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EVALUATION OF THE STRESSED-STRAINED STATE OF CROSSINGS OF THE 1/11 TYPE TURNOUTS BY THE FINITE ELEMENT METHOD

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Проведено оцінку напружено-деформованого стану хрестовин марки 1/11 методом скінченних елементів у програмному середовищі Ansys. Досліджено величини деформацій та контактних напружень у характерних перетинах хрестовини. Встановлено вплив площі контакту на величину контактних напружень. Отримано характер руйнування поверхні металу вусовика та сердечника хрестовини при високому рівні контактних напружень

Ключові слова: хрестовина, стрілочний перевід, метод скінченних елементів, контактні напруження, рухомий склад залізниць

Проведена оценка напряженно-деформированного состояния крестовин марки 1/11 методом конечных элементов в программной среде Ansys. Исследована величина деформаций и контактных напряжений в характерных сечениях крестовины. Установлено влияние площади контакта на величину контактных напряжений. Получен характер разрушения поверхности металла усовика и сердечника крестовины при высоком уровне контактных напряжений

Ключевые слова: крестовина, стрелочный перевод, метод конечных элементов, контактные напряжения, подвижной состав железных дорог

1. Introduction

Switches or turnouts are an integral part of railway transport, which are at the same time one of the weakest components of track, since their design includes moving components and a crossing [1]. The crossing is a more complicated element that is exposed to large dynamic forces caused by its longitudinal and transverse profiles [2, 3].

Results of processing statistical data by the tonnage carried through revealed that the current profile in line with GOST 28370-89 does not meet performance criteria because the average time of crossings operating cycle on all the examined directions of the Ukrainian Railroads (Ukraine) does not reach the warranty period of operation [4]. In most cases, the performance of crossings does not actually reach the half-way term of a failure-free operation.

An analysis of the removed crossings by the tonnage carried through showed that the largest percentage of crossing failures occur due to the reasons of chipping and

wear of the crossing core and wing rails (Fig. 1). In particular, up to 50 % of the total number of removed crossings are the defects of wear in the core and wing rails, 30 % – core and wing rails chipping, 20 % – other defects [2].

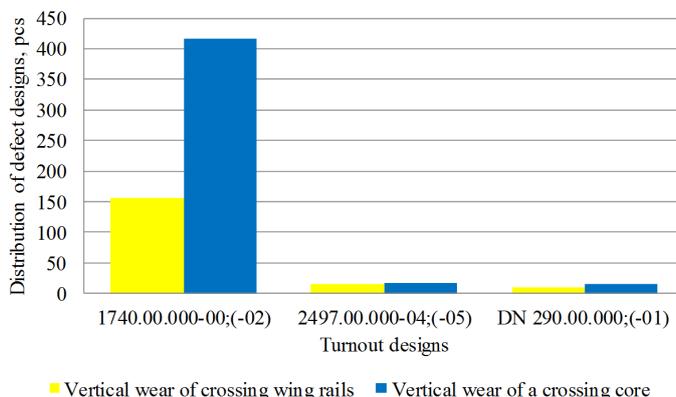


Fig. 1. Distribution histogram of defects in turnouts [2]

The studies conducted, aimed at establishing the causes for the occurrence of such defects, are necessary and relevant for today. Appropriate monitoring will make it possible to devise timely engineering solutions for improving bearing capacity of the crossings of turnouts.

2. Literature review and problem statement

The main factor that affects the level of dynamic forces on the crossings is the irregularity of their vertical plane, which is caused by the profile of wing rails and core [5, 6]. As a result, the longitudinal and transverse profiles of the crossing affect the intensity of wear of its elements, the formation of defects, and ultimately the term of its service.

The phenomenon of contact fatigue of material of surface layers of the crossing occurs as a result of multiple elastic or elastic-plastic deformation of the material in the regions of contact wheel – crossing. This leads to the occurrence of micro- and then macro cracks around the contact surfaces of crossing and wheel. The intensity of the process of accumulation of fatigue damage is affected by a multitude of factors. The main are the frequency of loading, the level and character of distribution of contact stresses. In this case, the character of the distribution of residual stresses depends on the temperature and condition of the surface layer of crossing.

Exploring the reasons for the occurrence of such defects, the mechanisms of their development during operation will contribute to the development of technical control systems. This will make it possible to develop effective measures to extend the terms of operation of the crossings of turnouts.

However, conducting such an analysis is possible only during detailed studies and calculations of the stressed-strained state of the crossing in the region of its contact interaction with a wheel [7]. As is known, classical theory of the calculation of contact stresses by the methods outlined in articles [8, 9] is based on the significant simplifications of actual picture of contact interaction between a wheel and a core, which is why results of the calculations in most cases differ from experimental data. The main reason for the differences in theoretical and experimental results is the existence of plastic deformations during real interaction between a wheel and a core, which leads to a change in the magnitude of contact area [9]. Author of article [7] paid attention to this and he essentially corrected the classic procedure for the calculation of contact stresses. In order to calculate the strength and durability of the core in the region of contact interaction with the wheels of rolling stock, it is necessary to know kinetics of the stressed-deformed state of metal in the given region. It will depend on the mutual position of rolling surface of the crossing core and the wheel during motion of rolling stock, as well as during changes in the transverse profiles of the wheel and the core, which happens in the process of their operation and wear.

This problem cannot be solved using known analytical methods of calculation and known analytic formulae. The exact determining of local stresses and deformations in the region of core and wing rails of the crossing is possible only under condition of the application of modern numerical methods of calculations with the use of electronic computing machines.

A large number of contact models are currently used, from the simple ones to the more complex, which in general can be divided into four groups. The first group includes empirical

and heuristic approaches that are based on the preliminary assumption about elliptical shape of the contact spot and the law of Hertz concerning the distribution of normal stresses in it. The most commonly used are the linear and nonlinear empirical Kalker's models and the Muller method [6, 9, 10], Vermeulen and Johnson theory, Polah method [11]. Their shortcoming is the lack of precision in terms of a two-contact interaction and underestimation of all possible profiles of the wheel and the crossing.

The second group is the rapid and approximated methods that consider the distribution of tangential stresses at a contact spot, but the distribution of normal stress is accepted by the Hertz law. Also known are the method of first order accuracy FASTSIM [5, 6], the method of second order accuracy FASTSIM2 [6, 11], the method of approximating tables [6], the semi-analytical method FASTSIM_A [6]. Such methods make it possible to take into account the actual shape of surfaces of the wheel and the crossing, at relatively simple calculations, but they are not suitable for detailed examination of the contact problem.

The third group consists of the methods for accurate determining of the contact interaction, based on the approaches of elasticity theory, in which contact surfaces are considered as half-spaces. Such approach is employed by the method CONTACT [9, 12]. These methods do not introduce additional assumptions about the distribution of normal stresses, arbitrary shape of contact surfaces is considered. The disadvantage is cumbersome calculations that prevent using such methods as part of the models of interaction between the track and the rolling stock.

The fourth group includes the finite-element approaches, which to the largest extent match the real picture of interaction between the wheel and the crossing; nevertheless, these are the most difficult ones in terms of solving practical tasks on interaction between the track and the rolling stock.

Contact interaction between the core and the wheel physically represents a three-dimensional distribution of stresses and deformations in the body of core and in the contact region of wheel [13, 14].

When shifting the load to the edge of of a rail head, results of calculations according to the classical theory may yield significantly reduced values of both normal (σ_{\max}) and tangential stresses (t_{\max}). Convergence in the results of stresses may vary within the range of 2.0–2.5 times compared with the actual picture of the stressed state received by employing the MKE method.

Normal stresses on the rolling surface of rail and wheel depend on the wheel load on the rail, the curvature radii of the rolling surfaces of wheel and rail, the properties of their materials. Note that the Hertz contact theory holds under assumptions about the materials of contact bodies being uniform and isotropic. In this case, friction forces in the contact area are missing while dimensions of the contact area are small compared with the size of conjugated bodies.

Based on the performed analysis, it was established that the prediction of crossing wear based on the mathematical models of material deformation runs into several difficulties. The approaches that are available to date for establishing the causes for the development of defects in the crossings of turnouts are based on cumbersome computations, not considering actual conditions of the interaction between the wheel and the profile of crossings. As a result, it was found that the assessment of the stressed-strained state of

turnout crossings is possible when using the finite-element approaches that most accurately describe the real picture of interaction between the wheel and the crossing.

3. Research goal and objectives

The goal of present work is to evaluate the stressed-strained state of the 1/11 type crossings of turnouts by the method of finite elements using the Ansys programming environment.

To accomplish the set goal, the following tasks should be solved:

- to develop a finite-element model of the crossing in the Ansys programming complex;
- to determine the kinetics of development of contact-fatigue damage on the rolling surface of the crossing;
- to establish main causes for the occurrence of defects and damage to the crossings of turnouts.

4. Development of the finite-element model “wheel-crossing”

We designed a three-dimensional crossing and a wheel of the rolling stock. Geometric model of the crossing is shown in Fig. 2; diagram of the forces that load the wheel pair is presented in Fig. 3.

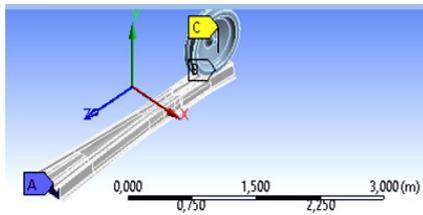


Fig. 2. Geometric model crossing-wheel

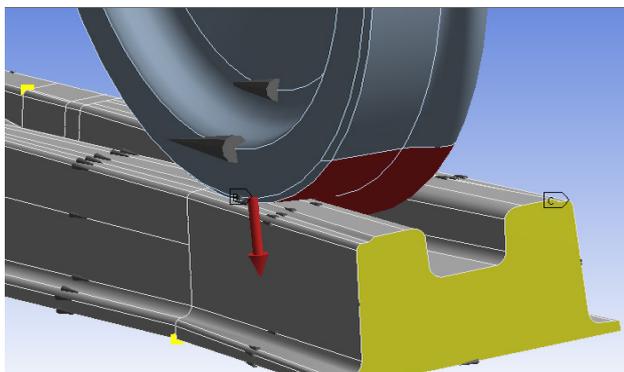


Fig. 3. Diagram of the forces applied to a crossing

When building the model “wheel-crossing” and performing the calculations, we considered the R65 type turnout, 1/11 brand; in this case, we examined the unworn profile of the brace and the crossing. Mechanical properties of the brace and the crossing that were used for the calculation are given in Table 1.

We split in the global Cartesian coordinate system the wheel pair into tetrahedra-like finite elements. The finite-element grid of the model “wheel-crossing” is shown in Fig. 4.

Values of the required functions in the grid nodes are marked as unknown, the discretized problem is stated rela-

tive to them [12, 13]. The dimensions of finite elements are significantly reduced in the examined region to improve accuracy in the calculation of stresses and deformations. A snippet of the grid of finite elements into which the crossing and the wheel brace were split in the estimated cross section is shown in Fig. 4.

Table 1
Mechanical properties of brace and crossing

Mechanical properties	Brace	Crossing
Density, kg/m ³	7850	7850
Young modulus, Pa	2.1·10 ¹¹	2.1·10 ¹¹
Poisson ratio	0.3	0.3

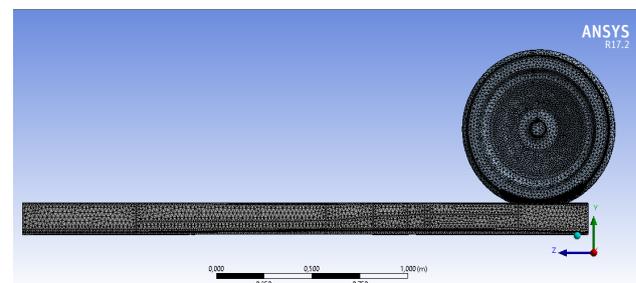


Fig. 4. A finite-element model of the pair “wheel-crossing”

In order to simulate contact surfaces, we determined the contact pairs as a result of preliminary analysis of behavior of the crossing at its deformation. In the contact asymmetric pairs “surface-surface”, one of the surfaces is accepted as the target (TARGET170), another – as the contact (CONTACT174). Simulation of contact pairs (Fig. 5) is typically performed under interactive mode by using the feature Contact Wizard.

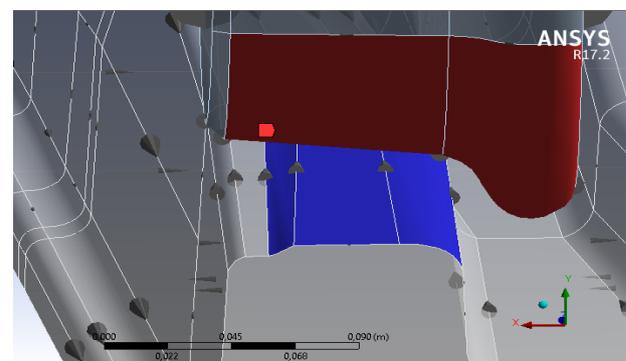


Fig. 5. Creation of the contact pair in Ansys Mechanical; assigning the target and the contact surfaces (TARGET – CONTACT)

As a result of discretization, the stationary (that is, such that does not depend on time) continual problem is reduced in order to solve it to a system of linear or nonlinear algebraic equations relative to the values of the desired function in the grid nodes.

When solving the problem in question, we accepted as the main unknowns under the assigned conditions of mechanical load the search for the fields of stresses and deformations using formula (1):

$$\sigma_{ij} = (x, y, z); \quad \varepsilon_{ij} = (x, y, z); \quad i, j = 1, 2, 3, \quad (1)$$

as well as the three components of displacement vector – formula (2):

$$U(x, y, z); \quad V(x, y, z); \quad W(x, y, z). \quad (2)$$

The process of solving comes down to establishing the connection between fifteen unknown functions: six components of the stress tensor, six components of the deformation tensor, and three components of the displacement vector.

When calculating the equivalent stresses, the given solution to the problem employed a general theory of boundary states [14] (general theory of strength). For the case when the stressed state is determined not in the system of the main stresses areas, in order to calculate equivalent stresses, formula (3) is applied:

$$\sigma_{eqv} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)}, \quad (3)$$

for the calculation of equivalent deformations, the following formula is used (4):

$$\varepsilon_{eqv} = \frac{1}{\sqrt{2(1+\mu)}} \times \sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + \frac{3}{2}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{xz}^2)}. \quad (4)$$

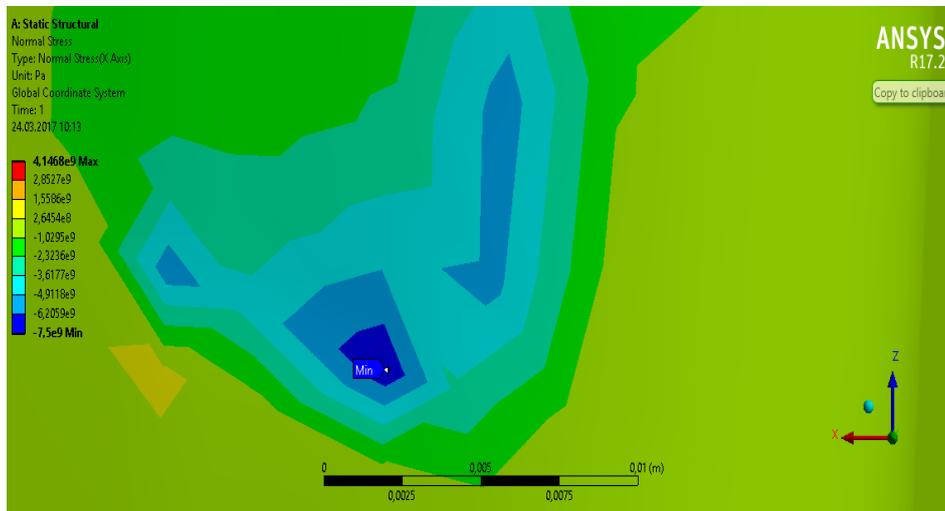
Using the given approaches, we shall proceed to the calculation of contact stresses in the region of contact between the wheel and the crossing.

5. Results of calculation of the contact stresses “wheel-crossing”

Fig. 6 shows the distribution of contact pressure along the contact area “wheel-crossing”. It demonstrates that the shape of the contact area is elliptical, but very close to a circle (dimensions of the ellipse axes are 11.1 mm in longitudinal direction and 13.25 mm in transversal direction). The ellipse of the contact in this case is placed close to the middle of crossing head while the center of the contact area under the given conditions is shifted by a few millimeters from the rail axis towards the side of the working edge.



a



b

Fig. 6. Distribution of contact pressure along a crossing surface: a – in the region of a crossing core; b – in the region of a crossing wing rail

Note that considering the plastic deformations leads to an increase in the size of the contact area, and this, in turn, reduces the maximum magnitude of contact pressure. Compared with data of the analytical solution (ideally elastic), considering the plastic deformations increases the area of contact zone by around 10.6 % and reduces maximum pressure by around 10.7 %. However, for determining the qualitative impact of the shape (profile) of wheel rolling surface on the magnitude of contact stresses, it will suffice to solve this problem in elastic statement.

For the normal stresses, values that are maximal by absolute magnitude correspond to the moment when the wheel passes the estimated cross section. For the tangential stresses, on the contrary, at the moment when the wheel is over the estimated cross section, their magnitude is close to zero.

As the results of calculations show, the maximal normal compressive contact stresses σ_z (Fig. 7) act along the direction of the applied vertical load – along the z axis. Their magnitude reaches $\sigma_z = -414.6$ MPa (in the center of contact ellipse on the crossing's rolling surface), which significantly exceeds yield limit of the crossing's steel $\sigma_T = 330$ MPa.

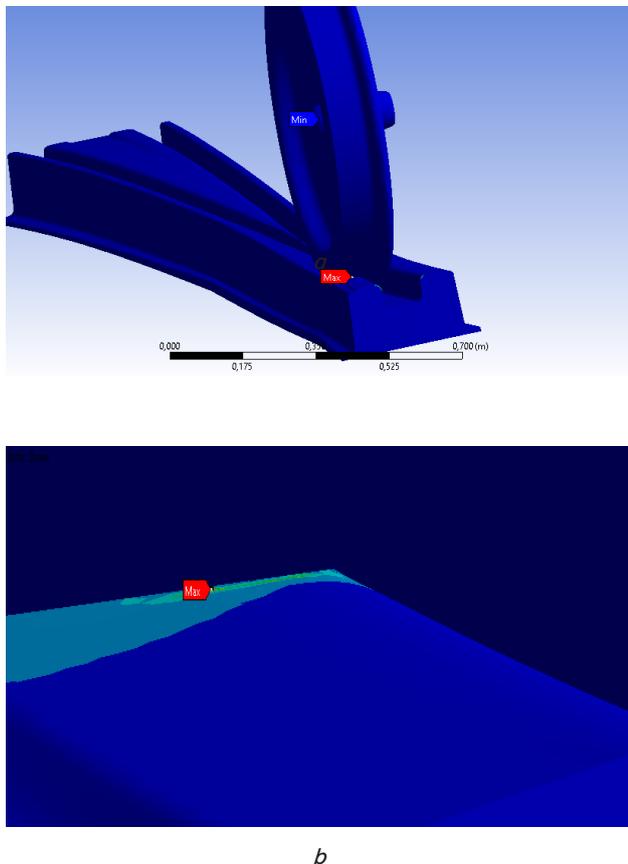


Fig. 7. Distribution of contact stresses along the surface of crossing: *a* – general view of the distribution of contact stresses; *b* – area of concentration of maximal contact stresses in the zone of contact between the wheel and the crossing

The point at which the largest equivalent stresses are implemented should be named dangerous. In the vicinity of

the dangerous point, the largest shear stresses are also implemented, and this the occurrence of plastic deformations is possible.

In contrast to the components of normal stresses tensor that have only negative values in the region of contact, some components of linear deformation tensor are stretching. Thus, linear deformations (Fig. 8) in the transverse direction to crossing ϵ_y on the surface are stretching while those under the surface are compressive.

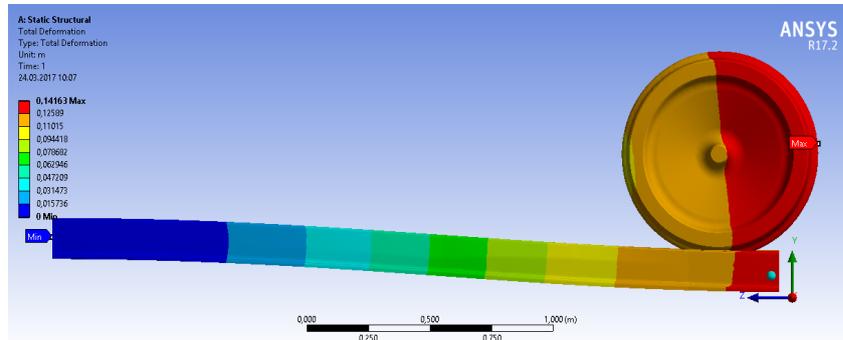


Fig. 8. Deformation distribution on a crossing

Linear deformations along the z vertical axis are compressive over the entire contact area. Linear deformations along the longitudinal x axis of the crossing on the rolling surface are negative while those beneath the surface are positive.

6. Kinetics of the defect development at contact interaction «wheel-cross»

One of the dominating factors that define kinetics of the contact interaction phenomena is the condition of contacting surfaces. A kinetic indentation method has been used in the studies into surface properties of materials.

Underlying the method is the principle of receiving compression deformation of the surface layer of a metal at a constant speed, which is equivalent to the uniaxial stretching at the level of plasticity limit. A condition of equivalence in this case is the equality between relative extension at compression and relative contraction at stretching, which arises from retaining the volume at plastic deformation. The ratio of yield limit to the mean contact pressure is approximately 0.3.

Let us consider interaction between a wheel and a crossing. Under the influence of the applied force from the wheel to the crossing, there occurs a deformation of two types: elastic and plastic. The ratio between them and the degree of their development depend on the magnitude of the applied load, the radius of wheel r , crossing surface curvature radius r' , properties of the material and coordinates of the given element of volume that is deformed. First, we shall examine the processes that take place at elastic deformation. It follows from the solution of Hertz problem on the elastic collision of two bodies that:

$$\rho = N^{1/3} \left(D \frac{rr'}{r+r'} \right)^{1/3}, \tag{6}$$

where

$$D = \frac{3}{4} \left[\left(\frac{1-\mu^2}{E} \right) + \left(\frac{1-\mu'^2}{E'} \right) \right],$$

μ , μ' , E , E' are the Poisson's ratios and Young modulus of the wheel of rolling stock and of the crossing of turnouts, respectively, N is the load.

The magnitude of convergence of two bodies, which is pre-determined by elastic deformation, is determined from formula:

$$h_y = N^{2/3} \left(D^2 \left(\frac{1}{r} + \frac{1}{r'} \right) \right)^{1/3}. \quad (7)$$

Using this equation, we obtain

$$h_y = \frac{ND}{\rho}. \quad (8)$$

Mean contact stresses are determined by ratio (9):

$$\bar{\sigma} = \frac{N}{\pi \rho^2}. \quad (9)$$

Then convergence at elastic deformation during unloading of the conjugated bodies can be written in the form (10):

$$h_y = \pi^{1/2} D \bar{\sigma}^{1/2} N^{1/2}. \quad (10)$$

At elastic-plastic deformation:

$$h_y = \frac{2N}{\pi C_1^2 r \bar{\sigma}}, \quad (11)$$

where coefficient $C_1 = 1,98$.

Due to the existence of surface irregularities, the interaction of bodies will be discrete in character. In this case, the spots of real contact form whose total area is the actual area A_r . Contact spots are located in the areas whose total area forms the contour contact area A_c . Based on this, we shall introduce a coefficient that links both of these magnitudes. Using a formula for the surface of a spherical segment and by assuming that at one and the same magnitude A , the convergences are the same both in the case of smooth and in the case of rough surfaces, we shall write equation for the elastic deformation (12):

$$A_r = \beta 2\pi r h_y. \quad (12)$$

Similarly, for the actual area of interaction at elastic-plastic deformation we obtain:

$$A_r = \beta 2\pi r h_{y-n}. \quad (13)$$

Introduce relations $R/R_0 = A_0/A$, where the magnitudes with a zero index are related to the initial state, while the magnitudes without indexes are the current values. We introduce coefficient $\beta = A_0 R_0$. Thus, the dependence of kinetics of contact stresses on the wheel contact with the crossing can be written by formula (14):

$$R = \frac{\beta}{\frac{N}{C\sigma_s} + \frac{3}{4} I \pi r (\pi N C \sigma_s)}. \quad (14)$$

Here, the constant C shows by how many times the actual pressure exceeds the yield limit and is determined by the properties of the surface ($C \approx 3,0$).

It is worth noting that under conditions of multiaxial load, in addition to the fatigue damage, quasi-static damage may also accumulate. This is connected to the phenomenon of a one-side accumulation of deformations in any direction – cyclic creep. The kind of damage by the defect code 11.1 and 11.2, which develops on the surface of a wing rail under the influence of contact stresses, is shown in Fig. 9.

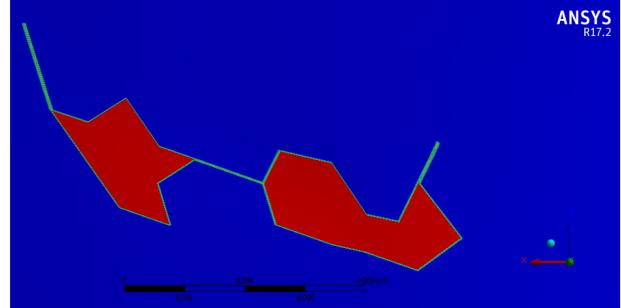


Fig. 9. Metal damage on the surface of a wing rail of the crossing

Under conditions of three-axial compression, at large stresses of vertical compression, the cracks of multi-cycle fatigue develop. It is initiated when a contact area approaches the edge of the working face of the crossing's core, which is the consequence of non-uniform longitudinal-transverse compression of surfaces of the wheel and the crossing.

7. Discussion of research results on the evaluation of the state of strength of the crossings of turnouts

Determining the optimal hardness of a wheel rolling surface is a challenging task whose solution should be found systematically taking into account different parameters. The point at which the largest equivalent stresses are implemented should be named dangerous. In the vicinity of the dangerous point the largest shear stresses are also implemented, and thus the occurrence of plastic deformations is possible.

When a wheelset passes, each of the stresses in the estimated section of the rail varies according to its inherent law. For the normal stresses, a pulse cycle is realized with a small coefficient of asymmetry, for the tangential stresses – sign-alternating cycle with a large coefficient of asymmetry (close to -1). For the normal stresses, values that are maximal by absolute magnitude correspond to the moment when the wheel passes the estimated cross section. For the tangential stresses, on the contrary, at the moment when the wheel is over the estimated cross section of the crossing, their magnitude is close to zero.

Under conditions of multiaxial load, in addition to the fatigue damage, quasi-static damage may also accumulate, which is connected to the phenomenon of a one-side accumulation of deformations in any direction – the so-called cyclic creep). The data obtained indicate the possibility of accumulation in the contact zone of the crossing of linear deformations, which subsequently may lead to the origination of cracks of multi-cycle fatigue.

The defined stressed-strained state of the crossings of turnouts provides the limits for the creation of normative documents on the reliability of crossings of turnouts, taking into account a set of initial factors of influence on their durability.

One of the shortcomings of the studies conducted is that the estimated model employed physical-mechanical properties of the crossing and the wheel of only one type of rolling stock and turnout crossing. That is why in the further scientific-research work the evaluation of durability of the crossings of turnouts might be directed with regard to different types and brands of crossings and types of rolling stock of railways. Those studies would require further work, to form the evaluative criteria of reliability and functional safety of transport facilities of the railway track.

8. Conclusions

1. Maximal normal compressive contact stresses act along the direction of the applied vertical load – along the z axis. Their magnitude in the wing rail of a crossing reaches – 414.6 MPa. In this case, the stresses are localized within the limits of a small area of contact between the wheel and the

crossing. Dimension of the area can be compared with the size of the contact spot.

2. Determining and analysis of the formation mechanism and development kinetics of the contact-fatigue damage by the code DS 14.1-14.2 on the rolling surface of the cast part of a wing rail and the crossing's core allow us to argue that the main reason for the occurrence of such defects is too high contact stresses near the edge of the working face of a wing rail and the crossing's core. They occur in this region in the form of a cyclically repeated and sign-alternating normal and tangential stresses from the cyclically recurring power impacts from the wheels of rolling stock.

3. The cause of defects and damage to the crossings of turnouts is the process of accumulation of linear deformations in the microvolumes of metal of the crossing. They depend on the character of cyclic loading, which arises from the railroad rolling stock on the surface of the crossing, as well as approaching of the ellipse of contact to the edge of working surfaces of the wing rail or the crossing's core.

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