



The Optimal Thickness of the Surface Plasma Hardening Layer of Functional-Gradient Parts with Symmetrical Stress Concentrators

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Abstract. Based on the approaches of continuous mathematical modeling, a computational methodology is proposed for establishing one of the most important operational parameters of surface engineering technologies, which determines the life of parts - the thickness of the hardened layer. For the first time, using non-local models of mechanics and the Fenics finite element analysis package, a scheme for finding the optimal hardening depth of parts depending on the load parameters and characteristics of stress concentrators has been built. Using the example of wheel rims of locomotives, it is shown that in the presence of stress concentrators, the increase in the life cycle of parts is achieved by plasma strengthening to a depth of 2.5... 3.5 times larger than the radius of the concentrators. The methodology proposed in the work is necessary for describing the behavior of the material of parts under the action of force loads in the presence of stress concentrators for selecting the optimal technological modes of hardening the surface layers. The established relationship between the change of properties of local volumes of wheels of locomotives at operational loadings and characteristics of concentrators of tension is used to obtain products with the defined life cycle, computer designing of the technological process of plasma strengthening of surfaces of contact interaction of details.

Keywords: Mathematical modeling · Finite element analysis · Stress concentrator · Surface engineering technology · Contact strength · Plasma hardening · Weakened zone

1 Introduction

Managing the operational parameters of parts operating under intense contact loads is basic at maintaining the specified life cycle of structures [1].

In conditions of the inhomogeneous stress state of parts, which may have a structural, technological, or operational origin, these issues are particularly acute [2].

One of the sources of formation of this state is surface stress concentrators, the appearance of which significantly reduces the durability of structures [3].

The existence of stress concentrators on the surface of parts leads to the appearance of local areas with differences in stress levels, which significantly changes the parameters of contact strength and other performance characteristics of products [4–6].

To reduce the negative impact of inhomogeneous stress on the life of structures in mechanical engineering, hardening treatments are used, which are obtained by various methods of surface engineering. As a result, 3-D functional-gradient structures are formed in the body, which reduces the negative impact of surface stress concentrators [7].

However, the question of choosing the optimal modes of surface treatment technologies depending on the size of stress concentrators and their placement on the surfaces of parts remains open [8–10].

In this regard, this work aims to develop a computational and experimental methodology for establishing optimal surface treatment modes to increase the life cycle of parts with stress concentrators.

2 Literature Review

One of the most effective methods of establishing optimal surface engineering technologies is mathematical modeling [2].

The first mathematical methods of calculating and selecting modes of surface engineering technologies were the methods of multicriteria optimization and mathematical statistics [2, 11].

These methods allowed the use of limited computing resources to obtain the desired results but did not justify the standpoint of the use of approaches to mechanics, materials science, mechanical engineering. Insufficient efficiency of such approaches is also because they lack physical models of bodies and processes that occur during technological processing, particularly the redistribution of stresses, strains, and temperature fields.

In modern mechanics, analytical or numerical methods are used in the calculations and analysis of the stress-strain state and temperature fields of bodies [11]. The former allows to obtain a sufficiently accurate solution but has a significant limitation – they can be used only for bodies with simple geometry and uniform distribution of properties. Considering the gradient of the distribution of properties in the material of bodies requires the use of such a complex mathematical apparatus that analytical calculations become almost impossible.

In this regard, numerical methods are most often used in engineering practice, namely - the finite element method and the corresponding software packages that allow you to solve a wide range of problems [11]. However, their use for modeling and solving modern problems in technological mechanics, which is called Integrated Computational Materials Engineering, is quite limited due to the confidentiality of the code of most paid licensed software products.

Therefore, to solve problems, which must also consider the heterogeneity of the structural and energy state of the material, used free, open-source software [11, 12].

This type of software has a number of advantages [11, 13]: a) allows you to customize the software to your specific needs and tasks; b) improve and expand their functionality;

c) has a high degree of interoperability and scalability, which allows it to be used in applied calculations.

Determination of the stress-strain state and its optimization were performed using a finite element analysis package Fenics, and its implementation in Python [12–14].

From the standpoint of mathematical modeling of problems of technological mechanics in the Fenics package, there are the following functionalities and advantages [12, 13]:

1. this environment allows you to automatically solve stationary and non-stationary differential equations in areas of complex shape using variational formulations;
2. at a high level of abstraction, you can write discrete models;
3. implement the boundary conditions of the 1-st, 2-nd, and 3-rd kind;
4. maintain a large number of approximating elements for areas of different dimensions, for continuous and discontinuous quantities;
5. provide various partition adaptations for discrete domains;
6. this environment allows you to work with subregions and materials that have different thermophysical properties;
7. The Python interface will enable you to additionally use such effective mathematical software packages as NumPy, SciPy, MatPlot Lib.

Post-processing of the finite element analysis results was performed using the free MatPlotLib package for Python [14].

3 Research Methodology

To analyze the contact strength of bodies with stress concentrators, consider the following generalized mathematical formulation:

1. Let the studied body (detail) occupy the area of space X , denote the body surface ∂X .
2. In the body, we set the area Y_1, \dots, Y_n which are sources of stress concentration ($Y_i \subset Y, i = 1..n$).
3. The body's surface is represented as a set of disjoint sets $\partial X = \partial X_1 \cup \partial X_2 \cup \partial X_3$, where ∂X_1 - an area of the body in which the specified power loads, ∂X_2 - body area, with specified restrictions on movement, ∂X_3 - unloaded surface. Areas $\partial X_1, \partial X_2, \partial X_3$ in turn may consist of simple sub-areas: $\partial X_1 = \bigcup_{i=1}^n \partial X_1^i, \partial X_2 = \bigcup_{j=1}^m \partial X_2^j, \partial X_3 = \bigcup_{l=1}^k \partial X_3^l$.
4. In each of the elementary sub-areas ∂X_1^i given power vector \vec{F}_1^i , and on ∂X_2^j - vector of displacements \vec{u}_2^j which in ∂X have the following distribution $\vec{F}_1^i = \vec{F}_1^i(x), \vec{u}_2^j = \vec{u}_2^j(x), x \in \partial X$.

When considering the behavior of parts under contact loads, we use a mathematical model of the elastic continuum with distributed damage, which is given in [15]:

$$\begin{aligned} & \vec{\nabla} \cdot \left(\frac{K(x)}{1 - \sqrt{\alpha_1 (\omega(x))^2 + \alpha_2 \left(\left| \vec{\nabla} \omega(x) \right| \right)^2 + \alpha_3 \left(\frac{1}{V_0} \int_V \omega dV \right)^2}} \left(\vec{\nabla} \cdot \vec{u} \right) \hat{I} \right. \\ & \left. + \frac{2G(x)}{1 - \sqrt{\alpha_1 (\omega(x))^2 + \alpha_2 \left(\left| \vec{\nabla} \omega(x) \right| \right)^2 + \alpha_3 \left(\frac{1}{V_0} \int_V \omega dV \right)^2}} \left(\vec{\nabla} \otimes \vec{u} - \frac{1}{3} \left(\vec{\nabla} \cdot \vec{u} \right) \hat{I} \right) \right) = 0, \end{aligned} \quad (1)$$

where $\alpha_1, \alpha_2, \alpha_3$ - numerical constants, $\omega(x)$ - damage level, \vec{u} - displacement vector, \hat{I} - single tensor, $\vec{\nabla}$ - Hamilton differential operator, $K(x)$ - volume compression module, $G(x)$ - shear module, \otimes - tensor product, \cdot - scalar product, V_0 - the characteristic size of the material area at the meso-level is equal to the three-grainmeters.

As a result of power loads in part, a stress state is formed, characterized by a tensor at each point $x \in X$.

Since the nominal stresses do not uniquely determine the operational parameters of structures [16], for the analysis of contact strength was used the expression:

$$k(x) = 1 - \frac{\sigma_m(x)}{\sigma^*(x)}, \quad (2)$$

where $k(x)$ - coefficient of margin of contact strength, $\sigma_m(x)$ - equivalent representation of the stress tensor in the form of Mises, $\sigma^*(x)$ - strength properties of the material at the point. This parameter is dimensionless.

Size $k(x)$ varies in the interval $(-\infty; 1]$, at $k(x) = 1$ - no weakening, when $k(x) \leq 0$ - the weakening has passed.

The task of increasing the contact strength of the body X from a given set of voltage concentrators $\{Y_1, \dots, Y_n\}$ is reduced to the search for such a structural and energy state of the body, which ensures the stability of the state of weakening of the structure:

$$L = \frac{\dim(k(x) \leq 0)_{x \in V}}{\dim(x \in V)} \rightarrow const, \quad (3)$$

where $\dim(\dots)$ - the size of the area that satisfies the specified condition. As can be seen from relation (3), the parameter L is dimensionless.

As an example for research, we will consider one of the most widespread technologies of surface engineering - plasma strengthening of a bandage of a wheel of a locomotive which changes a structural and power condition of material both in surface layers and in-depth at the expense of purposeful thermal influence on a detail [17]. As a result, gradient structures are formed in the near-surface zones, the properties of which differ significantly in different zones.

In the study, we consider a two-dimensional rectangular body with a symmetrical distribution of stress concentrators with radius R_1 , located at a distance Z_1 from the zone of friction loads (Fig. 1).

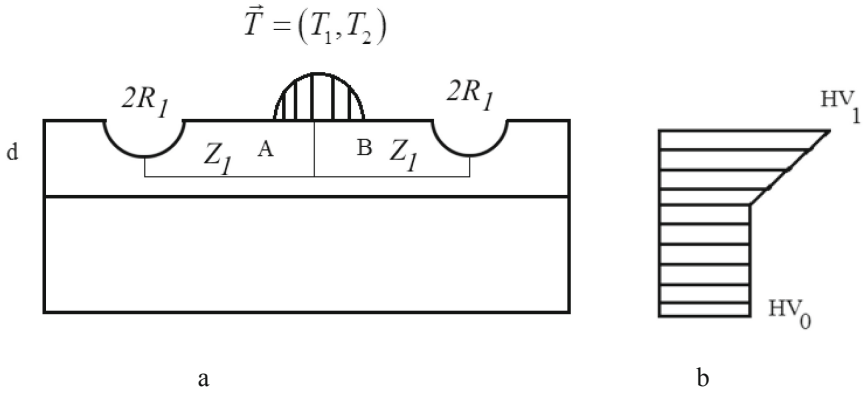


Fig. 1. The object of research: AB - contact spot (area of the contact interaction), $\vec{T} = (T_1, T_2)$ - load vector, d - thickness of the hardening zone, R_1 - radius of stress concentrators, Z_1 - the distance between the contact zone and stress concentrators; b) plot of the distribution of microhardness in-depth: HV_1 - the value of microhardness on the surface, HV_0 - the importance of microhardness in the depth of the material.

4 Results

It was found that directly on the surface of the locomotive wheel bandage as a result of plasma hardening microhardness is $HV_1 = 3900$ MPa, the original microhardness - $HV_0 = 2600$ MPa, which is found at a depth $d = 2$ mm from the surface, the value of the initial force load was equal to $12.5 \cdot 10^4$ N, and the coefficient of friction is 0.25. The size of the investigated area was: width - 100 mm, depth - 50 mm.

The value of the load is $12.5 \cdot 10^4$ N, and the coefficient of friction is 0.25. For calculations, we additionally take the value of $R_1 = 3$ mm, and $Z_1 = 10$ mm. This formulation of the problem corresponds to the contact interaction of the wheel-rail in the presence of damage to the wheel in the form of symmetric stress concentrators of operational or technological origin [18, 19].

As a result of calculations, we will receive a stress field which will have the form shown in Fig. 2.

Since the stress state clearly does not determine the strength of the object, as a characteristic of its performance properties, we take the weakening of the structure according to relation (2) (Fig. 3).

We will study the effect of the size of stress concentrators on the weakening of the structure depending on the depth of strengthening of the material layer. To do this, we choose functional (3) as an optimization criterion, and under the control parameter, we consider the thickness of the reinforced layer d . The value of d will be changed from 0 to 10 mm, and the size of the radius of the stress concentrator R_1 from 1 to 3 mm. As a result, we obtain the following dependence on softening from d (Figs. 4, 5 and 6).

Analysis of the calculations indicates that depending on the size of the surface stress concentrator and the coefficient of friction; there is a value of the depth of the hardened layer d^* , which is approximately 2.5...3.5 times larger than the radius of stress concentrators, at which the softening parameter does not change significantly. Further

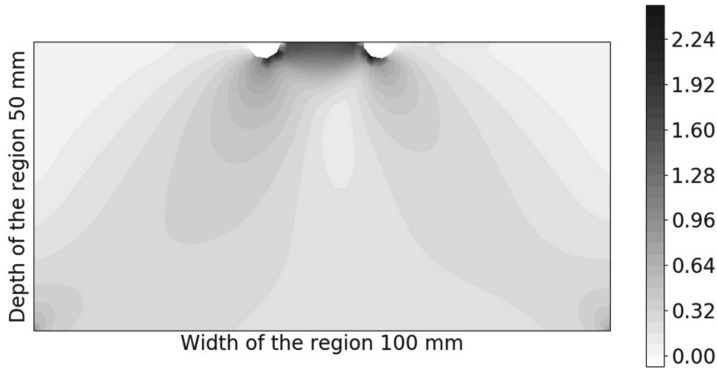


Fig. 2. Stress distribution in the investigated structure in the presence of symmetrical concentrators at a load of $12.5 \cdot 10^4 \text{ N}$, 10^9 Pa .

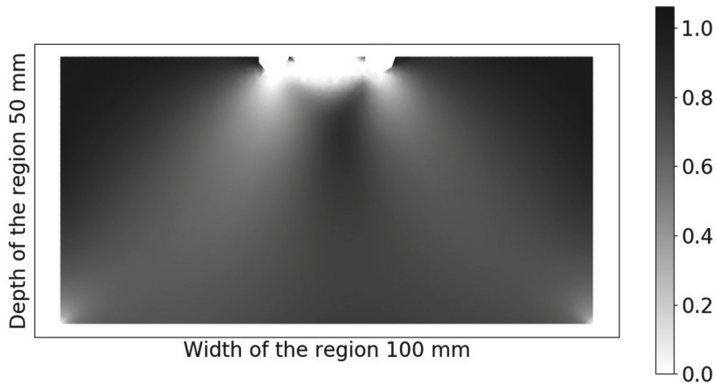


Fig. 3. Distribution of the amount of weakening in the body in the presence of symmetric stress concentrators.

hardening will be impractical because with the same parameters of operational strength increases energy consumption for the technological process of surface treatment.

Additionally, there is a nonlinear dependence of the growth of the softening zones with a change in the size of the stress concentrators: with increasing R_1 from 1 to 3 mm, the increase in the size of the softening occurs 2 times.

There is also a change in the cushioning parameter depending on the coefficient of friction, which indicates the influence of processes occurring in the zone of frictional contact on the strength parameters of the wheels. To ensure the functioning of the wheelsets in conditions where the coefficient of friction will be close to 0.5, it is necessary to increase the thickness of the surface hardening layer by an additional 10...15%.

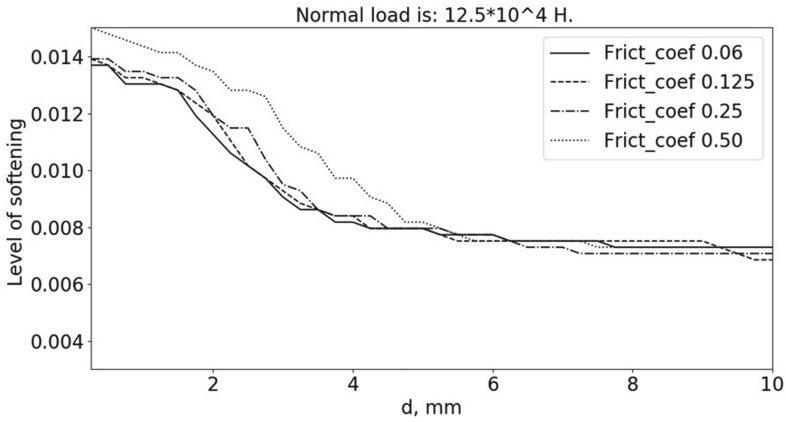


Fig. 4. The level of the softening for normal load $12.5 \cdot 10^4$ N, and the size of the stress concentrators $R_1 = 1$ mm.

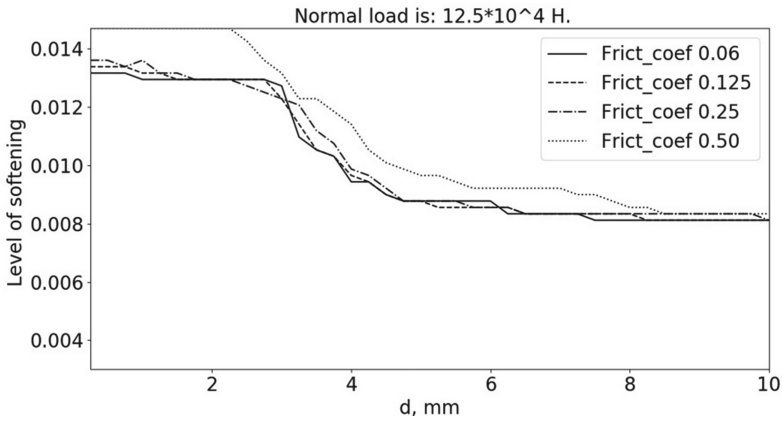


Fig. 5. The level of the softening for normal load $12.5 \cdot 10^4$ N, and the size of the stress concentrators $R_1 = 2$ mm.

The developed approach allows choosing the modes of plasma hardening to control the wheel pairs' operating parameters with the presence of stress concentrators. Optimization of processing modes ensures the strength and durability of parts following the safety conditions of the railway infrastructure, which are specified in the relevant EU regulatory documents (EN 12663-1:2010) and scientific literature [20, 21].

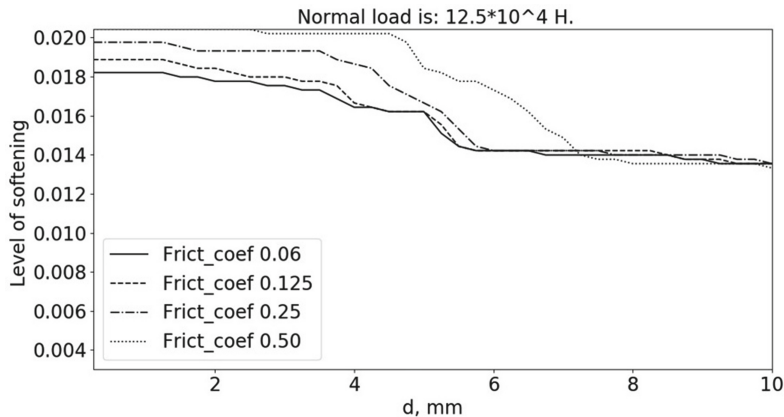


Fig. 6. The level of the softening for normal load $12.5 \cdot 10^4$ N, and the size of the stress concentrators $R_1 = 3$ mm.

5 Conclusions

Based on computational and applied mechanics approaches, the problem of optimization of operational parameters of locomotive wheel tires with the symmetrical distribution of stress concentrators after plasma hardening is considered.

Using the package of FEM-analysis Fenics and its implementation in Python, analyzed the stress state of the structure at load modes that are close to operating.

It is established that depending on the size of the stress concentrator. It is necessary to strengthen the wheel rims to different depths, which should be 2.5...3.5 times larger than the radius of the concentrators. Strengthening the tires of locomotive wheels to a greater depth is impractical because the same parameters of operational strength increase the cost of the technological process.

To ensure the functioning of the wheelsets in conditions where the coefficient of friction will be close to 0.5, it is necessary to increase the thickness of the surface hardening layer by 10...15%.

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