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FACTORS INFLUENCING THE MODELLING
OF TRANSPORT FLOW DYNAMICS IN CITIES

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Abstract

The article defines the regularities in the influence on the traffic flows dynamics in cities using mathematical modelling, taking into account the historical features of this process, as well as the most relevant issues. In the queuing theory obstruction of free movement is considered a service, and the duration caused by this delay is called service duration, which is not applicable to the delay of transport at intersections. By means of mathematical modelling, the dependence of the influence of increasing the congestion degree on the increase in vehicle delays was determined. Results of studying transport flow characteristics in large cities should be considered when determining loading of urban passenger vehicles and calculating duration of delivering passengers to their destination. Application of the proposed mathematical model will help to reduce the probability of traffic congestion in the urban transport network of large cities, providing high-quality public transport services to clients.

Key words: transport flow, modelling, urban network, distribution density, street crossing

Introduction. The traffic management system is intended to become a key element of intelligent transport systems. However, before proceeding with the implementation of such a system, it is necessary to determine the patterns in the influence on the dynamics of transport and logistics flows in cities using mathematical modelling.

The authors revealed that in the queuing theory, the obstruction of free movement is considered a service, and delay duration is called service duration. However, these concepts can be directly applied only to a small range of issues related to the traffic flow theory.

Without modelling stage, it is impossible to adequately ensure the selection of approaches to the synchronization of the traffic light system, as well as to provide the dynamic distribution of free urban space.

Modelling of transport flows has a long history. In general, it should be noted that this issue was observantly studied in the middle of the last century. The use of stochastic fundamental diagrams allows taking into account the transport flow heterogeneity, which was the focus of the research by WANG et al. [1]. Modern macroscopic models are based on fundamental researches by LIDTHILL and WHITHAM [16], RICHARDS [17], etc. and on modern approaches considered in the research by HELBING and JOHANSSON [2]. Models of operational behaviour based on the use of neural networks have been investigated by PANWAI and DIA [3], and by KHODAYARI et al. [4].

It was experimentally proved by SUGIYAMA et al. [5] that emergence of traffic congestion is a collective phenomenon, such as “dynamic” phase transitions and pattern formation in a non-equilibrium system. Developments related to the definition of a continuous model for vehicles based solely on the time interval are particularly relevant. These developments involve the determining of a model by differential system to which a sequential numerical scheme is associated, that was discussed by TORDEUX et al. [6]. There is also considerable scientific interest in modifying intelligent driver models. The proposed procedure by KURTC and ANUFRIEV [7] includes the obtained local conditions of stability and takes into account the dynamics of the vehicle, driver behaviour and weather conditions. In this case, nonlinear constraints are generated by the numerical integration scheme.

A microscopic approach, offered by CHECHINA et al. [8], is used in mathematical modelling of transport flows in urban road networks, which allows simulating traffic on various types of road fragments and forecasting consequences of various decisions regarding changes in the road infrastructure. Calibrating microscopic traffic models is extremely important in further studies and it has been improved without noticeable effect on the capability of reproducing reality by using sensitivity analysis based on dispersion and number of other developments, as stated by PUNZO et al. [9].

Recent studies by WANG et al. [10,11] deal with modelling issues in cargo flows management emphasizing on importance of effectiveness improvement. GALKIN et al. [12,13] studied interaction aspects in providing transportation services.

Results. As noted by KASHTALINSKIY et al. [14], the study of transport flow properties is an integral part of the process of implementing adaptive and intelligent traffic management systems in cities, which requires the development of appropriate mathematical models.

To determine the patterns of influence on the dynamics of transport flows, the following cases were considered. Let us assume that a pedestrian arrives to the junction at time $t = 0$ and he needs time equal to T_n to cross a street. Let the length of time to the moment of the first car arrival t_1 have distribution density $g_0(x)$, distribution function $G_0(x)$, and Laplace transform of distribution density $\varphi_0(s)$. Then after the first car passing, lengths of time between sequential arrival moments of vehicles are equal to t_2, t_3, \dots, t_n , respectively, and each of them has distribution density $g(x)$, distribution function $G(x)$ and Laplace transform of distribution density $\varphi(s)$.

$W(x)$ is the probability of waiting duration for the start of street crossing less than or equal to x . Distribution density of a random variable x is denoted by $\omega(x) = W'(x)$, and Laplace transform of distribution density is denoted by $\psi(s)$. In case of $t_1 > T_n$, pedestrian will not wait, therefore, distribution has a discrete component at the start point:

$$(1) \quad P(t_1 > T_n) = 1 - G_0(T_n)$$

and continuous component determined in area $(0, \infty)$. Let us assume that opportunity to cross the street appears for the first time in time interval t_{n+1} , $n > 0$. Then waiting time will be equal to $\sum_{i=1}^n t_i$, which is the usual sum of random variables with known distribution densities. In this case, before carrying out convolution directly, it is important to keep in mind that according to the accepted hypothesis, a pedestrian is not able to cross the street at any of the time intervals $t_1, t_2, t_3, \dots, t_n$ and thus each of these time intervals must be at least T . Therefore, distribution density of random variable t_1 is not equal to $g_0(x)$, as in the case considered earlier, when such restrictions were not imposed, but it is equal to:

$$(2) \quad \frac{g_0(x)}{G_0(T)}, \quad 0 < x < T.$$

Herewith, area of determining distribution density of the first interval duration is limited by value of T . Similarly, distribution density of other intervals duration will be as follows:

$$(3) \quad \frac{g(x)}{G(T)}, \quad 0 < x < T.$$

So, if the street crossing occurs in the $(n + 1)$ -th interval, then the continuous component of distribution density $\omega(x)$ is:

$$(4) \quad \left[\frac{g_0(x)}{G_0(T)} \right] \cdot \left[\frac{g(x)}{G(T)} \right]^{(n-1)*}.$$

To obtain the unconditional continuous density component $\omega(x)$, it is necessary to multiply expression (3) by probability of the street crossing in $(n + 1)$ -th interval. Primarily, area of defining n -fold convolution given by expression (3) is examined. Since the duration of each time interval t_i is limited by the value of T , the sum of n intervals should be in the interval $(0, nT)$. So, area of determining convolution given by expression (4) will vary for different values of n .

Obviously, probability of the street crossing in $(n + 1)$ -th time interval will be:

$$(5) \quad G_0(T)G^{n-1}(T)[1 - G(T)].$$

By multiplying expressions (4) and (5), summing the result by n and taking into account the discrete component at the start point, the following was obtained:

$$(6) \quad \omega(x) = [1 - G_0(T)]\delta(x) + [1 - G(T)] \sum_{n=1}^{\infty} g_0(x) \cdot g^{(n-1)*}(x).$$

It is considered that $g^0 = 1$, which means that convolutions have different definition areas. Taking the Laplace transforms of function $\omega(x)$, the following was obtained:

$$(7) \quad \psi(s) = [1 - G_0(T)] + \frac{[1 - G(T)]\varphi_0(s; T)}{1 - \varphi(s; T)},$$

where

$$(8) \quad \varphi_0(s; T) = \int_0^T g_0(x)e^{-sx} dx$$

and

$$(9) \quad \varphi(s; T) = \int_0^T g(x)e^{-sx} dx.$$

They are “partial” transformations of functions g_0 and g corresponding to the value of T . By differentiating equality (7) and assuming $s = 0$, the following expression for average waiting time was obtained:

$$(10) \quad \text{Average waiting time} = \int_0^T xg_0(x) dx + \frac{G_0(T)}{1 - G(T)} \int_0^T xg(x) dx.$$

In formula (6), distribution density of waiting time was expressed in terms of distribution of time intervals length between sequential vehicles. Let us assume that $(r - 1)T < x < rT$. Then:

$$(11) \quad \begin{aligned} W(x) &= 1 - G_0(T) + [1 - G(T)] \sum_{n=r}^{\infty} \int_0^x g_0 * g^{(n-1)*} dx \\ &= 1 - G_0(T) + [1 - G(T)] \sum_{n=r}^{\infty} \bar{Q}_{n-1}(x), \quad (r - 1)T < x < rT. \end{aligned}$$

Frequently, more convenient expressions for distribution of the number of vehicles are obtained in the case of synchronous calculation. The distribution of the number of vehicles with a synchronous calculation is as follows:

$$(12) \quad W(x) = 1 - G_0(T) + [1 - G(T)] \frac{1}{\gamma} \int_0^x Q_{r-2}(t) dt, \quad (r-1)T < x < rT.$$

The presence of r in formulas (11) and (12) clearly shows that distribution function of the waiting time is piecewise continuous. In formula (12), this fact was not explicit.

Let us assume that arrival times of pedestrians form a homogeneous Poisson process with parameter λ . Then the results can be applied regardless of the transport flow nature.

In particular, Laplace transform of the waiting distribution density must satisfy the expression in which denotation ψ corresponds to φ :

$$(13) \quad \pi(s) = \sum s^n P(n),$$

where $P(n)$ is the probability of waiting n pedestrians.

The function $\beta(s)$ is Laplace transform of service duration distribution density. In this case, the service duration is equal to zero, since crossing the street T is not considered an integral part of the waiting time. So:

$$(14) \quad \beta[\lambda(1-s)] = \int_0^\infty e^{-\lambda(1-s)x} \delta(x) dx = 1.$$

By formula (7), the function applied for distribution of the number of pedestrians in the group was found. In the simplest case (a purely random transport flow), this function will be:

$$(15) \quad \frac{A - Bs}{C - Ds + Pe^{Qs}}.$$

Constant expressions can easily be expressed in terms of average values λ and μ , as well as time for crossing the street T . However, it is impossible to easily write the formula for the coefficients at s^n in the series expansion of expression (15).

It should be noted that the contribution of KASHTALINSKIY and PETROV [15] to the development of solutions for transport flows issues in cities confirms the abovementioned results obtained by the authors.

Conclusion. As can be seen from the results of mathematical modelling and based on a number of studies, the transport flow as a management object has a number of features, including randomness, which manifests itself in the unevenness of changes in intensity, traffic along highways, and crossing of intersections. The patterns of the influence of transport flow irregularity on the degree of intersection congestion of regular connections with vehicles were considered. Based

on the mathematical modelling, the dependence of the influence of increasing the congestion degree on the increase in vehicle delays was identified. Application of the given mathematical model will reduce the probability of traffic congestion in the urban transport network of large cities, and will also make possible to establish the duration of passenger transportation by urban passenger transport on urban routes, thereby ensuring high-quality public transport services. Thus, in the formation of an integrated intelligent transport system of cities and the traffic control system as its element, the use of the proposed mathematical model is appropriate.

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