotational re-
sistance in the slewing gear of construction ma-
chines, which allows projecting more advanced 
gears and machines in general. Existing techniques 
are based on empirical dependencies and experi-
mental coefficients that reduce the accuracy of cal-
culations, increase the size and cost of work. It is 
proposed to improve the accuracy and simplify the 
process of determining the rotational resistance and 
the magnitude of the reduced rotational resistance 
coefficient of the building tower cranes. More pre-
cise definition of the rotational resistance in the 
slewing gear of construction machines leads to sav-
ing the machine manufacturing and operation costs 
[21], as well as reduction of their harmful impact on 
the service staff and the environment [17–20].
Methodology

Now the formulas of Tabor (1) and (2) can be written as follows:

– for a linear contact:

\[ k = \frac{2b}{3\pi} e^{-1.13R}, \]  

(6)

– for a point contact:

\[ k = \frac{3b}{16} e^{0.23R}. \]  

(7)

With formulas (6) and (7), we can solve the set problems analytically.

In [7] it is indicated that the value of hysteresis losses \( \alpha \) in Tabor formulas is small. We can use formula (5) for its determination and (6), (7) for determination of the resistance.

Findings

1. Wheel rolling resistance. For a linear contact, we can take \( [\sigma] = 800 \) MPa (steel 65G, crane operating mode 4M [11]), the elastic modulus \( E = 2.1 \times 10^5 \) MPa, the Poisson factor is 0.3.

When the value of the pressure restraining force \( P \) [4]

\[ P = \frac{BR[\sigma]^2}{0.418 E}, \]  

(8)

the half-width of the contact pattern will be

\[ b = 1.526 \sqrt{\frac{PR}{BE}}, \]  

(9)

where \( B \) – wheel width, m; while the rolling friction coefficient can be determined by the formula (6).

For the point contact we can take \( [\sigma] = 1040 \) MPa, the radius of the bull-head rail \( R_e = 300 \) mm. Similarly to the formulas (8) and (9) we can determine the values for the point contact

\[ P = \frac{R^2 R_e^2 [\sigma]^3}{0.245 n_p^3 E^2 (R + R_e)^2}, \]  

(10)

\[ b = 1.397 n_p \sqrt{\frac{P}{E} \frac{RR_e}{R + R_e}}, \]  

(11)

where \( n_p \) – coefficient depending on the tangent ellipse equation coefficient \( A/B = R_e / R_i \); \( R_i \) – rail rounding radius.

Depending on the wheel radius of the pressure restraining force, the coefficient of hysteresis losses, the coefficient of rolling friction and resistance are shown in Fig. 1.

Since the rolling friction coefficient for the wheels of the construction cranes corresponds to their certain radius, it can be assumed that the relationship between the force of rolling resistance and the load on the wheel is linear. But the rolling friction coefficient is determined by the half-width of the contact pattern, depending on several parameters not linearly, therefore, it is necessary to establish the dependence of the wheel rolling resistance on the load.

Fig. 1. Dependence on the wheel radius for linear a) and point contact b)
(points show the reference values of the rolling friction coefficients):

1 – wheel pressure restraining force; 2 – coefficient of hysteresis losses;
3 – rolling friction coefficient; 4 – rolling resistance coefficient
For this, the load \( P \) on the wheels with the radii \( R_1 = 500 \) mm and \( R_2 = 100 \) mm can be divided on two wheels in the ratio \( P_2 = \frac{P_1}{P} \).

Dependences of the coefficients of rolling friction, loading and rolling resistance of the wheel and the total resistance of the wheels are shown in Fig. 2

![Fig. 2. Dependences of the ratio of the applied forces for linear (a) and point (b) contacts:](image)

1, 1. \( I_1 \), \( I_2 \) – total value of the pressing force and the force acting on each wheel; \( 3_1 \), \( 3_2 \) – rolling friction coefficients; \( 4_1 \), \( 4_2 \) – total rolling resistance value and rolling resistance of each wheel;

lower position of curves for wheel \( R = 100 \) mm, upper for wheel \( R = 500 \) mm

Analytic dependencies (6) and (7) are used to determine the coefficients rolling friction, so it is possible to restore one lacuna in the reference literature. Losses in roller bearings are found by the coefficient of friction reduced to the shaft (ball \( \mu = 0.01...0.015 \), roller \( \mu = 0.015...0.02 \) [2]). However, this does not take into account, which race is rotating, inner or outer one.

Assuming that the deviation in the coefficient is negligible, it should be borne in mind that the number of locally positioned bearings may be significant (conveyors, vehicles), as well as an increase in the efficiency from 0.99 to 0.995 per ten bearings gives it an increase in more than 5%.

2. Ball bearings. The tasks to be clarified when calculating resistance:

1) To take into account the difference in the coefficients of rolling friction during rolling of the ball on the inner and outer races, since for calculating their size we take them equal, and the tangential force acting on the ball (Fig. 3, a) is defined as [8]

\[
F_i = \frac{P k}{r_e};
\]

2) To take into account the rotation of the race, since the special feature of the roller bearings design is that the balls (rollers) pass different lines during one revolution of the inner or outer race.

Under the simplified scheme of the bearing, the problem is solved as follows. If the outer race rotates at an angular velocity \( \omega_o \) (Fig. 3, b), then the speed of point 1 as the point belonging to the outer race will equal:

\[
v_o = \left( r_i + 2r_b \right) \omega_o = 2\pi n \left( r_i + 2r_b \right), \quad (12)
\]

Where \( \omega \), \( i \), \( b \) are the letters of the indices of sizes and speed of outer, inner races and ball; \( n \) – frequency of rotation of both inner and outer races.
Fig. 3. Elements of bearings:

- scheme for determining the tangential force during the rotation of inner race [1],
- scheme for determining the speed of points of outer race and ball; 
- contact pattern

Naturally, that the instantaneous velocity center of this race is located at point 2 of the ball touch.

Assuming that there is no slip between the outer race and the ball, then

\[ v_1 = v_2. \]

The length of the ball rolling track on the outer race

\[ l_o = 2\pi r_o, \]

and on the inner race

\[ l_i = 2\pi r_i \]

and the length difference will be \( \Delta l = 2\pi(r_o - r_i) \), that is, on this track there will be ball sliding on the inner race.

In case of rotation of the inner race with the fixed outer race the difference \( \Delta l \) is evident that the ball will pass the outer race track that equals the inner race track.

We find the load on the balls based on their number [8]:

\[ z = 2.9 \frac{D + d}{D - d}. \quad (13) \]

The force acting on the most loaded ball is:

\[ P_0 = \frac{5Q}{z}. \quad (14) \]

For further calculations, the radius of the ball (without rounding to the standard one) and of the rolling bearing track will be equal [8, 9]:

\[ d_b \approx 0.3\left(D - d\right); \]

\[ r_i \approx 1.03r_b. \]

For the number of balls \( z \geq 10 \) the load on the bearing \( Q \) (for example, if \( z = 10 \)) [8]:

\[ Q = P_0\left(1 + 2\cos^{5/2} \gamma + 2\cos^{5/2} 2\gamma\right), \quad (15) \]

where \( \gamma \) is the angle between the balls (here \( \gamma = 36^\circ \)). Based on this, the load on the side balls

\[ P_1 = P_0 \cos^{5/2} \gamma, \quad P_2 = P_0 \cos^{5/2} 2\gamma. \quad (16) \]

The values of the half-width of contact patterns in formulas (9) and (11) are determined from expressions (17) and (18). When rolling the ball on the inner ring:

\[ b_i = 1.397 n_{ov} \sqrt{\frac{P}{E} \frac{1}{1 - \frac{1}{r_b} - \frac{1}{r_i} - \frac{1}{r_0}}}, \quad (17) \]

where \( n_{ov} \) is the coefficient, depending on the tangency ellipse equation

\[ A = \frac{1}{\frac{1}{r_b} + \frac{1}{r_i} + \frac{1}{r_0}}. \]

In formulas (13) – (17) \( D \) – outer bearing diameter; \( d \) – inner bearing diameter; \( r_i \approx 0.5d + r_b \) radius of the track of the inner race.

At \( b_i \) for the most loaded ball, it is necessary to set optionally the value of \( P \), and for the side balls \( P_1 \) or \( P_2 \) depending on the number of balls.

When rolling the ball on the outer race:

\[ b_o = 1.397 n_{ov} \sqrt{\frac{P}{E} \frac{1}{1 - \frac{1}{r_b} - \frac{1}{r_i} - \frac{1}{r_0}}}, \quad (18) \]

where \( n_{ov} \) is determined as a function;

\[ A = \frac{1}{\frac{1}{r_b} - \frac{1}{r_o}}; \quad r_o \approx 0.5d + 3r_b \] is the radius of the outer race track.
3. Influence of resistance in bearings on wheel rolling resistance. Let us consider two rolling bearings of one series, but of essentially different sizes.

3.1. Ball bearing of 304 series. Calculation output data: bearing of 304 series, \( d = 20 \) mm, \( D = 52 \) mm, static load \( Q = 7.94 \) kN, average diameter \( D_{av} = 0.5(D + d) = 36 \) mm, \( d_b = 9.6 \) mm, number of balls \( z = 7 \) at \( \gamma_i = (360°) : 7 = 51.4° \), \( r_i = 14.8 \) mm; \( r_o = 242.4 \) mm; \( r_t = 4.944 \) mm.

Half-width contact pattern of the ball, loaded with force \( P_i = 1740 \) N, with the inner race \( b_{ir} = 0.23 \) mm for \( n_i = 0.38 \), with the outer race \( b_{or} = 0.3 \) mm for \( n_o = 0.42 \). Correspondingly, the side balls loaded by force \( P_i = 1740 \) : \( b_{il} = 0.155 \) mm; \( b_{ol} = 0.202 \) mm. Resistance to rolling of the most loaded ball: on the inner race \( W_{ir} = 44.45 \) N, with the rolling friction coefficient \( k_{ir} = 0.0434 \) mm, on the outer race \( W_{or} = 57.77 \) N with \( k_{or} = 0.0564 \) mm; two side balls on the inner race \( W_{i1} = 18.30 \) N with \( k_{i1} = 0.029 \) mm and \( W_{ol} = 23.90 \) N with \( k_{ol} = 0.038 \) mm.

Let us determine the work of the rolling friction forces during one rotation of the inner and outer races.

During rotation of the inner race, Nm:
\[
A_i = 2\pi r_i (W_{ir} + W_{i1} + W_{or}) = 13.4; \quad (19)
\]

During the rotation of the outer race, Nm:
\[
A_o = 2\pi \left[ r_o (W_{or} + W_{o1}) + r_t (W_{ir} + W_{i1}) \right] + 2\pi f (P_i + 2P_i)(r_o - r_i) = 14.99 + 5.83 + 59.98 = 20.82 + 59.98 = 80.8. \quad (20)
\]

Thus, during the rotation of the inner race, the rolling friction force work during one rotation equals \( A_i = 13.4 \) Nm, in case of the outer race rotation \( A_o = 20.82 \) Nm (1.55 times higher), and taking into account sliding \( A_{alsl} = 20.82 + 59.98 = 80.8 \) Nm, that is, 6 times higher.

In this case, the value of the conditional coefficient of friction reduced to the shaft is equal to: during the rotation of the inner race \( \mu_i = \frac{A_i}{2\pi Q r_i} = 0.018 \), for the recommended value \( \mu = 0.010...0.015 \), and during the rotation of the outer race \( \mu_o = \frac{A_o}{2\pi Q r_o} = 0.081 \).

3.2. Ball bearing of 2306 series. Calculation output data for the bearing of 2306 series: \( d = 30 \) mm, \( D = 72 \) mm, static load \( Q = 20.6 \) kN, roller diameter \( d_i = 0.25(D - d) = 10.5 \) mm, roller length \( l_t = d_i, \ d_t = 10.5 \) mm, number of rollers \( z = \frac{5(D + d)}{(D - d)} = 12 \) at \( \gamma_i = 360° : 12 = 30° \), bearing track radius on the inner race \( r_i = 0.5d + 0.5d_t = 20.25 \) mm, track radius on the outer race \( r_o = 0.5d + 1.5d_t = 30.75 \) mm.

The force acting on the most loaded and side rollers is determined from formulas (15) and (16).

It was proved in [9] that if a load is applied to a group of bodies according to the cosine law, then to determine the resistance to their rolling, all loads can be applied to one body, that is, the rolling resistance of all five rollers on the inner race for the linear contact is determined from the expression:
\[
b_i = 1.522 \sqrt{\frac{Q}{BE}} \cdot \frac{r_i \cdot r_t}{r_t + r_t}, \quad (21)
\]
and on the outer race:
\[
b_o = 1.522 \sqrt{\frac{Q}{BE}} \cdot \frac{r_i \cdot r_t}{r_t - r_t}. \quad (22)
\]

According to formula (6), the rolling friction coefficient will be respectively \( k_i = 0.0636 \) mm, \( k_o = 0.0876 \) mm. The rolling resistance of rollers: on the outer race \( W_o = 343.7 \) H, N, and on the inner race \( W_i = 249.6 \) N.

Work of rolling and sliding friction forces on the inner and outer races, Nm:
\[
A_i = 2\pi r_i (W_i + W_o) = 75.4, \quad A_o = 2\pi (r_i W_i + r_t W_o) + 2\pi Qf (r_o - r_t) = 98.1 + 135.8 = 233.9. \quad (23)
\]
for the friction sliding coefficient of rollers on the inner race } f = 0.1 .

The motion resistance coefficient is: during the rotation of the inner race } \omega_i = \frac{W_i}{Q} = 0.012 , during the rotation of the outer race } \omega_o = \frac{W_o}{Q} = 0.017 , for the recommended value [9] for the wheel with up to 700 mm diameter } \omega = 0.02 .

4. Ball-bearing slewing gear (SG). The formula for determining the greatest pressure on the ball, given in [11], contains two unknowns: the average diameter of the rolling circle and the number of balls.

If the first unknown can be set on the basis of constructive considerations, then the number of balls can be set after finding their diameters. In addition, this formula is acceptable only if the reaction from the moment does not go beyond the support contour.

We propose finding the moment of the friction forces in the following sequence.

4.1. The slewing ring is broken, for example, into 10 sectors with a central angle } \gamma = 36^\circ and for constructive reasons the average radius of the ball centers is taken } R_{av}.

4.2. We apply the load to one conditional ball in the sector, similar to the ball bearing (15), we find the maximum vertical pressure on it from the moment, Nm:

\[ N_{0m} = \frac{M}{2R_{av}(1+2\sin \gamma_1 \cos \gamma_1 + 2\sin 2\gamma_1 \cos 2\gamma_1)}. \quad (24) \]

Under the known value of vertical pressure } V , the pressure on the balloon will be:

\[ N_{0t} = \left( N_{0m} + V \frac{\gamma_1}{2\pi} \right) \frac{1}{\cos \beta} \text{ Nm,} \quad (25) \]

where } \beta is the angle between the reaction of the ball and the vertical line (usually } \beta = 45^\circ ) (Fig. 4).

Maximum pressures on conditional side balls, Nm:

\[ N_{0t1} = N_{0t} \cos \gamma_1 , \]
\[ N_{0t2} = N_{0t} \cos 2\gamma_1 . \quad (26) \]

4.3. Maximum pressure on the opposite (left) conditional ball:

\[ N_{00} = \left( -N_{0m} + V \frac{\gamma_1}{2\pi} \right) \frac{1}{\cos \beta} \text{ Nm.} \quad (27) \]

The pressure on the left conditional side balls is found in the same way as for the right ones.

4.4. After the value } R_{av} , we roughly take the diameters of the ball } d_b = 0.4R_{av} .

4.5. We find the number of balls in one sector with geometric conditions: } n = \frac{\gamma R_{av}}{d_b + 5} .

4.6. Maximum pressure on one ball of the right sector } P_0 = \frac{N_{0m}}{n} and the ball radius, based on Hertz contact pressure for the track radius } r_u = 1.2 r_b \text{ mm:}

\[ r_b = 0.1 n_t \sqrt{\frac{P_{Er} E^2}{\sigma^3}} \text{ Nm}, \quad (28) \]

where } n_t is the value depending on the ratio of tangible ellipse equation factors } \frac{A}{B} = \frac{r_b}{r_u} ;} \frac{1}{r_b} ; \sigma - boundary contact stresses depending on the steel grade, contact type and Brinell-hardness; for } r_u = 1.2 r_b \text{ mm, } n_t = 0.86 \text{ (} n_s = 1.96 ; \quad n_i = 0.59 \text{ [4]).}

4.7. We find the final diameter of the ball, while proceeding from conditions 4.4 and 4.6 and determine the number of balls.

4.8. Based on the equations (25), (26), (27) and the number of balls, we determine the pressure on one ball per sector, the rolling resistance by the formula (7) of one of the balls of 10 sectors.

We find the rolling resistance of } z \text{ balls and total pressure as the sum of values obtained by the formulas (25) and (27).

We find the rotational resistance coefficient as the ratio of the total rotational resistance to the total pressure.

Calculations are carried out according to the following data: the greatest moment acting on the slewing ring } M = 427 \text{ kNm, the largest vertical reaction } V = 178 \text{ kN, the average diameter of the}
ball centers \( D_{av} = 1500 \text{ mm} \) \( (R_{av} = 750 \text{ mm}) \).

In this case, the vertical pressure from the moment, taking into account the side balls (24) will be \( N_{0av} = 112 \text{ kN} \), and pressure on the right and left conditional balls (25) and (27) \( N_{0av} = 183.6 \text{ kN} \), having taken the ball diameter \( d_b = 0.4R_{av} = 30 \text{ mm} \) and the maximum pressure on one ball, we will have:

\[
P_{00} = \frac{N_{0av}}{n} = \frac{183.6}{13} = 14.12 \text{ kN},
\]

where \( n \) is determined from geometric conditions \( n = \frac{\gamma R_{av}}{d_b + 5} \approx 13 \). Let us check the taken ball radius

\[
r_b = 0.1\sqrt{\frac{P_{00}E_2}{\sigma}} \text{ Nm,} \tag{29}
\]

where \( r_b = 15 \text{ mm} \) for \( [\sigma] = 3000 \text{ MPa} \) (surface hardened steel 45 [2]).

For \( n = 0.59 \) taking into account the number of balls in the sector and pressure on the conditional central and side balls (26), we find the half-width of the contact pattern:

\[
b = 1.397n_i\sqrt{\frac{P_1}{E}\frac{r_e \cdot r_b}{2r_e - r_b}}. \tag{30}
\]

The rolling friction coefficient is determined by the formula (7), and the rolling resistance subject to two rolling surfaces, i.e. \( W = \frac{2kP}{r_b} \).

The distribution of pressure per ball on the ring length and the rolling resistance of each ball in the form of graphs are shown in Fig. 5 [5, 12-13].

When adding all the pressures on the balls and their resistance to rolling and disision of \( W = 17.42 \text{ kN} \) by \( P = 1025.3 \text{ kN} \), we obtained the value of the reduced rotational resistance of the crane \( \omega = 0.017 \), that significantly exceeds the recommended value \( \omega = 0.01 \).

The reasons for this discrepancy may be: a) irrelevance of the value adopted here \([\sigma]\) to the valid one; b) understated value \( M \) during the experiment.

It can be emphasized that in the examples of SG calculations, given in [11, 12], the coefficient \( \omega \) is taken in relation to these quantities, and in [13] \( \omega = 0.04 \).

Let us find the value of the rotational resistance coefficient, which falls on sliding during rolling along the ring. Usually it is taken into account only when moving along a cylinder ring. However, in the case of ball contact with both the plane and the bearing track, the contact pattern is not a point, but the ellipse with the axes \( 2a \) and \( 2b \), the length of which is determined from the Hertz contact deformation formulas.

The average pressure per ball during its rotation by \( 360^\circ \) is \( P_1 = 8.6 \text{ kN} \). Herewith, the minor axis of the ellipse is \( a = 2.2 \text{ mm} \).

Concentrating the pressure at 4 points, we find that the pressure at the points \( v \) and \( n \) (see Figure 3) is \( P_0 = 8.6 \text{ kN} \). In this case, the vertical axis of the ellipse [4]:

![Fig. 4. Design diagram of the ball-bearing SG](image-url)
The distance from these points to \( R_{av} \) is \( 3a/16 \), i.e. 0.41 mm.

The difference in the distance travelled by one rotation of the ring is \( 4\pi a = 5.15 \text{ mm} \). For \( 2P_1 = 4.3 \text{ kN} \), \( f = 0.15 \) (steel on steel, no lubrication), the work of sliding friction forces will be \( A_{sl} = 2P_1f\ell = 3.32 \text{ Nm} \). Expressing the work of normal forces \( 2P_1 \) through the reduced coefficient, we can obtain \( A_{sl} = 2\alpha P_1 \cdot 2\pi R_{av} \), wherefrom and is about 0.01 of the recommended value of the reduced rotational resistance coefficient of the building cranes. However, it should be borne in mind that the denominator of the formula defining \( \omega_c \), includes the average radius of the ball centres \( R_{av} \).

The distribution of pressure per ball on the ring length and the rolling resistance of each ball in the form of graphs are shown in Fig. 5.

Fig. 5. Distribution of pressures per one ball and resistance to its rolling along the ring

5. Roller slewing gear. For the calculation example, we considered the slewing gear of the construction tower crane with a fixed pillar and fixed rollers (Fig. 6).

Calculation output data: construction tower crane; average diameter of the thrust ball bearing \( d_{av} = 97.5 \text{ mm} \); diameter of the bearing ring \( D = 2R = 1500 \text{ mm} \); horizontal reaction \( H = \frac{M}{h} = 21.4 \text{ t} \), where \( M = 87 \text{ t} \cdot \text{m} \) – the resulting moment of the rotary part in the vertical plane; \( h = 4 \text{ m} \) – the distance between the line of application of reactions \( H \) and the journal; vertical reaction \( V = 18 \text{ t} \).

Fig. 6. Design diagram of the slewing gear of a construction tower crane with fixed rollers on roller bearings, thrust bearing and top slide journal

5.1. Calculation of support rollers. Load on the roller, located on the line of force \( H \)

\[
P_0 = \frac{H}{1 + 2\cos \gamma} = 88.65 \text{ kN}, \text{ the force acting on each of the two side rollers is the same and is equal to } P_1 = \frac{P_0}{\cos \gamma} = 125.39 \text{ kN}.
\]

For the roller width \( B = 0.25D_i \text{ mm} \) and
[\sigma] = 750 \text{ MPa (steel 75, mode of operation 5M)}, its radius is determined from the Hertz contact forces formula:

\[ R_t = \frac{0.418^2 P_E}{R \sigma^2} + \left( \frac{0.418^2 P_E}{R \sigma^2} \right)^2 + \frac{0.418^2 P_E}{0.5 \sigma^2}, \]

and it is 130 mm.

Half-width of the contact pattern

\[ b = 1.322 \frac{P_t}{BE} \frac{RR_t}{R - R_t}, \]

and it is 1.75 mm.

Rolling coefficient of the side roller (6) \( k_i = 0.32 \text{ mm}, \) and that located on the horizontal axis \( k_o = 0.23 \text{ mm}. \)

Rolling resistance of three rollers:

\[ W = W_0 + 2W_1 = 154 \cdot 617 = 771 \text{ N}. \]

Due to the high pressure on the rollers and the impossibility of selecting the appropriate roller bearing, it is possible to apply in the construction tower cranes the bearings with friction sliding coefficient \( \mu = 0.1 \) and with the journal diameter \( d_j = 0.25D_t = 65 \text{ mm}. \)

Friction resistance in roller journals:

\[ W_j = \frac{0.5d_j}{0.5D_t} (2P_i + P_0) = 16.97 \text{ kN}. \]

For the top journal diameter \( d_t = 150 \text{ mm} \) the sliding resistance in it

\[ W_{j1} = H \mu = \mu \frac{M}{h} = 21.4 \text{ kN}. \]

5.2. Resistance in thrust bearing. According to the value of static load on the bearing \( V = 180 \text{ kN} \) we take the bearing of 8216 series with \( d = 80 \text{ mm}, \ D = 115 \text{ mm}, \) ball diameter \( d_b = 14 \text{ mm}, \) number of balls \( z = 20, \) track radius \( r_t = 0.54 \text{ mm}. \)

When loading one ball \( P = V/20 = 9 \text{ kN}, \) the half-width of the contact pattern (11) \( b = 0.447 \text{ mm} (n_1 = 0.49, n_2 = 2.7). \)

The rolling friction coefficient according to formula (7) is \( k = 0.084 \text{ mm}. \)

The rolling resistance of 20 balls \( W_0 = W_1 \cdot 20 = 108 \cdot 20 = 2160 \text{ N}. \) The ball sliding resistance coefficient in accordance with (32) for \( f = 0.1 \) (thick lubrication) \( \omega_c = 0.01 \) and for its rolling resistance value is \( W_c = 0.5\omega_V V = 900 \text{ N}, \) which is about half of the rolling resistance and requires consideration when calculating the units with thrust bearings.

The total moment of frictional forces during the turning of a construction crane with a fixed tower consists of the resistances:

- rolling of support rollers 771 N (20% of the total);
- in the roller journals 17 N (0.4% of the total);
- in the top journal 21 N (0.5% of the total);
- in the support roller from the rolling of balls 2 160 N (56% of the total) and their sliding 900 N (23% of the total) for a total rotational resistance value of 3 870 N.

6. Rotational resistance of SG rollers with stationary and fixed rollers. In some construction cranes, the support rollers are stationary (Fig. 7, a) or movable (Fig. 7, b). Load per a roller

\[ P = \frac{H}{2 \cos \alpha}. \]

It is obviously that for cylindrical rollers, the values of the maximum contact stresses will be different, and the diameters of the rollers and their rolling support on the slewing ring will have different values as well.

For calculations we take the same radius, as in the previous example, \( D = 2R = 1500 \text{ mm}, \) the horizontal reaction is equal to \( H = 25 \text{ kN}, \) the boundary contact stress is \( [\sigma] = 750 \text{ MPa}, \) the roller width \( B = 0.25D_t = 0.5R_t. \)

The radius of the roller for the diagram \( a \) (Fig. 7) can be found from formula (33), by replacing \( P_i \) with \( H/\cos \alpha. \) According to the taken values \( R_t = 60 \text{ mm}, \) the journal radius is taken to be equal to \( r_j = 15 \text{ mm} \) for cases \( a \) and \( b \) (Fig. 7).

The radius of the roller for the diagram \( b \) (Fig. 7) can be found by the same formula (33) in the case of change of the sign under the radical to the inverse, and it will be equal to \( R_j = 50 \text{ mm}. \)

Half-width of the contact patterns is found by the formula (34) with the change of the sign

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Half-width of the contact patterns (Fig. 7):
− according to the diagram $a\ a = 0.11$ mm, according to the diagram $b - a = 0.12$ mm;
− rolling resistances of the two rollers are respectively $W = 17.8$ N and $W = 23.4$ N.

The resistance in the journals of the two rollers according to the first and second diagrams $f = 0.1$ is $W_j = 2\frac{H}{\cos \alpha}f = 5.77$ kN, that is more than two orders higher than the rolling resistance of rollers.

**Originality and practical value.** The paper proposes to use analytical dependences to determine the reduced rotational resistance coefficient for linear and point contacts using Hertz contact deformations theory and Tabor partial analytic dependencies. The obtained dependencies will allow to design new types of slewing gear assemblies of the construction machines and to find additional rotational supports, which depend on the overall dimensions, shape and type of material from which the components of the assembly are made and do not contain any empirical data.

![Fig. 7. Design diagram of the slewing gear of cranes:](image)

Fig. 7. Design diagram of the slewing gear of cranes:

$\text{a} - \text{with stationary rollers; } \text{b} - \text{with moving rollers}$

**Conclusions**

The analysis of the dependencies and graphs obtained makes it possible to draw the following conclusions and suggestions:
− rolling friction coefficient and rolling resistance of the crane type wheels practically linearly depends on the wheel radius, and the coefficient of hysteresis losses linearly decreases from 0.9 to 0.6 per linear contact and linearly increases from 1.01 to 1.06 per point contact;
− the friction coefficient value of rolling bearings, reduced to the shaft, depends on whether inner or outer race rotates;
− during the outer race rotation on a different path, passed by a ball or a roller on the tracks of the outer and inner races, the friction coefficient reduced to the shaft, which falls on pure rolling, is 1.3...1.5 times higher than its value during the inner race rotation, and taking into account the sliding of balls or rollers on the inner race, it is 4...6 times higher than its size in ball bearings and 3...4 times higher – in roller bearings;
− due to the high value of friction in case of outer race rotation, during the design of rolling assemblies, it is necessary to avoid such solutions, and, if it is impossible, to take into account this fact both for the determination of resistance and for the lubrication of the assembly;
− the given value of the resistance of the construction crane with the ball-bearing sewing gear is obtained analytically, is 70% higher than the one recommended by supplier;
− in case of construction cranes with a turning tower, the greatest resistance to rotation falls on support rollers (about 80%).

**LIST OF REFERENCE LINKS**


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АНАЛИТИЧЕСКОЕ ОПРЕДЕЛЕНИЕ ПРИВЕДЕННОГО КОЭФФИЦИЕНТА СОПРОТИВЛЕНИЯ ВРАЩЕНИЮ МЕХАНИЗМОВ ПОВОРОТА СТРОИТЕЛЬНЫХ МАШИН

Цель. Проектирование новых образцов строительных машин тесно связано с разработкой механизмов поворота, а те, в свою очередь, имеют привод, мощность и габариты которого зависят от сопротивления поворота и приведенного коэффициента трения в узлах. Отсутствие аналитических зависимостей для определения приведенного коэффициента трения вращению строительных машин, во-первых, ограничивает возможности конструктора в выборе материалов, а во-вторых, не дает возможности принимать оптимальные конструктивные решения. Поэтому цель статьи – найти аналитические решения для определения сопротивления вращению в механизмах поворота строительных машин, что позволит проектировать более совершенные механизмы и машины в целом. Существующие методики опираются на эмпирические зависимости...
мости и экспериментальные коэффициенты, уменьшающие точность подсчетов, увеличивающие габариты и стоимость работ. Предлагается повысить точность и упростить процесс определения сопротивления поворота и величину приведенного коэффициента сопротивления вращению строительных башенных кранов.

**Методика.** Достичь поставленной цели можно с помощью аналитических зависимостей для определения коэффициентов трения качения при линейном и точечном контактах. Это позволит точнее найти величину коэффициента сопротивления вращению, а конструктору при расчетах принять целенаправленные меры по его уменьшению, используя механические константы материалов узлов качения и их геометрические параметры. Расчет основывается на теории контактных деформаций Герца и теории плоского движения точек тела.

**Результаты.** Полученные зависимости позволяют аналитически найти сопротивление качению роликов в строительных машинах с неподвижной и вращающейся колоннами, с круговыми поворотными устройствами, а также в шариковых и роликовых опорно-поворотных кругах. Найденные значения коэффициентов сопротивления вращению для некоторых типов механизмов дают близкие значения с рекомендуемыми, а для некоторых – существенно отличаются и требуют их уточнения в справочниках величин.

**Научная новизна работы** заключается в использовании аналитических зависимостей для определения приведенного коэффициента сопротивления вращению для линейного и точечного контактов с использованием теории контактных деформаций Герца и частично аналитических зависимостей Табора.

**Практическая значимость.** Полученные зависимости позволят проектировать новые типы узлов вращения механизмов поворота строительных машин и выявлять дополнительные опоры вращению.

**Ключевые слова:** строительная машина; сопротивление; вращения; поворот; поворотный круг; рельс; трение качения

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