20th INTERNATIONAL CONFERENCE
„CURRENT PROBLEMS IN RAIL VEHICLES - PRORAIL 2011"
September 21 – 23, 2011, Žilina, Slovakia

USAGE OF QUATERNIONIC MATRICES TO DEFINE INERTIA MOMENTS OF WHEELSET TAKING INTO ACCOUNT MANUFACTURING AND ASSEMBLY ERRORS

NÁZOV PRÍSPEVKU

Aleksandr Kharchenko, Tamila Kravets, *)

1 INTRODUCTION

Inertial forces carry out essential influence on dynamics and firmness of a high-speed rolling stock, on dynamic loading of structure elements and track, causing intensive vibration of a car body, raising of wheels and rails deterioration. One of sources of these negative phenomena is asymmetry of wheel set construction and change of its inertial characteristics connected with it. This asymmetry is caused by a succession of random factors which lead to displacement of the center of weight relative to axis of symmetry,and skewness of the main axes of inertia from geometrical axes. Negative dynamic phenomenas like noise, vibrations, fluctuations, raised deterioration are caused by centrifugal and gyroscopic forces which increase proportionally to a square of speed. Their estimation and account during designing and operation of a high-speed rolling stock get great value. The inertia matrix is the basic indicator of inertial properties of wheel set. Therefore it is very important to work out an effective algorithm of reduction of a matrix of inertia to structurally convenient center and base trihedral. Thus manufacturing and assembly should be considered.

In this work the advanced method of calculation of matrix of inertia of wheel set which is based on quaternionic matrixes is offered. It takes into account specified technological manufacturing and assembly errors. Methods of solving of this problem that are known now leads to cumbersome formulas which are inconvenient for programming and calculations on computer. Quaternionic matrixes are made up on basis of Rodriguez-Hamilton parameters. These parameters consider a skewness of main central axes of inertia of a wheels and axis of wheel set from geometrical symmetry axes.

*) **Scientific supervisor Aleksandr Kharchenko**, Research and Development Establishment of Running Gear, Railway Track and Constructions, Dnipropetrovsk National University of Railway Transport named after ac. V Lazaryan. Ukraine, Dnipropetrovsk, Lazaryana str. 2, post code 49005, +380503211460, Kharchenko.ndi@gmail.com
Assistant Tamila Kravets, Department of theoretical mechanics, +380679211067

2 Problem formulation

The layout scheme of wheel set is shown in Fig.1

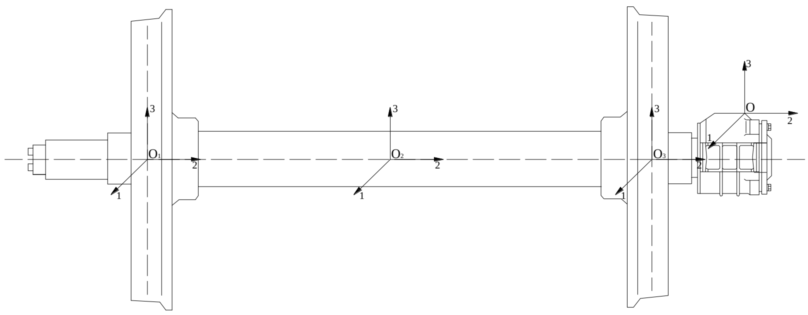

***Fig.1**. Layout scheme of wheel set and system of coordinates which are connected with the geometrical centers and symmetry axes.*

O − the pole of base trihedral which is located on axle-box knot;

O_1, O_2, O_3 − geometrical centers of symmetry of wheels and axis of wheel set, and also corresponding systems of coordinates.

The basic geometrical parameters of wheelset are set in nominal sizes. The entered systems of coordinates which are connected with each wheel (pole O_1, O_3) and an axis (O_2) in the geometrical centers of symmetry. Axes of the entered systems of coordinates are focused on geometrical axes of symmetry of elements. Let's consider that base trihedral (pole O) is located on axle-box knot according to figure. Then coordinates of poles of wheels and axis in base trihedral are $x_{oij} = (i = 1, 2, 3, j = 1, 2, 3)$.

2.1 Manufacturing errors

We will designate nominal sizes of mass of wheels and axis as m_1, m_2, m_3, and $m_1 = m_3$. Due to random factors the actual size of these mass can change within manufacture tolerance $\pm\Delta m_i$, i.e. $m_i \pm \Delta m_i$ $(i = 1, 2, 3)$. The manufacturing and assembly error leads to a deviation of actual coordinates of the centers of mass of wheels and axis from the geometrical centers of mass within tolerances $\pm\varepsilon_j^{ci}$ $(j = 1, 2, 3)$. Let's consider that sizes of the central moments of inertia of wheels and axis also change within the set manufacture tolerances $I_{jj}^{ci} \pm \Delta I_{jj}^{ci}$, and $I_{11}^{ci} = I_{33}^{ci}$, and also $I_{jj}^{c1} = I_{jj}^{c3}$, $\Delta I_{jj}^{c1} = \Delta I_{jj}^{c3}$. Nominal sizes of the main moments of inertia of wheels and axis are set by diagonal matrixes. Their orientation relative to geometrical axes of symmetry we will define by Euler-Krylov angles which are in small sizes and get out within the set tolerances $\pm\Delta\alpha^{ci}$, $\pm\Delta\beta^{ci}$, $\pm\Delta\gamma^{ci}$ (fig.2)

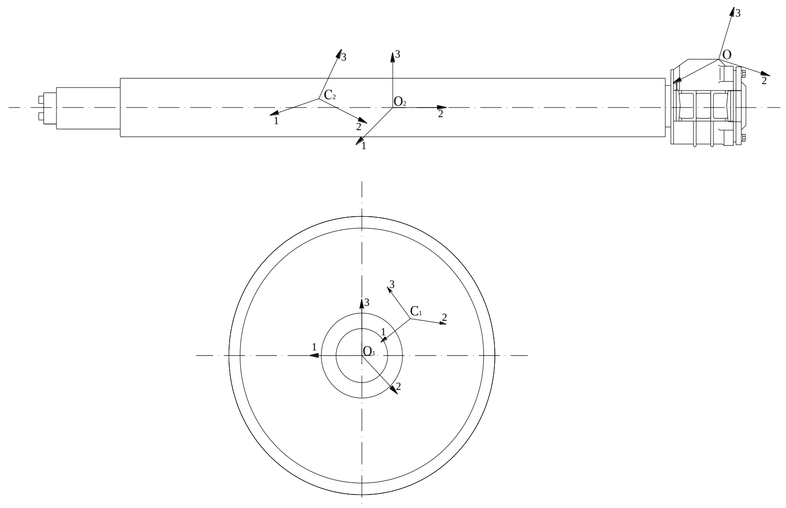

Fig 2. *The scheme of an arrangement and orientation of the central axes of inertia of wheels and axis of wheel set.*

C_1, C_2, C_3 – mass centers of wheels, axis and corresponding main central axes of inertia

$\pm \varepsilon_j^{ci}\ (i, j = 1,2,)$ – deviation of coordinates of mass centers of wheels $(i = 1,3)$ and axis $(i = 2)$ from the nominal position

$\pm \Delta\alpha^{ci}, \pm\Delta\beta^{ci}, \pm \Delta\gamma^{ci}$ – Euler-Krylov angles, which are characterized the deviation of actual main central axes of inertia of wheels and axis from the associated coordinate systems

$\pm\Delta m_i$ – deviation of actual mass from the nominal values

$\pm \Delta I_{jj}^{ci}$ – deviation of actual main central moments of inertia of wheels and axis from the nominal values.

The matrix formula of reduction of the main central moments of inertia of wheels and axis to geometrical axes of symmetry is the following:

$$2\bar{I}_{Oi} = 2\Delta_{ci} \times {}^t\Delta_{ci} \times \bar{I}^{ci} \times \Delta_{ci}^t \times {}^t\Delta_{ci}^t + E_{ci}\left(E_{ci}^t + {}^tE_{ci}^t\right) \tag{1}$$

The formula takes into account transfers and rotations in three-dimensional space.

Here:

$$\bar{I}^{ci} = \frac{1}{m_i}\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & I_{11}^{ci} & 0 & 0 \\ 0 & 0 & I_{22}^{ci} & 0 \\ 0 & 0 & 0 & I_{33}^{ci} \end{Vmatrix} \tag{2}$$

$$E_{ci} = \begin{Vmatrix} 0 & \varepsilon_1^{ci} & \varepsilon_2^{ci} & \varepsilon_3^{ci} \\ -\varepsilon_1^{ci} & 0 & -\varepsilon_3^{ci} & \varepsilon_2^{ci} \\ -\varepsilon_2^{ci} & \varepsilon_3^{ci} & 0 & -\varepsilon_1^{ci} \\ -\varepsilon_3^{ci} & -\varepsilon_2^{ci} & \varepsilon_1^{ci} & 0 \end{Vmatrix} \tag{3}$$

$$\Delta_{ci} = \begin{Vmatrix} \delta_0^{ci} & \delta_1^{ci} & \delta_2^{ci} & \delta_3^{ci} \\ -\delta_1^{ci} & \delta_0^{ci} & -\delta_3^{ci} & \delta_2^{ci} \\ -\delta_2^{ci} & \delta_3^{ci} & \delta_0^{ci} & -\delta_1^{ci} \\ -\delta_3^{ci} & -\delta_2^{ci} & \delta_1^{ci} & \delta_0^{ci} \end{Vmatrix} \tag{4}.$$

Rodriguez-Hamilton parameters $\delta_j^{ci}\left(j = 0,1,2,3\right)$are determined by Euler-Krylov angles:

$$\begin{Vmatrix} \delta_0^{ci} \\ \delta_1^{ci} \\ \delta_2^{ci} \\ \delta_3^{ci} \end{Vmatrix} = \begin{Vmatrix} cos\frac{\Delta\gamma^{ci}}{2}cos\frac{\Delta\beta^{ci}}{2}cos\frac{\Delta\alpha^{ci}}{2} - sin\frac{\Delta\gamma^{ci}}{2}sin\frac{\Delta\beta^{ci}}{2}sin\frac{\Delta\alpha^{ci}}{2} \\ cos\frac{\Delta\gamma^{ci}}{2}cos\frac{\Delta\beta^{ci}}{2}sin\frac{\Delta\alpha^{ci}}{2} + sin\frac{\Delta\gamma^{ci}}{2}sin\frac{\Delta\beta^{ci}}{2}cos\frac{\Delta\alpha^{ci}}{2} \\ -sin\frac{\Delta\gamma^{ci}}{2}cos\frac{\Delta\beta^{ci}}{2}sin\frac{\Delta\alpha^{ci}}{2} + cos\frac{\Delta\gamma^{ci}}{2}sin\frac{\Delta\beta^{ci}}{2}cos\frac{\Delta\alpha^{ci}}{2} \\ sin\frac{\Delta\gamma^{ci}}{2}cos\frac{\Delta\beta^{ci}}{2}cos\frac{\Delta\alpha^{ci}}{2} + cos\frac{\Delta\gamma^{ci}}{2}sin\frac{\Delta\beta^{ci}}{2}sin\frac{\Delta\alpha^{ci}}{2} \end{Vmatrix} \tag{5}$$

Considering that turn of the main central axes of inertia relative to geometrical axes is small, then:

$$\begin{Vmatrix} \delta_0^{ci} \\ \delta_1^{ci} \\ \delta_2^{ci} \\ \delta_3^{ci} \end{Vmatrix} = \begin{Vmatrix} 1 - \frac{\Delta\gamma^{ci}}{2}\frac{\Delta\beta^{ci}}{2}\frac{\Delta\alpha^{ci}}{2} \\ \frac{\Delta\alpha^{ci}}{2} + \frac{\Delta\gamma^{ci}}{2}\frac{\Delta\beta^{ci}}{2} \\ \frac{\Delta\beta^{ci}}{2} - \frac{\Delta\gamma^{ci}}{2}\frac{\Delta\alpha^{ci}}{2} \\ \frac{\Delta\gamma^{ci}}{2} + \frac{\Delta\beta^{ci}}{2}\frac{\Delta\alpha^{ci}}{2} \end{Vmatrix} \tag{6}$$

Neglecting small values of the second order and above, we will receive:

$$\begin{Vmatrix} \delta_0^{ci} \\ \delta_1^{ci} \\ \delta_2^{ci} \\ \delta_3^{ci} \end{Vmatrix} = \begin{Vmatrix} 1 \\ \frac{\Delta\alpha^{ci}}{2} \\ \frac{\Delta\beta^{ci}}{2} \\ \frac{\Delta\gamma^{ci}}{2} \end{Vmatrix} \tag{7}$$

2.2 Assembly errors

In process of assembling of wheelset due to some random factors some displacements of the centers of symmetry of wheels and axis of wheelset and skew of their geometrical axes relative to the base trihedral occurs (fig.3).

á)

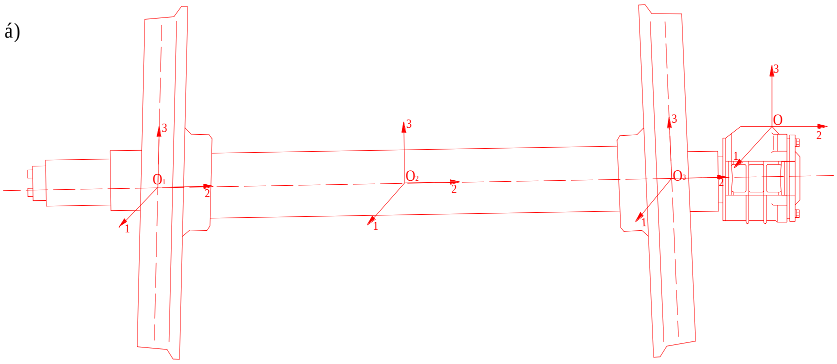

Fig. 3. The scheme of displacement and a skew of geometrical axes of symmetry during assembly.

The matrix formula of reduction of moments of inertia of wheels and axis to base trihedral is the following:

$$2\bar{I}_{oi}^{б} = 2A_i \times {}^tA_i \times \bar{I}_{oi} \times A_i^t \times {}^tA_i^t + \\ + \left(X_{ci} + {}^tX_{oi}^t\right)\left(X_{oi}^t + {}^tX_{oi}^t\right) + X_{oi}\left(X_{ci}^t + {}^tX_{ci}^t\right), \quad (8)$$

where

$$X_{ci} = X_{oi} + A_i \times E_{ci} \times {}^tA_i^t, \\ X_{ci}^t = X_{oi}^t + {}^tA_i \times E_{ci}^t \times A_i^t, \\ {}^tX_{ci}^t = {}^tX_{oi}^t + A_i \times {}^tE_{ci}^t \times {}^tA_i^t. \quad (9).$$

$X_{oi}, X_{oi}^t, {}^tX_{oi}^t, {}^tX_{oi}$ − quaternionic matrices, which are made up on basis of the coordinates of poles $x_{joi}\ (i = 1, 2, 3;\ j = 1, 2, 3)$, and take into account the displacement of wheelset components during assembly;

$X_{ci}, X_{ci}^t, {}^tX_{ci}^t, {}^tX_{ci}$ − quaternionic matrices, which are made up on basis of the coordinates of mass centers x_{jci}, of wheelset components in base trihedral;

$A_i, A_i^t, {}^tA_i^t, {}^tA_i$ − quaternionic matrices, which are made up on basis of Rodriguez-Hamilton parameters $a_{joi}\ (i = 1, 2, 3;\ j = 0, 1, 2, 3)$ and take into account skew of elements of wheelset during assembly relative to the base trihedral;

The resultant matrix of inertia of wheel set taking into account errors of manufacturing and assembly which is led to base trihedral is defined under the formula:

$$\bar{I}_o^{б} = \sum_{i=1}^{3} \frac{m_i}{m_1 + m_2 + m_3} \bar{I}_{oi}^{б} \quad (10)$$

3 CONCLUSION

It is offered to use computing algorithm of transformation of matrix of inertia at rotation in three-dimensional space. This algorithm is based on quaternionic matrixes. Quaternionic matrixes are made up on basis of Rodriguez-Hamilton parameters which consider a skew of the main central axes of inertia of wheels and axis of wheel set relative to geometrical axes of inertia. And also on coordinates of characteristic points which consider a deviation of actual coordinates of mass centers from the geometrical centers of symmetry within specified tolerances on manufacturing and assembly. Use of quaternionic matrixes allows to receive compact symmetric formulas and provides convenience of realization on computer.

References

[1] Gernet M.M., Ratobilskiy V.F. Defining moments of inertia. – M.: Machinery, 1969. − 247 p. **[2] Ishlinskiy A.Yu.** Mechanics of relative movement and force of inertia. – M.: Science, 1981. – 191p. **[3]Kravets V.V., Kravets T.V.** About an estimation of centrifugal and gyroscopic forces at high-speed movement of railway cars // Applied mechanics – 2008. – 44, № 1. − p.123 − 132. **[4] Larin V. B.** On the Problem of Control of a Compound Weel Vehicle // Int. Appl. Mech.− 2007. − 43, N 11.− P. 1297 − 1302. **[5] Lobas L. G., Koval'chuk V. V., Bambura O. V.** Theory of Inverted Pendulum with Follower Force Revisited // Int. Appl. Mech. − 2007.− 43, N 6.− P. 690 − 700.

♠♠♠