Dynamics of Reinforced-Concrete Spans with Railway Track Eccentricity

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Significance effects of railway track eccentricity on railway span vibration character by locomotive moving with various speeds are investigated. The mathematical model of the eccentricity of a railway track on the span of railway bridges is proposed. The dynamic analysis of three-dimensional girder systems based of the Newton-Euler differential nonlinear equations modeling aspects are represented.

Keywords: **vibrations of railway bridges, railway track eccentricity, reinforced-concrete spans, Euler-Lagrange differential equations, solid bodies system dynamics, inertia moments of asymmetric element.**

INTRODUCTION

During operation of the reinforced-concrete spans with a ballast bridge road, they are subject to defects of various kinds leading to the increase of loading on separate load-bearing structure elements. A displacement of the rail track axis relative to the bridge axis (an eccentricity) is one of the most widespread defects. The information on dynamic work of such spans is practically absent in the scientific literature that requires the conduction of additional research.

EULER-LAGRANGE EQUATIONS OF MOTION

The girder structure of a span is divided into parts within which the cross-sections of elements and bending stiffnesses are considered as constants. Each part of a girder is simulated by a solid body with corresponding geometrical and inertial characteristics and can have, in general, spatial translation and rotary motions. Solid bodies are connected to one another by means of elastic bracings – rods with stiffness characteristics of an initial girder design (Fig. 1).

The mathematical model of vibrations for the s-th body of a "bridge-train" system is led to the first order differential equations based on the energy conservation laws in the form of Euler-Lagrange [1]:

$$\left(\frac{\partial T}{\partial \omega_s}\right)^{\cdot} + \sum_{r=1}^{n}\sum_{t=1}^{n} \gamma_{t,s}^{r} \frac{\partial T}{\partial \omega_r}\omega_t - \frac{\partial T}{\partial \pi_s} = Q_s , \qquad (1)$$

where T is kinetic energy of a system; n is quantity of generalized coordinates; ω_s is quasi-velocity; Q_s is the generalized force referred to quasicoordinate π_s; $\gamma_{t,s}^{r}$ are Boltzmann three-index symbols [1].

The supplementary conditions considering the kinematics of system motion are necessary for a solution of the system of differential equations (1):

$$\dot{q}_r = b_{r1}\omega_1 + \ldots + b_{rn}\omega_n = \sum_{s=1}^{n} b_{rs}\omega_s , \qquad (2)$$

where q_s are displacements of centers of mass of system elements (generalized coordinates); b_{rn} are coefficients for expression of quasi-velocities components on directions of generalized coordinates.

Let us take the linear and angular velocity vector components to the axes of a global coordinate system as quasicoordinates [2]:

$$\begin{cases} \omega_1 = v_{c,x}; \\ \omega_2 = v_{c,y}; \\ \omega_3 = v_{c,z}, \end{cases} \quad \begin{cases} \omega_4 = \omega_{c,x}; \\ \omega_5 = \omega_{c,y}; \\ \omega_6 = \omega_{c,z}. \end{cases} \qquad (3)$$

The partial differentials of kinetic energy of a system element with respect to the mentioned quasi-velocities ω_s are estimated as:

$$\frac{\partial T_i}{\partial \omega_1} = \frac{\partial T_i}{\partial v_{c,x}} = \frac{1}{2} m_i \frac{\partial v_{c,x}^2}{\partial v_{c,x}} + m_i \frac{\partial v_{c,x}}{\partial v_{c,x}} \omega_{c,y} \cdot r_{i,z}^{(c)} =$$
$$= m_i v_{c,x} + m_i \omega_{c,y} \cdot r_{i,z}^{(c)} . \qquad (4)$$

$$\frac{\partial T_i}{\partial \omega_4} = \frac{\partial T_i}{\partial \omega_{c,x}} = \frac{1}{2} m_i v_{c,z} \frac{\partial \omega_{c,x}}{\partial \omega_{c,x}} \cdot r_{i,y}^{(c)} - $$
$$ - \frac{1}{2} m_i v_{c,y} \frac{\partial \omega_{c,x}}{\partial \omega_{c,x}} \cdot r_{i,z}^{(c)} + \frac{1}{2} J_{i,x} \frac{\partial \omega_{c,x}^2}{\partial \omega_{c,x}} = $$
$$ = \frac{1}{2} m_i \left(v_{c,z} \cdot r_{i,y}^{(c)} - v_{c,y} \cdot r_{i,z}^{(c)} \right) + J_{i,x} \omega_{c,x} . \qquad (5)$$

Their derivatives with respect to time will be equal to:

$$\begin{cases} \dfrac{d}{dt} \dfrac{\partial T_i}{\partial \omega_1} = m_i \dot{v}_{c,x} + m_i \dot{\omega}_{c,y} \cdot r_{i,z}^{(c)} ; \\ \dfrac{d}{dt} \dfrac{\partial T_i}{\partial \omega_2} = m_i \dot{v}_{c,y} + m_i \dot{\omega}_{c,z} \cdot r_{i,x}^{(c)} ; \\ \dfrac{d}{dt} \dfrac{\partial T_i}{\partial \omega_3} = m_i \dot{v}_{c,z} + m_i \dot{\omega}_{c,x} \cdot r_{i,y}^{(c)} ; \end{cases} \qquad (6)$$

$$\begin{cases} \dfrac{d}{dt} \dfrac{\partial T_i}{\partial \omega_4} = \dfrac{1}{2} m_i \left(\dot{v}_{c,z} \cdot r_{i,y}^{(c)} - \dot{v}_{c,y} \cdot r_{i,z}^{(c)} \right) + J_{i,x} \dot{\omega}_{c,x} ; \\ \dfrac{d}{dt} \dfrac{\partial T_i}{\partial \omega_5} = \dfrac{1}{2} m_i \left(\dot{v}_{c,x} \cdot r_{i,z}^{(c)} - \dot{v}_{c,z} \cdot r_{i,x}^{(c)} \right) + J_{i,y} \dot{\omega}_{c,y} ; \\ \dfrac{d}{dt} \dfrac{\partial T_i}{\partial \omega_6} = \dfrac{1}{2} m_i \left(\dot{v}_{c,y} \cdot r_{i,x}^{(c)} - \dot{v}_{c,x} \cdot r_{i,y}^{(c)} \right) + J_{i,z} \dot{\omega}_{c,z} . \end{cases} \qquad (7)$$

As a result, the coordinate mode of the relative motion equations (1) - (2) in a matrix form is given [3]:

$$\begin{cases} m_k \tilde{U}_k \tilde{S}_k = F_k ; & (8) \\ 2 \dot{a}_k = A_k \omega_k ; & (9) \\ \dot{X}_k = {}^Z R^Y v_k , & (10) \end{cases}$$

where

$$\tilde{U}_k = \left\| U_k^* \mid S_k^* U_k^* \right\| ; \; \tilde{S}_k = \left\{ \ddot{S}_k \mid \dot{S}_k \right\} , \qquad (11)$$

$\dot{S}_k = \{ \omega_k \mid v_k \}$ is a block vector of derivatives of kinematic parameters; $\omega_k = \{ \omega_x \quad \omega_y \quad \omega_z \}$ and $v_k = \{ v_x \quad v_y \quad v_z \}$ are vectors of angular and linear velocities of the local coordinate system O_Y linked with the element relative to the global coordinate system O_Z; $X_k = \{ x \quad y \quad z \}$ and $a_k = \{ a_0 \quad a_1 \quad a_2 \quad a_3 \}$ are a position vector of the coordinate system O_Y relative to O_Z and Rodrigo-Hamilton parameters, which define orientation of the system O_Y.

It is possible to represent block matrices U_k^* and S_k^* with the inertial and kinematic components contained into expressions (11) in the following form:

$$U_k^* = \left\| \begin{array}{c|c} J_k & U_k \\ \hline U_k^t & E \end{array} \right\| ; \; S_k^* = \left\| \begin{array}{c|c} \dot{\Phi}_k & \dot{U}_k \\ \hline [0] & \dot{\Phi}_k \end{array} \right\| , \qquad (12)$$

where U_k, Φ_k, A_k are matrices with elements of the linear and angular displacements, Rodrigo-Hamilton parameters of the coordinate system O_Y relative to O_Z; J_k is a matrix of inertia of an asymmetric solid body in the local coordinate system O_Y:

$$U_k = \begin{Vmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{Vmatrix} ; \; \Phi_k = \begin{Vmatrix} 0 & -\varphi_z & \varphi_y \\ \varphi_z & 0 & -\varphi_x \\ -\varphi_y & \varphi_x & 0 \end{Vmatrix} ; (13)$$

$$A_k = \begin{Vmatrix} a_0 & -a_3 & a_2 \\ a_3 & a_0 & -a_1 \\ -a_2 & a_1 & a_0 \end{Vmatrix} ; \; J_k = \begin{Vmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{Vmatrix} . \quad (14)$$

Using of quaternion matrices (13) - (14) allows clear representing a symmetry of the equations (8) - (10) in the matrix form.

Thus, the dynamical equations of a solid body three-dimensional motion are a basis for simulating the complex-oriented spatial girder structures and the rolling stock as a system of the coupled solid bodies taking into account inertial and stiffness properties, elastic and inelastic impedances.

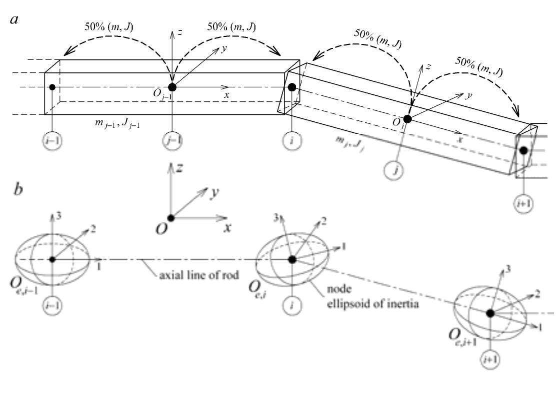

Fig. 1. Three-dimensional rod system: a) – the scheme of reducing the section inertia moments to nodes; b) – inertia ellipsoids of nodes

In determination of inertial characteristics of a node it is necessary to keep in mind that the node is an abstract concept and in a real system only a rod as solid (deformable or non-deformable) body can have a moment of inertia. Therefore, by analogy with stiffness characteristics the inertial parameters of the *j*-th rod are also to be determined in the local coordinate system O_Y combined with its center of gravity (Fig. 1, a). In so doing, each node joined by a rod gets a half of its concentrated mass.

The inertia tensor form (14) is suitable for description of inertial characteristics of both separate rod of a structure and a node. In case of any oriented rod it is necessary to rotate its tensor of inertia by means of the corresponding rotational matrix [4] and, using the Huygens-Steiner theorem [5], transfer to the node, then all the tensor components will correspond to the axes of global coordinate system O_Z.

Summing over the tensors of inertia for all rods converging in the *i*-th node leads to a matrix (14). However using its components in the equations (8) is only possible for systems with the symmetry in all directions. In this case the tensor of inertia is defining axial moments of inertia only, and at the same time they are the principal moments of inertia (centrifugal ones equal to zero):

$$J_1 = J_{xx}\,; \quad J_2 = J_{yy}\,; \quad J_3 = J_{zz}\,. \tag{15}$$

Various factors can cause the occurrence of centrifugal moments of inertia. For example, if in a three-dimensional rod system all elements have principal moments of inertia only but at least one of nodes has coordinates $x \neq 0$, $y \neq 0$, $z \neq 0$ then in the adjacent nodes we shall obtain all nine nonzero moments of inertia ("full" tensor) (14). In this case for search of the principal moments the cubic equation for matrix of inertia eigenvalues are to be used [5]:

$$\begin{vmatrix} J_{xx} - J^* & -J_{xy} & -J_{xz} \\ -J_{yx} & J_{yy} - J^* & -J_{yz} \\ -J_{zx} & -J_{zy} & J_{zz} - J^* \end{vmatrix} = 0, \tag{16}$$

where J^* is one of three desired principal moments of inertia (the equation root).

Values J_1, J_2, and J_3 are solutions of the equation (16). This process is also called as a matrix diagonalization. The new tensor containing only principal moments of inertia looks like:

$$J' = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}, \tag{17}$$

or in the vector form:

$$J' = \left\{ J_1 \; J_2 \; J_3 \right\}. \tag{18}$$

It is significant that after the fulfilled transformations the principal axes of inertia (the conjugate ellipsoid diameters) and the axes of global coordinate system O_Z do not always coincide. For determination of the principal axes of inertia the system of algebraic equations is to be used [5]:

$$\left.\begin{aligned} &\left(J_{xx} - J^*\right)x - J_{xy}y - J_{xz}z = 0; \\ &-J_{yx}x + \left(J_{yy} - J^*\right)y - J_{yz}z = 0; \\ &-J_{zx}x - J_{zy}y + \left(J_{zz} - J^*\right)z = 0, \end{aligned}\right\} \tag{19}$$

where the meaning of value J^* remains the same as in (17).

By sequential substituting into (19) the earlier found values of principal moments of inertia J_1, J_2, J_3 instead of J^*, one can get three groups of coordinates x, y, z of points lying on the principal axes of inertia. The unitary isomeric rotation matrix $R_{e,i}$ is consisting of cosines of angles between these vectors and the of global coordinate system axes. By rotating a vector (18) with its help, we have:

$$J^{(o)} = R_{e,i}^T J' = \left\{ J_1^{(o)} \; J_2^{(o)} \; J_3^{(o)} \right\}, \tag{20}$$

where T is a symbol of matrix transposition.

Taking into account (20) and solving each of the equations (8):with respect to an angular acceleration vector, we get:

$$\left.\begin{aligned} &\dot{\omega}_1 = \varepsilon_1; \quad \varepsilon_1 = \left[M_1 - \left(J_3^{(o)} - J_2^{(o)} \right) \omega_2 \omega_3 \right] / J_1^{(o)}; \\ &\dot{\omega}_2 = \varepsilon_2; \quad \varepsilon_2 = \left[M_2 - \left(J_1^{(o)} - J_3^{(o)} \right) \omega_3 \omega_1 \right] / J_2^{(o)}; \\ &\dot{\omega}_3 = \varepsilon_3; \quad \varepsilon_3 = \left[M_3 - \left(J_2^{(o)} - J_1^{(o)} \right) \omega_1 \omega_2 \right] / J_3^{(o)}. \end{aligned}\right\} \tag{21}$$

Using formulas (19), (20) of the general tensor of inertia it is possible to obtain the principal moments of inertia of separate elements and to consider them in the converted form (21) in motion equations (8).

SIMULATION OF SPAN WITH TRACK ECCENTRICITY

The generalized force (1) contains components of exterior forces a principal vector, the principal moment of forces and responses of the connections applied to a node of system "bridge-train". Interior forces and responses in elements of a superstructure are convenient for defining by means of a finite element method noted in the form of a displacement method.

In this case it is necessary to add the common algorithm of calculation (8) - (10) with following operations: prepared local stiffness matrixes of beam elements dividing a superstructure on sections of the equal length; prepared the common stiffness matrix of system, evaluated a flexibility matrix of system; solving system of the canonical equations of a method of transitions; finding nodal transitions.

The introduced approach is realized in bundled software for calculation dynamic of bridge constructions taking into account a velocity of a load motion and various dynamic factors of disturbance [4].

In order to construct a design model of girder span with rail track eccentricity let's choose from the whole structure a rod connecting nodes $i = 1, 2$ of the structure. And let the generalized concentrated force factor $F_p = \{F_x \; F_y \; F_z \; M_x \; M_y \; M_z\}$ moves with the set constant velocity within the rod length (Fig. 2).

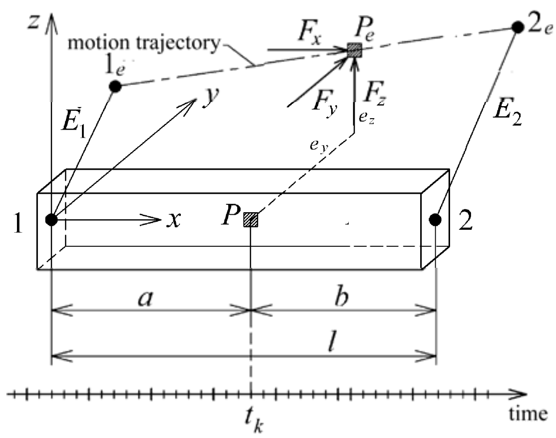

Fig. 2. Loading on a rod having eccentricity

The force factor trajectory of movement is a straight line, which generally does not coincide with the axial line of a rod [6].

The node points 1_e, 2_e of trajectory are shifted from the base nodes of structure $i = 1, 2$ by the distances corresponding to space vectors $\bar{E}_1$, $\bar{E}_2$. In the fixed instant t_k the force factor position in some point P_e is known from its law of movement. The force transfer to a rod in the point P is to be substituted by the set of equivalent force factors taking into account eccentricity $\bar{e}$, presented in the coordinate form:

$$\begin{cases} F_{x,P} = F_x; \\ F_{y,P} = F_y; \\ F_{z,P} = F_z; \end{cases} \quad \begin{cases} M_{x,P} = M_x + F_z e_y - F_y e_z; \\ M_{y,P} = M_y - F_z e_x + F_x e_z; \\ M_{z,P} = M_z + F_y e_x - F_x e_y. \end{cases} \tag{22}$$

Upon finding out the loading point P located on the axis line of a rod, the components of force (22) are to be added to the general vector of external loads F_0.

THE MOVING LOADING ACCOUNT

As an example, let us consider the motion of a locomotive VL8 on a span. Taking into account [7] for longitudinal, lateral and vertical contact forces between a span and wheelsets of locomotive VL8, we have got:

$$\begin{cases} F_{x,P} = F_{rf}^{(v)} - F_{tr}^{(v)}; \\ F_{y,P} = 0{,}5 P_{1,y} \left(1 + \cos\left(2\pi \nu_y t + \varphi_y\right)\right); \\ F_{z,P} = P_{1,z} \left(1 + A_{1,z} \cos\left(2\pi \nu_{1,z} t\right) + \right. \\ \qquad \left. + A_{2,z} \cos\left(2\pi \nu_{2,z} t\right)\right); \\ M_{x,P} = M_x + F_{z,P} e_y - F_{y,P} e_z; \\ M_{y,P} = M_y - F_{z,P} e_x + F_{x,P} e_z; \\ M_{z,P} = M_z + F_{y,P} e_x - F_{x,P} e_y. \end{cases} \tag{23}$$

Thus, the locomotive effect on a bridge span is modeled by force components $F_{1,x}, F_{1,y}, F_{1,z}$. When a locomotive enters on a span (a time step of calculation t_k), these forces initiate the transition of the span into stressed-and-strained state [8].

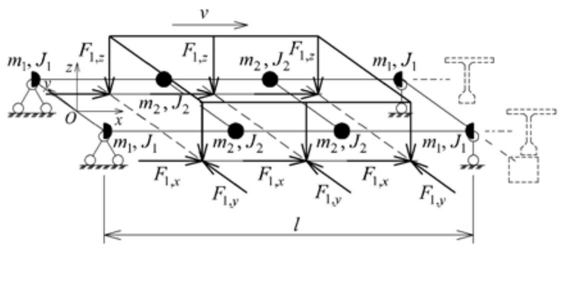

Fig. 3. Spatial design model of a reinforced-concrete span

The design model of a reinforced-concrete span with a length of 22.9 m is accepted in the form of the three-dimensional rod system composed of 13 rods and 10 nodes (42 degree of freedoms, 120 differential equations of movement) that can make vertical, lateral, longitudinal, and torsional vibrations (Fig. 3). The step of integration is 10^{-4} seconds.

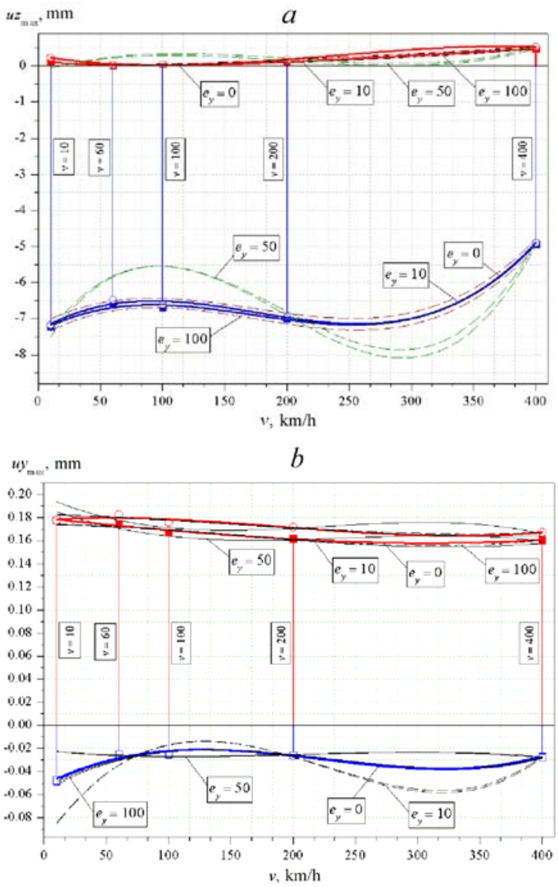

Fig. 5. Extremes of the vertical (a) and lateral (b) span displacements

Let's consider a variant of even displacement of a rail-sleeper lattice in the transverse direction. In this case the horizontal eccentricity $e_y^{(1)}, e_y^{(2)}$ on girders B1, B2 is identical (Fig. 4).

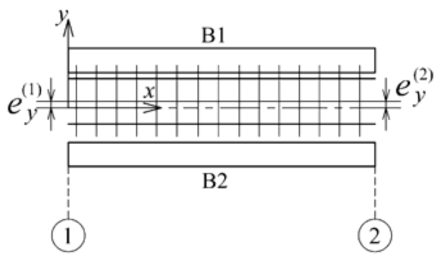

Fig. 4. The scheme of horizontal eccentricity on a bridge span

The eccentricity magnitude is accepted equal $e_y = 0; 10; 50; 100$ mm [9].

For determination of effect of the track eccentricity on the character of vibrations of a span each value e_y is to be analyzed within the range of loading movement velocity $10\ldots400$ km/h.

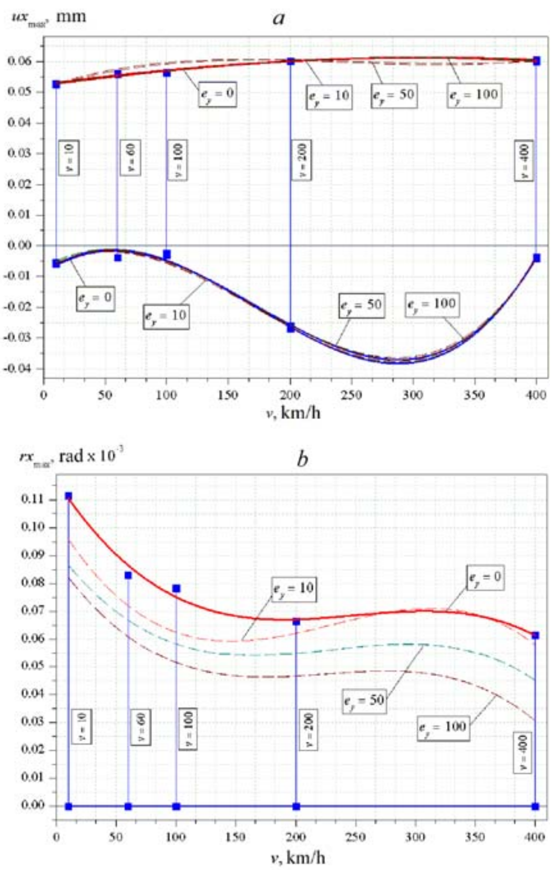

Fig. 6. Extremes of the longitudinal (a) and torsion (b) span displacements

THE ANALYSIS OF CALCULATION RESULTS

The results gained testify that the vertical span shifts *uz* (deflections) at the simultaneous accounting the spatial dynamic behavior of a structure and loading movement velocity have nonlinear dependence on the magnitude of rail track eccentricity (Fig. 5, a). In all the calculations the relatively high shifts are obtained at the loading movement velocity of 200 km/h. The greatest range of extremes of vertical shifts is observed at the eccentricity $e_y = 50$ mm; in other cases the *uz* magnitude variations have relatively low amplitudes. The lateral span displacements *uy* have a maximum if the rail track shift magnitude $e_y = 10$ mm.

The range of maximum displacements in the longitudinal direction at the loading movement velocity $10 \ldots 100$ km/h can be considered as stationary. Further increasing the velocity (to $260 \ldots 270$ km/h) is accompanied by essential magnification of the ux displacements and the maximum amplitude of structure longitudinal vibrations (Fig. 6, a). The value of displacements decreases and returns to the initial values at velocity of 400 km/h. Generally, the track eccentricity magnitude practically does not influence the longitudinal and torsion vibrations of a girder bridge span.

CONCLUSION

In simulation of dynamic behaviour of a spatial rod system, some nonlinear components of motion of nodes can be lost without taking into account the moments of inertia of the nodes. In Euler-Lagrange differential equations the interconnection between an angular velocity and an angular acceleration of a node in components with respect to the principal axes of inertia that characterizes a certain connection between vertical, horizontal, and torsional vibrations of a structure and also the energy distribution in a system is determined.

The combined application of the finite-element method and the solid body dynamic equations in dynamic computations allows estimating the effect of rail track eccentricity on the space vibrations of reinforced-concrete bridge spans. The influence of mentioned features is shown at movement of the single locomotive with a speed more than 200 km/h.

The analysis of dynamic work of reinforced-concrete and metal spans with the rail track eccentricity during the motion of freight and passenger rolling stock, as well as the estimate of influence of moments of inertia of separate elements on spatial dynamics of bridge spans are planned in the further research.

REFERENCES

1. **Лурье, А. И.** *Аналитическая механика*. Москва : Физматлит, 1961. 824 с.
2. **Кравец, В. В.** Матричные уравнения пространственного полета асимметричного твердого тела. *Прикладная механика*, Днепропетровск, 1986, том 22, № 1, с. 105-110.
3. **Распопов, А. С., Русу, С. П., Артемов, В. Е.** Применение уравнений Эйлера-Лагранжа к решению задачи динамики системы “мост-поезд” *Вісник Дніпропетровського національного університету залізничного транспорту імені академіка В. Лазаряна : Збірник наукових праць,* Днепропетровск, 2007, вип. 16, с. 109-114.
4. **Распопов, А. С., Артемов, В. Е., Русу, С. П.** Моделирование колебаний балочных железнодорожных мостов в среде объектно–ориентированного программирования Delphi. *Вісник Дніпропетровського національного університету залізничного транспорту імені академіка В. Лазаряна : Збірник наукових праць*, Днепропетровск, 2010, вип. 33, с. 217-222.
5. **Никитин, Н. Н.** *Курс теоретической механики*. Москва : Высшая школа, 1990, 607 с.
6. **Распопов, А. С., Артемов, В. Е., Русу, С. П.** Воздействие подвижных нагрузок на балочный мост, моделируемый системой дискретных элементов. *Строительство, материаловедение, машиностроение : Сборник научных трудов Приднепровской государственной академии строительства и архитектуры*, Днепропетровск, 2008, вып. 47, с. 493-501.
7. **Распопов, А. С., Русу, С. П., Артемов, В. Е.** Применение топологических моделей в задаче динамического взаимодействия системы твердых тел. *Вісник Дніпропетровського національного університету залізничного транспорту імені академіка В. Лазаряна : Збірник наукових праць,* Днепропетровск, 2009, вип. 30, с. 198-203.
8. **Распопов, О. С., Артьомов, В. Є., Русу, С. П.** Динаміка балкових конструкцій мостів під рухомим навантаженням. *Механіка і фізика руйнування будівельних матеріалів та конструкцій : Збірник наукових статей.* Львів: Каменяр, 2009, вип. 8, с. 712-721.
9. **Борщов, В. І., Солдатов, К. І., Тарасенко, В. П.** *Правила визначення вантажопідйомності балкових залізобетонних прогонових будов залізничних мостів.* Дніпропетровськ: Видавництво Дніпропетретровського національного університетуту залізничного транспорту імені академіка В. Лазаряна, 2003 404 с.

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A. Raspopov, S. Rusu, V. Artyomov DYNAMICS OF REINFORCED-CONCRETE BRIDGES WITH RAILWAY TRACK ECCENTRICITY ON SPAN
Significance effects of railway track eccentricity on railway span vibration character by locomotive moving with various speeds are investigated. The mathematical model of the eccentricity of a railway track on the span of railway bridges is proposed. The dynamic analysis of three-dimensional girder systems based of the Newton-Euler differential nonlinear equations modeling aspects are represented.

А. С. Распопов, С. П. Русу, В. Е. Артемов ДИНАМИКА ЖЕЛЕЗОБЕТОННЫХ ПРОЛЕТНЫХ СТРОЕНИЙ С ЭКСЦЕНТРИСИТЕТОМ РЕЛЬСОВОГО ПУТИ

Исследовано влияние эксцентриситета пути на характер колебаний пролетных строений при различных скоростях движения одиночного локомотива. Предложена математическая модель для учета эксцентриситета рельсового пути на пролетных строениях железнодорожных мостов. Рассмотрены некоторые аспекты динамического расчета стержневых систем с использованием нелинейных дифференциальных уравнений Эйлера-Лагранжа.

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