# SIMULATION OF WHEELSET MOVEMENT IN CAR DYNAMICS PROBLEMS

**Manashkin L. A., Doctor of Engineering, Professor,[1]**

**Myamlin S. V., Doctor of Engineering, Professor,[2]**

**Pshinko A. N., Doctor of Engineering, Professor [2]**

**Prikhodko V. I Professor. [3]**

[1] New Jersey Institute of Technology (the USA),

[2] Dnipropetrovsk National Railway University named after V. Lazaryan (Ukraine),

[3] Kryukovsky Car Building Works PJSC (Ukriane)

myamlin@nzn.diit.edu.ua

*The movement of the wheelset as a whole is analyzed as the forward and rotary movement of wheel rims, the forward and rotary movement of the wheelset axis together with the wheels naves, and the torsion of the wheelset. Recommendations on the choice of the torsion parameters are given.*

When solving the car dynamics problems, the wheelset movement relatively to the unsprung part of the bogie is usually neglected. The wheelset is considered as a part of unsprung solid body. In those cases when the wheelset movement relatively to the side frame is analyzed, it is considered as an individual solid body. However, when determining the forces of the wheelset and the rails interaction occurring due to the imperfect track and wheelset tread surface, this approach excludes from the consideration quite intensive short-term acting forces. These forces influence on the body oscillation is insignificant because the unsprung masses and the flexible body suspension filter these forces. The disturbance is assumed as applied to the spigot shaft of the wheelsetbecause only the wheels and rails interaction influence on the acceleration and axle-box movement values is considered in process of identification of such disturbance statistical parameters. At the same time, we normally overlook the specificity of immediate interaction of the wheelset and the rails. So, in case the wheel has quite real slid flat 20 mm long and 0.1 mm “deep” relatively to the wheel tread, the duration of such wheel rolling over will be 1 msec when the motion speed is 20 m/sec. These figures describe the vertical move frequency of the contact equal to 0.5 kHz (considering the impact duration corresponds to the half of the sinusoidal oscillation period). The acceleration of the contact point under the impact interaction conditions described above makes 1,000 m/sec2. Now suppose, it is only the wheel rim, which approximate weight is about 0.21 tons, that moves with such vertical acceleration; the inertia force of such rim only would comprise 210 kN (with the static loading of the rail equal to 108 kN). At the same time, the own wheel rim turn frequency relative to the axis normal to the car wheel center line makes just 187Hz, the own wheel rim shift frequency along the wheel axis is 340 Hz, and the lowest rim flexure forms are 366 and 947 Hz. The own frequencies of the wheelset torsional oscillations with type E120 electric locomotive drive elements are 16 and 50 Hz. On the other hand, for example, in case of breakage of the locomotive wheel and rail adhesion when the car wheel slide against the rail, quite high-frequency torsional oscillations of the wheelset occur together with the wheel bearing point vibration relatively to the rail in longitudinal direction. Taking into account the fact that these oscillations are intensified by vertical forces both unloading and loading the contact, these type of wheel and contact movement may significantly affect the wheelset movement stability, the traction effort realized by the wheel, the tread erosion and wear, and the rails erosion and wear including corrugations of rails. In case of agitation of torsional oscillations in the wheelset or wheel rim, the friction force occurring at motion will contribute to the rail wear with the frequency twice bigger than the own frequency of torsional oscillations. The vertical interaction force oscillations will modulate the friction force oscillations. As a result of such modulation, the spectrum of the friction force wearing the rail will include the components with doubled torsional oscillations frequency of the wheelset and with the frequencies equal to the sum and difference of doubled torsional oscillations frequencies and the frequencies of changing of the vertical interaction force of the wheel and rail. The oscillations with the smallest difference $\Omega$min (in Hz) of these frequencies will determine the maximum length $\lambda$max of the corrugation of rails at the given average car motion speed $\bar{V}$ in the track section. To find the length of the corrugation of rails $\lambda$max, relation $\lambda max=\overline{V}/\Omega min$ is used. Specific defects in rails provoke the actuation of rail wearing oscillations of all the wheels of a train in the given point of track.

Thus, the cases of the dynamic wheelset loading described above require the more detailed analysis of the wheelset oscillation mechanisms at its interaction with the rails.

This work analyzes the whole wheelset movement (Figure 1) as the spatial forward motion of the wheel rims relative to the wheel nave, the spatial forward

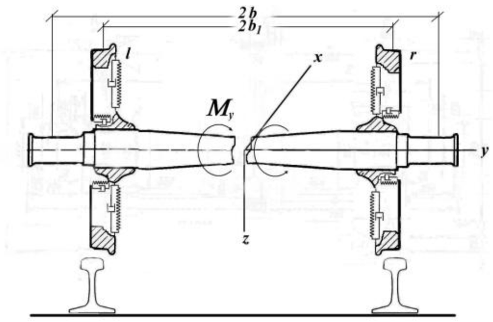


Fig. 1

motion of the wheelset axis together with the wheel naves and axle-boxes, the rotary motion of the whole wheelset relative to the vertical and longitudinal axes, and the rotary movements (including torsion) of wheels combined with the half of the wheelset axis relative to the its axis. The parameters of the wheelset parts coupling at their rotation around the wheelset axis may be determined as the wheelset axis torsion parameters and may be changed in time depending on the extreme torsion conditions. The wheels sliding against the rails determine these conditions. If only one wheel is in adhesion with the rail, the parameters correspond to the wheelset torsion under the condition of fixed right or left wheel and free opposite wheel. If both wheels slide, the parameters are chosen basing on the torsion conditions with two free wheels. To avoid the generalized forces values breakage at the moments of the wheelset torsion parameters changing when integrating the deferential equations of movement, the generalized forces are calculated at each time step of numerical integration by way of adding their increment at the given step to the force value of the previous step.

It shall be noted that analysis of the wheel rims as individual solid bodies was earlier described in relation to the locomotives with rubberized wheels. This work specifically analyzes the analogous model relatively to the wheelsets with solid metal wheels.

In whole, the freight car wheelset movement is described by 19 degrees of freedom, and the fright car itself is described by 98 degrees of freedom provided its mechanical system is not simplified. Depending on the problem set, the whole system may be quite reasonably simplified.

The equations of movement of a freight car shall be supplemented with the equations of track under the moving car according to the specific simulation pattern of its interaction with the car wheels. To simplify the system, this work assumes that the wheels interact with some bodies simulating the track inertia, which motion is limited by the track deformable ties. Hereinafter, we will refer to these bodies as reduced track masses.

Then in this paper, it is described the movement of just one wheelset with the attached bodies simulating the inertia and deforming properties of track. Such model is quite suitable for study of high-frequency and short-term pulse forces of the wheelset and the track interaction. Here, the body movement may be set in form of specific low-frequency slow changing functions derived experimentally in advance or when simulating the low-frequency car movements. Thus, 19 generalized coordinates may describe the wheelset movement.

The forces acting on the reduced masses from the basis side are calculated as derivatives of the expressions of the potential energy and the energy of the foundation dissipation under the car and track interaction forces basing on respective generalized coordinates.

In the text below as an example we will consider some possible wheelset oscillations.

**1. Constructed model of contact wheelset and rail.**

The main and possibly the most difficult element for simulation of dimensional model of railway vehicle is a model of contact and interaction wheel set and rail.

For valuation of correspondence of computer program with a real rolling stock, the mathematical model of railway vehicle spatial oscillation was developed [1-5]. On its basis the simplified mathematical model of wheelset movement consisting of 2 rails and a wheelset was created.

For the simulation, mathematical models [1-3] and the software program DYNRAIL [2,4,5], developed by Dnepropetrovsk National University of Railway Transport named after V. Lazaryan were used.

Objects parameters are described in Table 1.

Table 1.

**Parameters of constructed model**

| Object | Mass [t] | Momentums of inertia [t*m2] | | | Coordinates of mass center [m] | | |
|---|---|---|---|---|---|---|---|
| | | Jz | Jy | Jx | X | Y | Z |
| Foundation | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Left rail | 0,5 | 0 | 0 | 0 | 0 | -0,79 | 0 |
| Right rail | 0,5 | 0 | 0 | 0 | 0 | 0,79 | 0 |
| Wheelset | 1,37 | 1 | 0,1 | 1 | 0 | 0 | 0,475 |

In the paper below we describe constituent elements of the given model.

*Foundation* – is a motionless object, which doesn't have inertial parameters to which other objects are "attached".

*Rail (left and right)* – elements of railway track, which have reduced mass and situated on the left and on the right from the axle of track (coordinate Y). Both rails are "attached" by vertical, lateral and longitudinal connections to the Foundation.

*Wheelset* – is an object which simulates wheelset, and has inertial parameters and mass centre height equal to the radius of wheel. Wheelset is connected to rails with vertical-linear not bilateral constraint, and has connections with both rails in longitudinal and lateral directions, which are determined with forces of lateral and longitudinal creep. Besides at lateral movement of wheelset which surmount the clearance in track, a lateral spring linkage between flange and rail-track starts to act, in this case the influence of forces of lateral and longitudinal creep between flange of wheels and rail-track are also taken into account.

Coupling parameters of described model are summarized in Table 2.

Table 2.

**Model's coupling parameters**

| Coupling objects | | | | | |
|---|---|---|---|---|---|
| Foundation | | | Left rail (Right rail) | | |
| Coordinates of the points application of connection [m] | | | | | |
| X | Y | Z | X | Y | Z |
| 0 | 0 | 0 | 0 | 0 | 0 |
| Elements of connection | | | | | |
| Vertical | c = 86000kN/m β = 172kNs/m | | | | |
| Lateral | c = 20000kN/m β = 40kNs/m | | | | |
| Longitudinal | c = 80000kN/m β = 100kNs/m | | | | |
| Coupling objects | | | | | |
| Left rail | | | Wheelset | | |
| Coordinates of the points application of connection [m] | | | | | |
| X | Y | Z | X | Y | Z |
| 0 | 0 | 0 | 0 | -0,79 | 0,475 |

| Elements of connection | | | | | |
|---|---|---|---|---|---|
| Dimensional | c = 80000kN/m μк = 0,25 μг = 0,25 | | | | |
| Coupling objects | | | | | |
| Right rail | | | Wheelset | | |
| Coordinates of the points application of connection [m] | | | | | |
| X | Y | Z | X | Y | Z |
| 0 | 0 | 0 | 0 | 0,79 | 0,475 |
| Elements of connection | | | | | |
| Dimensional | c = 80000kN/m μк = 0,25 μг = 0,25 | | | | |

Coordinates of points of connections' application to the objects are set concerning to the objects' mass center. Coupling parameters are marked as: c – stiffness, β – ductility, μ – dry friction coefficient.

Coupling parameters of *Foundation – Left rail and Foundation – Right rail* are equal and have 3 linear viscoelastic elements: vertical, lateral and longitudinal.

Coupling parameters of *Left rail – Wheelset and Right rail* have just one (dimensional) element. Its parameters are - vertical stiffness, friction coefficient on tread area and friction coefficient of flange with rails (μк, μг).

Coordinates of the points of connections' application to the wheelset relatively to the mass center of wheelset in lateral direction differ in signs (Y), and in vertical direction are equal and make the wheel radius [1, 2].

**2. Nosing vibration frequency in a wheelset.**

There is known correlation which describes dependence of the wheelset's horizontal-lateral vibrations period at its move along motionless rail without slipping motion [1].

$$L = 2\pi\sqrt{\frac{sr_c}{n}}\,. \qquad (1)$$

It is evident, that during motion of wheelset with constant speed the frequency of lateral vibration will be defined by the following formula:

$$f = \frac{V}{2\pi\sqrt{\frac{sr_c}{n}}}\,, \qquad (2)$$

where $V$ – is a speed of forward movement of wheelset; $s$ –is a half distance between wheels tread; $r_c$ –is a wheel radius; $n$ –is a conicity of wheels tread.

With the aim to check it up, some calculations with the use of above-described model have been made, which simulated movement of wheelset by rail-track under the influence of step disturbance. In these calculations we used the same parameters as we used above in the paper.

By the results of calculations, frequencies of transverse vibrations of the wheelset were found. These results are summarized in Table 3.

Table 2.

**Calculation of frequency of lateral vibrations of wheelset**

| Wheel radius, [m] | Frequency, Hz (calculation) | Frequency, Hz (model) | Mistake, % |
|---|---|---|---|
| 0,475 | 0,581 | 0,585 | 0,688 |
| 0,525 | 0,553 | 0,558 | 0,904 |
| 0,625 | 0,506 | 0,512 | 1,186 |

As we can see from this table, simulation results are well-cohered with calculated values. Here it should be taken into account, that rails were not motionless in the model, and in coupling wheel – rail forces of lateral and longitudinal creep on the surface of rolling and between flanges of wheel and rails were taken into account, and, besides, lateral interaction of wheels flanges with rails during selection clearance in track was taken into account. All this, surely, effected on value of vibration frequencies.

For descriptive reasons on figure 1 an oscillogram of wheelsets' lateral movement with different diameters of wheels is shown.

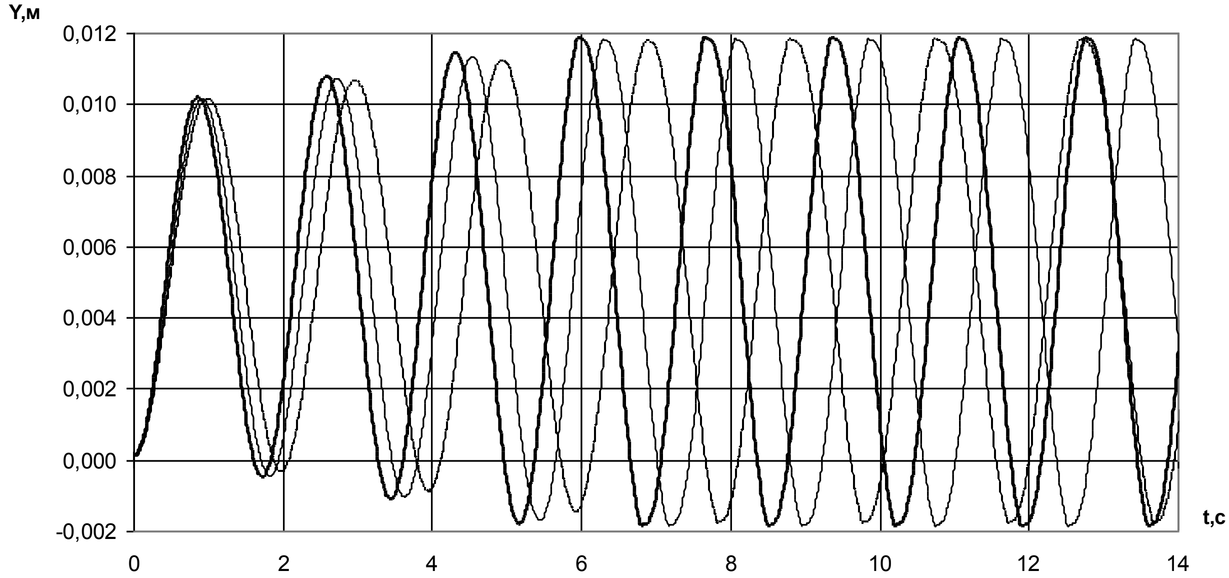


Fig. 1 Oscillogram of wheelsets lateral movement

**3. Elementary dynamic model of wheelset lateral vibrations.**

It is known that the wheelset moving along a straight track apart from the forward motion makes motion along the longitudinal axes and rotary movement along its vertical axes of motion.

Below we analyze an equation of nosing motion of wheelset along motionless rails without slipping motion given in [1]:

$$\frac{d^2y}{dx^2} + \frac{n}{sr_c}\,y = 0\,, \qquad (3)$$

Where $x$ - is a longitudinal movement of wheelset; $y$ – is a lateral movement of wheelset; $s$ – is a half distance between wheels tread of one wheelset; $r_c$ –is a wheel tread radius; $n$ –is a conicity of wheels tread.

Passing to the time domain we get:

$$\frac{d^2y}{dx^2} + \frac{V^2 n}{sr_c}\,y = 0\,. \qquad (4)$$

Perhaps, this equation gives most elementary description of nosing motion process of wheelset. Thus, the comparison of results we get using this model with results we get using the constructed model, which takes into account all details of the wheelset and the rails interaction is of interest.

**3.1 Wheelset move at nonzero-initial conditions on movement.**

Solution of equation (4) at initial conditions $y(0) = y_0,\ \dot{y}(0) = 0$ is:

$$y = y_0 \cos(\nu t)\,, \qquad (5)$$

where, $\nu^2 = \frac{V^2 n}{sr_c}$ - is intrinsic frequency of lateral oscillations of wheelset.

Calculations of wheelset movement along a straight track with a use of constructed model provided that $y_0 = 0{,}005\,m$ have been done. Consequently we got diagrams of wheelset lateral vibrations for 3 types of wheelsets used on Ukrainian main rolling stock with diameters of 950 mm, 1050 mm, 1250 mm, and conicity n = 0,05. In all calculations value s = 0,79 m, which corresponds to the gauge of railway 1520 mm, and a movement speed is 10 m/sec.

Results of our calculations are presented on figures 2 – 4. Here, solid line corresponds to the process calculated with the formula (5), points – are results of the calculations. Dashed line diagrams are described below.

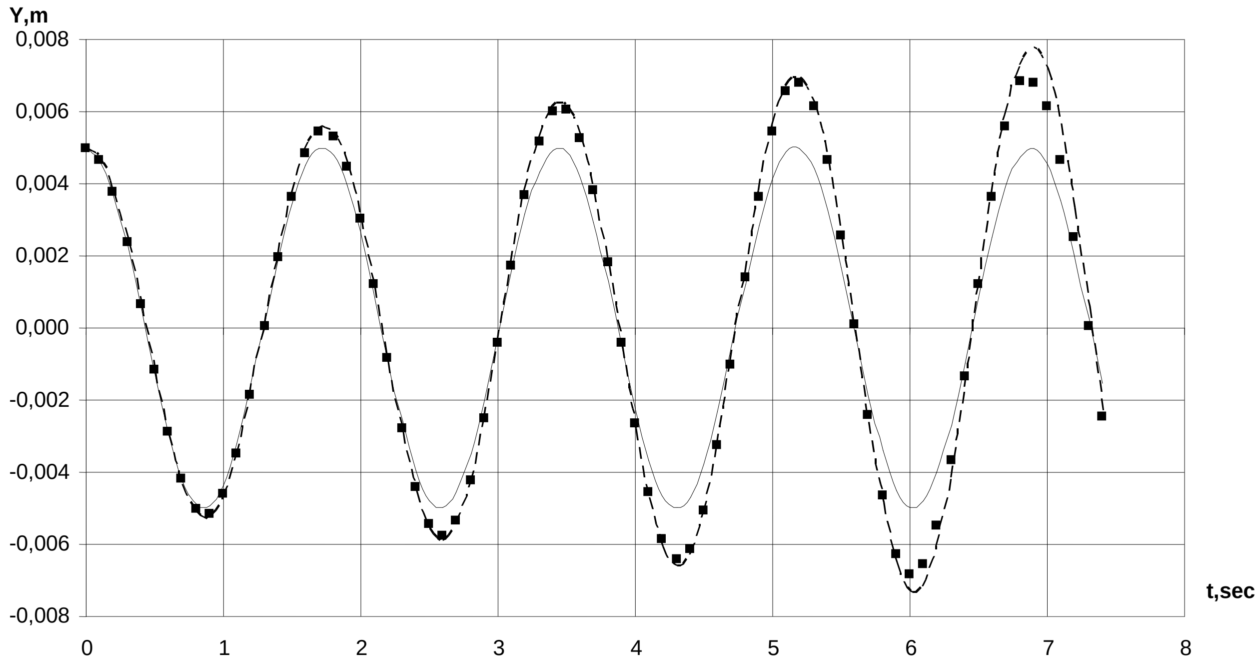


Fig. 1 Diagram of wheelset lateral vibrations ( $r_c = 0{,}475\,m$ ).

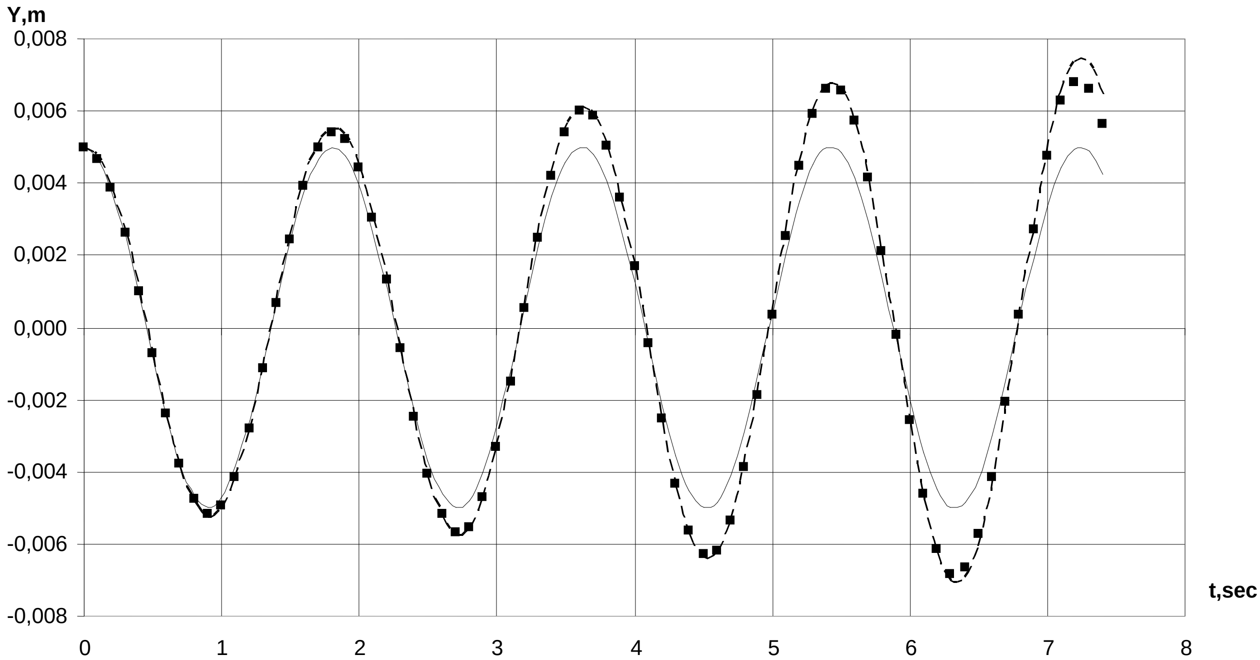


Fig. 3 Diagram of wheelset lateral vibrations ( $r_c = 0{,}525\,m$ ).

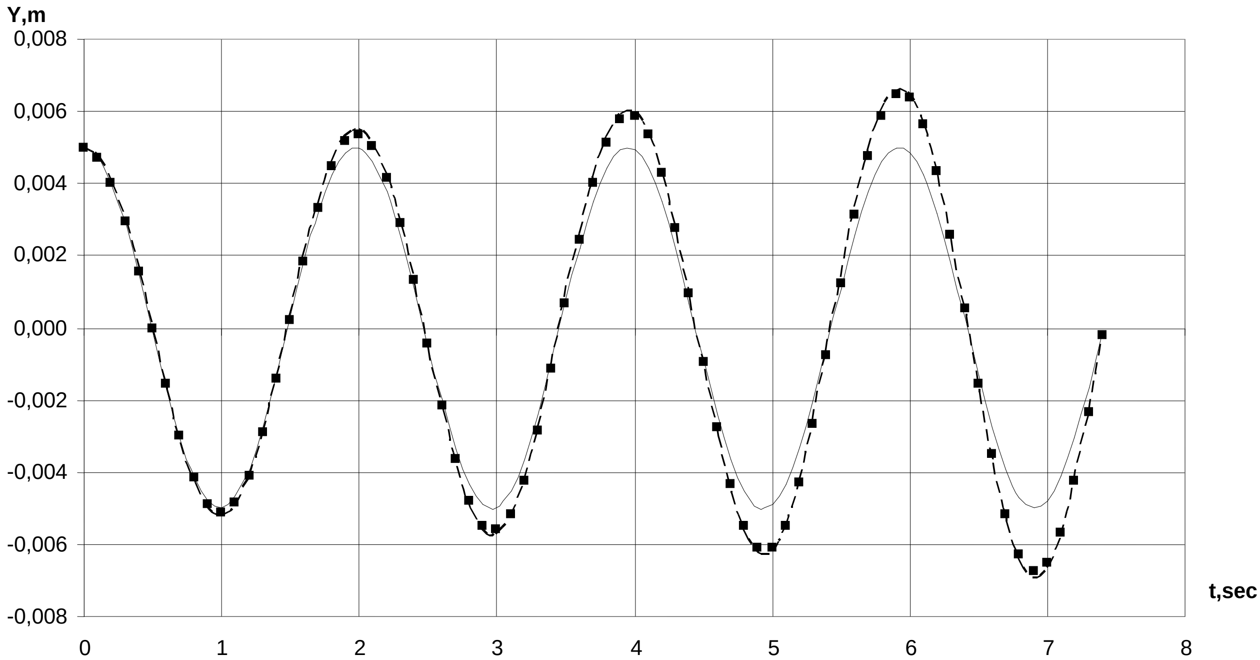


Fig. 4 Diagram of wheelset lateral vibrations ( $r_c = 0{,}625m$ ).

As we can see from these diagrams, oscillation period totally coincides with theoretical value, and amplitude rise can be explained by the fact that in the constructed model the nosing motion of wheelset is explained, apart from other factors, by its wobbling, which brings negative damping in the system, i.e. makes the system unstable.

Below we estimate damping factor (in our case growth factor) $h$ according to the model oscillation amplitude change by the following relation:

$$h = \frac{1}{T}\ln\left(\frac{A_{i+1}}{A_i}\right), \qquad (6)$$

where $T$ - is an oscillation period, $A_i, A_{i+1}$ - are neighboring amplitudes.

Calculated values $h$ for 3 variants of calculation are summarized in table 4.

Table 3

**Damping factors**

| Wheel radius, [m] | 0,475 | 0,525 | 0,625 |
|---|---|---|---|
| Value $h$, [1/sec] | 0,064 | 0,055 | 0,047 |

As we can see from this table, values of damping factors go down as values of wheels radius grow, i.e., oscillations of wheelset of bigger diameter oscillate longer. After determination of damping factor value it is possible to substitute complex model describing nosing motion for simplified model at a stage of wheelset amplitude of oscillation rise before track clearance selection:

$$y = y_0 e^{ht}\left(\cos(pt) + \frac{h}{p}\sin(pt)\right), \qquad (7)$$

where $p^2 = \nu^2 - h^2$.

Expression (7) is the solution of differential equation:

$$\ddot{y} - 2h\dot{y} + \nu^2 y = 0. \qquad (8)$$

Diagrams, resulted by formula (7) are shown on figures 2 – 4 by dashed lines and are in good agreement with calculations results. We want to underline once more that relation (7) works only at the stage of amplitude of oscillation rise to the value of track clearance, after that amplitude of lateral vibrations can be regarded as constant. In cases when we use model (7) we need to consider dependence of the value $h$ not only on wheel diameter but also on the speed of forward motion $V$.

**3.2 Wheelset move at nonzero-initial conditions on movement speed.**

If we give in differential equation (8) the following initial conditions $y(0) = 0,\ \dot{y}(0) = \dot{y}_0$, we get:

$$y = \frac{\dot{y}_0}{p} e^{ht} \sin(pt). \qquad (9)$$

At pulse lateral disturbance which has an effect on a wheelset from the track side, lateral vibrations are like in relation (9). In the paper below we check an opportunity to describe lateral vibrations of wheelset by different equation (8) of the constructed model.

As a disturbance we take an initial speed of lateral movement $\dot{y}_0 = 0{,}0005\ m/\sec$.

By the results of calculations frequencies of transverse vibrations of the wheelset were found. These results are summarized in Table 5.

Table 4

**Damping factors**

| Wheel radius, [m] | 0,475 | 0,525 | 0,625 |
|---|---|---|---|
| Value $h$, [1/sec] | 0,064 | 0,057 | 0,049 |

As we can see from this table, that accurate within mistake which occurs as a result of discrecity of results fixing we can consider values of damping factors equal to values in equation [1]. Thus, lateral vibrations at pulse disturbance can also be described by differential equation (8).

Results are shown on figures 5 – 7. Here points are for modeling results and solid lines are results received by formula (9).

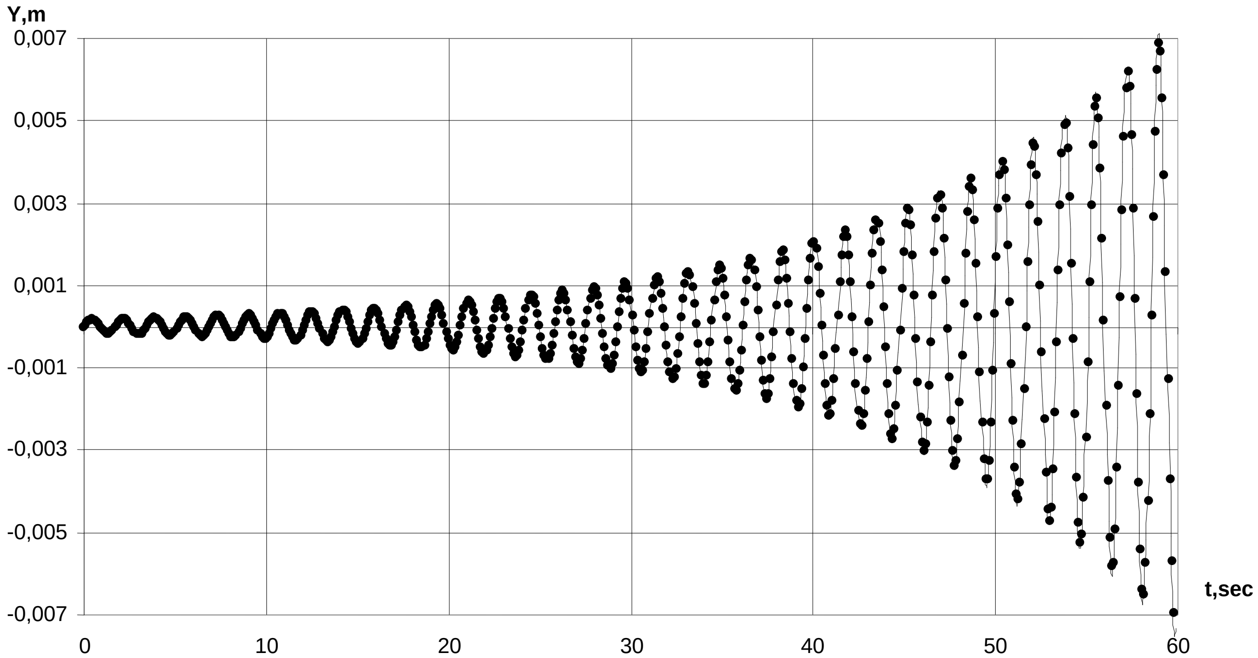


Fig. 5 Diagram of wheelset lateral vibrations ( $r_c = 0{,}475m$ ).

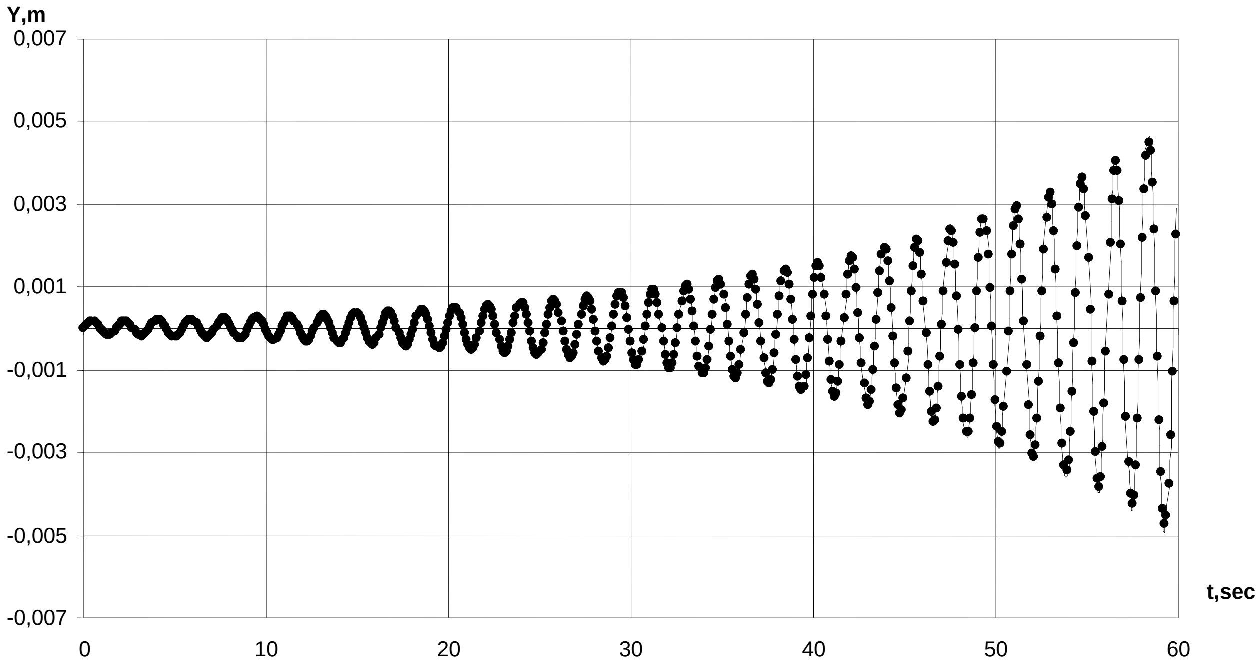


Fig. 6 Diagram of wheelset lateral vibrations ( $r_c = 0{,}525 m$ ).

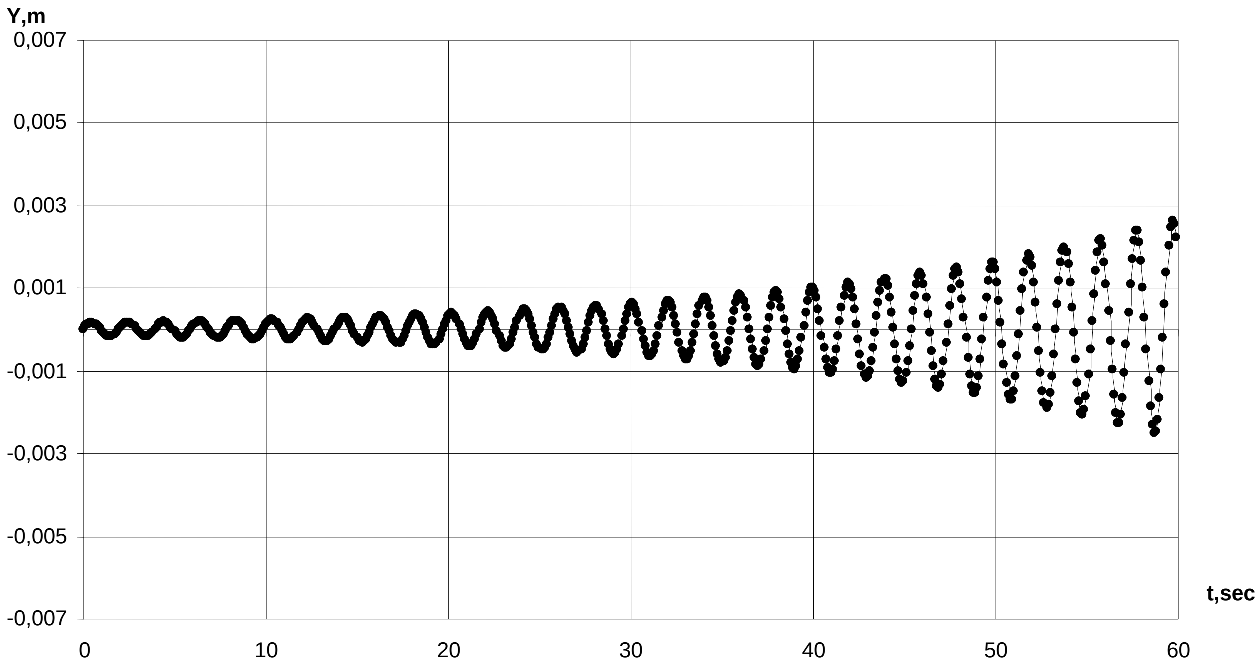


Fig. 7 Diagram of wheelset lateral vibrations ( $r_c = 0{,}625 m$ ).

We give our results in one scale intentionally to underline the fact that at pulse disturbance wheelsets with bigger diameter have smaller damping factor, thus their lateral vibrations rise for a longer period.

Thus, the mathematical model of the wheelset movement is developed. Integration is applied to the system of equations represented in canonical form relatively to the motion speed of the bodies comprising the wheelset model. After the motion speed values are obtained, a new system of canonical equations is formed for the bodies' movement against each other and for the angle of rotation. The offered mathematical model of the wheelset movement is used in more complex models of railway vehicles spatial oscillations which allow to determine both dynamic parameters of railway vehicle and wear factor of wheels.

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