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On the Question of Choosing the Optimal Strength Criterion of Soils and Rocks

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Abstract

Based on the analysis of solutions to the classical problems of soil mechanics and geomechanics on determining the critical height and angle of the slope, as well as active and passive pressure on the enclosing structures, to identify the optimal strength criteria and the limits of their application, which allow predicting the destruction of ideally free-flowing, ideally plastic rocks, as well as rocks characterized by internal friction and specific cohesion, which is necessary for calculating the strength and stability of underground and open mine workings in the supercritical region - after the destruction of the near-contour region.

The areas of application of some criteria for the strength of soils and rocks are outlined and substantiated. It is shown that, in contrast to the known strength criteria of Z.T. Benyavsky, Hoek - Brown and A.N. Shashenko Coulomb - Mohr strength criterion and our proposed modification of the strength criterion of A.N. Shashenko allow predicting the destruction of ideally - free flowing, ideally - plastic rocks, as well as rocks characterized by internal friction and specific cohesion.

KEY WORDS: *Mohr - Coulomb strength criteria, modified strength criterion A.N. Shashenko, ideally - loose soil; ideally plastic soil; soil, specific adhesion; internal friction angle.*

1. Introduction

Currently, the following strength criteria are most widely used in soil and rock mechanics [1-5, 11, 12]: Mora - Coulomb; Z.T. Benyavsky; Hoeka - Brown; A.N. Shashenko (including modified); Cod - Hill; Mises - Botkin; J. Watt; D.N. Rasskazova; other criteria incl. combined (taking into account several mechanisms of destruction, for example, tension and shear).

Each of these criteria describes well the behavior of the material (in the considered case of soil and rock) in a certain narrow range of pressures and physical and mechanical properties. At the same time, it is not clear how general and universal each of them is.

We also note that this article deliberately does not separate the terms "soils" and "rocks" in view of the fact that "From the point of view of construction, soil is called any rock used in construction as the foundation of a structure, the environment in which the structure is being erected, or material for construction ".

Based on the analysis of solutions to the classical problems of soil mechanics and geomechanics about determining the critical height and angle of the slope, identify the optimal strength criteria and the limits of their application, which allow predicting the destruction of ideally free-flowing, ideally plastic rocks, as well as rocks characterized by internal friction and specific cohesion ... This is necessary for calculating the strength and stability of aboveground, underground and open mine workings in the supercritical region - after the destruction of the near-contour region [1-15].

The research task was formulated as follows:

a. The strength characteristics of the soil or rock are known - specific cohesion c and the angle of internal friction φ or the compressive strength of the rock R_c and tension R_p.

b. The specific gravity of the soil (rock) γ is known.

c. It is required to identify the most acceptable strength criterion for solving problems of soil and rock mechanics. To do this, you need to do the following:

1) Reveal the features of using various strength criteria in the conditions of a one-dimensional and spatial problem.

2) Establish the possibility of describing, using one or another criterion of strength, properties of materials

with: specific adhesion; dry friction; specific adhesion and dry friction at the same time.

3) Compare the results of solving such classical problems of geomechanics, obtained using various strength criteria:

– determination of the critical angle of the slope from ideally loose soil;
– determination of the critical height of the vertical slope for soil with specific adhesion and internal friction.

2. Research

Due to the limited scope of the article, it contains the results of considering the most frequently used criteria.

In the course of theoretical studies, obtained by solving the above problems, the results were compared with the solutions obtained using the Mohr - Coulomb strength criterion. This is due to the fact that this strength criterion is the simplest and most widely used in solving problems of soil mechanics and geomechanics.

In addition, to ensure the concentration of attention on the problem under consideration and the convenience of presenting the material of the research, soils and rocks with an undisturbed structure were deliberately considered.

Stage 1. Features of the use of strength criteria in the conditions of one-dimensional and spatial problems.

1.1. Mohr - Coulomb strength criteria for the one-dimensional and spatial cases are known and have the form [1-5]:

– for the spatial case:

$$\frac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3+2\cdot c\cdot ctg(\varphi)}\le sin(\varphi);\ \sigma_1\ge\sigma_2\ge\sigma_3; \tag{1}$$

– for the one-dimensional case:

$$\tau\le\sigma\cdot tg(\varphi)+c, \tag{2}$$

where τ is the shear stress; σ is normal; $\sigma_1,\sigma_2,\sigma_3$ are the principal normal stresses for the case of the spatial problem; φ is the angle of internal friction; c − specific cohesion.

First, we investigate condition (1). When the specific cohesion is equal to zero (c = 0), we have:

$$\left.\frac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3}\le \sin(\varphi);\ \sigma_1\ge\sigma_2\ge\sigma_3\right\}. \tag{3}$$

Analysis of Eq. (3) allowed us to conclude that the Mohr - Coulomb strength condition (1) is applicable for ideally loose soils.

Next, we multiply both sides of inequality (1) by the denominator and set φ = 0. We have:

$$\sigma_1-\sigma_3\le 2\cdot c. \tag{4}$$

Thus, the Mohr - Coulomb spatial strength criterion is also applicable for ideally plastic soils.

Similar conclusions can be made by analyzing the one-dimensional version of the Mohr - Coulomb law - for c = 0 we have:

$$\tau\le\sigma\cdot tg(\varphi), \tag{5}$$

and at φ = 0:

$$\tau\le c. \tag{6}$$

Thus, the Mohr - Coulomb strength condition makes it possible to predict the destruction of such media:

– perfectly free-flowing;
– perfect plastic;
– environments with internal friction and specific adhesion at the same time.

In this case, according to [4, 5], using the Mohr - Coulomb law of strength, underestimated values of active pressure on the holding structures and overestimated values of passive pressure are obtained.

1.2. Strength criterion Z.T. Benyavsky for the spatial case has the form [1, 3, 6]:

$$\left.\sigma_1\le A\cdot(R_c)^{0,25}\cdot(\sigma_3)^{0,75}+R_c;\ \sigma_1\ge\sigma_2\ge\sigma_3\right\}, \tag{7}$$

where $A\in$ (0, .., 20) is an empirical constant.

In order to pass to the strength constants c and φ, let us take into account that in (7) tensile stresses should be taken with a minus sign, and compressive stresses with a plus sign. In addition, we use the well-known equalities (Florin V.A. Fundamentals of soil mechanics, v.1.- L.-M .: Gosstroyizdat, 1959, [4, 5]):

$$\left.R_c = -2\cdot c\cdot tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right);\;\; \psi = \frac{R_r}{R_c} = -\frac{1}{tg^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)}\right\}. \tag{8}$$

We have:

$$\left.\sigma_1 \le \left[2\cdot c\cdot tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right]^{0,25}\cdot\sigma_3^{0,75} + 2\cdot c\cdot tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right);\sigma_1\ge\sigma_2\ge\sigma_3\right\}. \tag{9}$$

In order to obtain a one-dimensional version of the strength condition Z.T. Benyavsky, we take into account the well-known relations [1, 3, 6]:

$$\left.\sigma = \frac{\sigma_1+\sigma_3\cdot}{2};\;\; \tau = \frac{\sigma_1-\sigma_3}{2}\right\}. \tag{10}$$

We have:

$$\tau \le -\sigma + A\cdot(\tau-\sigma)^{0,75}\cdot\left[2\cdot c\cdot tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\right]^{0,25} + 2\cdot c\cdot tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right). \tag{11}$$

Next, we investigate inequality (9). With $c = 0$ we have: $\sigma_1 \le 0$; $\sigma_1\ge\sigma_2\ge\sigma_3$. Thus, the application of the strength criterion Z.T. Benyavsky when solving the spatial problem is unacceptable for predicting the strength of ideally free-flowing soils, since (12) does not depend on the angle of internal friction φ.

For $\varphi = 0$ we have:

$$\left.\sigma_1 - A\cdot(2\cdot c)^{0,25}\cdot\sigma_3^{0,75} - 2\cdot c \le 0;\;\; \sigma_1\ge\sigma_2\ge\sigma_3\right\}. \tag{12}$$

Thus, the criterion of Z.T. Benyavsky (9) is quite acceptable for predicting the strength of ideally cohesive soils.

Next, we investigate equality (11). For $c = 0$ we have:

$$\tau \le -\sigma . \tag{13}$$

Thus, in one-dimensional problems, the criterion of Z.T. Benyavsky at $c = 0$ does not depend on the angle of internal friction φ and therefore is unacceptable for predicting the strength of ideally loose soils.

In addition, for $\varphi = 0$ we have:

$$\tau \le -\sigma + A\cdot(\tau-\sigma)^{0,75}\cdot(2\cdot c)^{0,25} + 2\cdot c . \tag{14}$$

In this case, the criterion Z.T. Benyavsky (11) is quite acceptable for predicting the strength of perfectly cohesive soils.

The foregoing allowed us to conclude that the strength criteria of Z.T. Benyavsky for spatial (9) and one-dimensional (11) problems allow us to predict the destruction of such media:

– perfect plastic;
– a medium that has both internal friction and specific adhesion.

Moreover, this criterion is absolutely unacceptable for predicting the strength of ideally free flowing media. In addition, relations (9) and (11) have a rather complicated form, which is also their disadvantage.

1.3. The Hoek - Brown strength criterion for soil and rock with undisturbed structure in the spatial case has the form [1, 3, 6]:

$$\left.\sigma_1 \le \sigma_3 + R_c\cdot\sqrt{m\cdot\sigma_3/R_c+1};\sigma_1\ge\sigma_2\ge\sigma_3\right\}, \tag{15}$$

where $m \in (0, .., 33)$ is an empirical constant.

In order to pass to the strength constants c and φ, let us take into account that in (15) tensile stresses should be taken with a minus sign, and compressive stresses with a plus sign. In addition, we use the well-known equalities (8). We have

$$\left.\sigma_1 \le \sigma_3 + 2\cdot c\cdot tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)\sqrt{\frac{m\cdot\sigma_3}{2\cdot c}\cdot ctg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)+1};\ \sigma_1 \ge \sigma_2 \ge \sigma_3\right\}. \tag{16}$$

In order to pass to the use of the Hoek - Brown criterion in a one-dimensional problem, we use relations (16) and (10). Then:

$$\left.\tau \le -\frac{1}{4}\cdot m\cdot c\cdot tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)+\frac{1}{4}\cdot\sqrt{c^2\cdot\left(m^2+16\right)\cdot tg^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)+8\cdot\sigma\cdot c\cdot m\cdot tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)}\right\}. \tag{17}$$

Next, we investigate the application of the Hoek-Brown strength criterion (16) when solving spatial problems. For $c = 0$ we have:

$$\tau \le 0\,. \tag{18}$$

Thus, the option of using the Hoek - Brown strength criterion when solving the spatial problem is unacceptable for predicting the strength of ideally free-flowing soils.

For $\varphi = 0$ we have:

$$\left.\tau \le -\frac{1}{4}\cdot m\cdot c+\frac{1}{4}\cdot\sqrt{c^2\cdot\left(m^2+16\right)+8\cdot\sigma\cdot c\cdot m.}\right\}. \tag{19}$$

Thus, the Hoek - Brown criterion (17) is quite acceptable for predicting the strength of ideally cohesive soils.

The foregoing allowed us to conclude that the Hoek - Brown strength criteria for spatial (16) and one-dimensional (17) problems allow predicting the destruction of such media:

- perfect plastic;
- characterized by internal friction and specific adhesion.

Moreover, this criterion is absolutely unacceptable for predicting the strength of ideally free-flowing media.

1.4. Strength criterion A.N. Shashenko for soil and rock with undisturbed structure in the spatial case has the form [1-3, 6]:

$$\left.-(1-\psi)\cdot(\sigma_1+\sigma_3)+\sqrt{\left[(1-\psi)^2\cdot(\sigma_1+\sigma_3)^2+4\cdot\psi\cdot(\sigma_1-\sigma_3)\right]}-2\cdot\psi\cdot R_c \le 0;\ \sigma_1 \ge \sigma_2 \ge \sigma_3\right\}. \tag{20}$$

In order to pass to the strength constants c and φ, let us take into account that in (20) tensile stresses should be taken with a minus sign, and compressive ones with a plus sign, and the parameter $\psi = \dfrac{R_r}{R_c}$ − in absolute value. Taking into account equalities (8), we have:

$$\left.\left(\sigma_1-\sigma_3\right)-2\cdot\sqrt{c^2+c\cdot\left(\sigma_1+\sigma_3\right)\cdot tg(\varphi)} \le 0;\ \sigma_1 \ge \sigma_2 \ge \sigma_3\right\}. \tag{21}$$

Using (10) and (21), we arrive at the one-dimensional case of the strength criterion of A.N. Shashenko. We have:

$$\tau \le \sqrt{2\cdot c\cdot\sigma\cdot tg(\varphi)+c^2}\ . \tag{22}$$

Next, we investigate the spatial version of using the strength criterion of A.N. Shashenko (21). For $c = 0$ we have:

$$\left.\left(\sigma_1-\sigma_3\right) \le 0;\ \sigma_1 \ge \sigma_2 \ge \sigma_3\right\}. \tag{23}$$

It follows from (23) that the spatial version of the application of the strength criterion A.N. Shashenko (21) is unacceptable for predicting the strength of ideally free-flowing soils, since (23) does not depend on the angle of internal friction. For $\varphi = 0$ we have:

$$-2 \cdot c \pm \left(\sigma_1 - \sigma_3\right) \le 0;\ \sigma_1 \ge \sigma_2 \ge \sigma_3 \Big\}. \tag{24}$$

It follows from (24) that the spatial variant of using the criterion of A.N. Shashenko (21) is quite acceptable for predicting the strength of ideally cohesive soils.

Next, let us analyze a one-dimensional version of the application of the strength criterion A. Shashenko (22). For $c = 0$ we have:

$$\tau \le 0. \tag{25}$$

It follows from (25) that the one-dimensional case of the strength criterion of A.N. Shashenko is unacceptable for predicting the strength of ideally free-flowing media. For $\varphi = 0$ we have:

$$\tau \pm c \le 0. \tag{26}$$

It follows from (26) that the one-dimensional case of the strength criterion of A.N. Shashenko is quite acceptable for predicting the strength of ideally plastic media.

In general, the analysis of one-dimensional and spatial strength criteria by A.N. Shashenko made it possible to draw the following conclusions:

1. When solving spatial problems, the strength criterion of A.N. Shashenko (21) allows predicting the destruction of such media: ideal-plastic; possessing internal friction and specific adhesion at the same time. At the same time, the use of the spatial strength criterion of A.N. Shashenko is impossible.
2. When solving one-dimensional problems, the strength criterion of A.N. Shashenko (22) makes it possible to predict the destruction of such media: - ideal-plastic; having internal friction and specific adhesion at the same time. At the same time, its use for predicting the strength of ideally free-flowing media is impossible.

It should also be noted that the strength criterion of A.N. Shashenko makes it relatively easy to take into account the nonlinearity of the Coulomb - Mohr envelope. In this regard, it makes sense to modify it in such a way that it would be possible to predict the strength of ideally - free flowing, ideally - plastic media, as well as media with internal friction and adhesion.

1.5. In order to modify the strength criterion A.N. Shashenko, we expand (22) in a Taylor series in the vicinity of some natural state $\tau(\sigma_0, c_0)$ and keep only linear terms in the expansion. We have:

$$\tau \approx \sqrt{2 \cdot c \cdot \sigma_0 \cdot tg(\varphi) + c^2} + \frac{tg(\varphi) \cdot (\sigma - \sigma_0) \cdot c}{\sqrt{2 \cdot c \cdot \sigma_0 \cdot tg(\varphi) + c^2}}. \tag{27}$$

Putting in (27) $\sigma_0 = 0$, we arrive at the form of writing the Coulomb - Mohr strength criterion for the one-dimensional case:

$$\tau \approx \sigma \cdot tg(\varphi) + c. \tag{28}$$

In conclusion, we note that the following asymptotic estimates also take place:

$$\lim_{\sigma \to 0}(\tau) = c;\ \lim_{c \to 0}(\tau) = \sigma \cdot tg(\varphi) \Big\}. \tag{29}$$

It follows from (27), (28) and (29) that at low stresses the strength laws of A.N. Shashenko and Kulona - Mora are the same.This fact was used by us to modify the strength criterion of A.N. Shashenko. Let us find such a minimum value of the specific cohesion of the soil (rock) at which the breaking shear stresses, calculated using the Coulomb strength criteria for loose soil and A.N. Shashenko for cohesive soil, coincide. In this case, we use the known actual values of the angle of internal friction of the soil φ and the normal stress σ_0. acting at the point under consideration. In view of the above, the solution to the problem is reduced to solving an algebraic equation of the form: $\sqrt{2 \cdot c_{res} \cdot \sigma_0 \cdot tg(\varphi) + c_{res}^2} - \sigma_0 \cdot tg(\varphi) = 0$, from where:

$$c_{res} = \left(\sqrt{2} - 1\right) \cdot \sigma_0 \cdot tg(\varphi). \tag{30}$$

Here c_{res} is the reduced specific cohesion, and σ_0 is the maximum normal stress.

Further, we require that at low values of the specific cohesion of the rock $c_{fact} < c_{res}$, its destruction occurs in accordance with the Coulomb - Mohr strength criterion. If the inequality $c_{res} < c_{fact}$ takes place, then the fracture occurs

in accordance with the classical strength criterion of A.N. Shashenko.

In view of the above, the modified strength criterion by A.N. Shashenko looks like:

$$\left.\begin{array}{c}\text{one dimensional case: } \tau=\sqrt{2\cdot c\cdot\sigma\cdot tg(\varphi)+c^2};\\ \text{spatial case: } (\sigma_1-\sigma_3)-2\cdot\sqrt{c^2+c\cdot(\sigma_1+\sigma_3)\cdot tg(\varphi)}\le 0;\ \sigma_1\ge\sigma_2\ge\sigma_3;\\ \text{where: } c=c_{fact}\cdot U\left(c_{fact}-c_{rez}\right)+c_{rez}\cdot\left[1-U\left(c_{fact}-c_{rez}\right)\right],\end{array}\right\} \tag{31}$$

where $U(x)$ – is the Heaviside unit step function [10]; c_{res} – the minimum specific cohesion that should be taken into account (reduced specific cohesion); c_{fact} – actual value of specific cohesion. In particular, the last equality (31) means that if the reduced specific cohesion c_{res} is greater than the actual one, then the specific cohesion c_{res} should be taken into account; otherwise, the actual specific cohesion should be taken c_{res}.

Next, we will consider solutions to some problems in soil mechanics and geomechanics obtained using the strength criteria discussed above.

Stage 2. Features of the use of strength criteria when determining the critical angle of the slope from loose soil (Fig. 1, a). This problem occurs when constructing embankments, dumps, soil cushions, dams, reclamation of soil, etc.

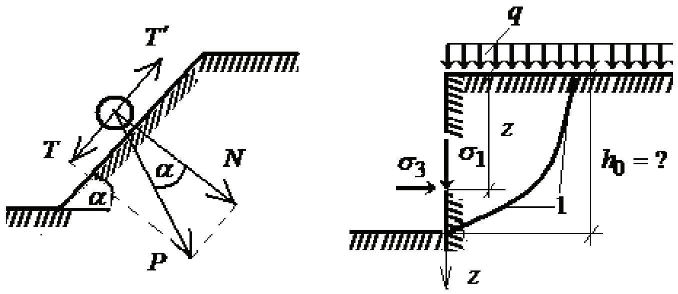

Fig. 1 Scheme for determining the stability of the slope: a – from loose soil; b – from soil with specific adhesion and internal friction

The research task was formulated as follows:

A. The criterion of rock (or soil) strength is known.
B. The rock (or soil) is absolutely free-flowing (ie, the specific cohesion is zero).
C. It is required to determine the critical angle of the slope, the design scheme of which is shown in Fig. 1, a.

2.1. Coulomb-Mohr strength criterion.

Consider a rock particle with weight P located on the slope. Let us decompose the weight P into the perpendicular surface of the slope N and the parallel shear force T. Next, we find the holding force T'. In addition, we equate the shearing and holding forces. We have:

$$\left.\begin{array}{c}N=P\cdot\cos(\alpha);\ T=P\cdot\sin(\alpha);\ T'=N\cdot tg(\alpha)=P\cdot\cos(\alpha)\cdot tg(\alpha);\\ T=T';\ P\cdot\sin(\alpha)-P\cdot\cos(\alpha)\cdot tg(\varphi)=0;\\ \alpha=\varphi.\end{array}\right\} \tag{32}$$

2.2. Strength criterion Z.T. Benyavsky.

In this case, for loose soil from condition (11) we have:

$$T'\le -N. \tag{33}$$

Repeating the reasoning presented in section 2.1, we find:

$$\left.\begin{array}{c}N=P\cdot\cos(\alpha);\ T=P\cdot\sin(\alpha);\ T'=-N=-P\cdot\cos(\alpha);\\ T=T'\Rightarrow P\cdot\sin(\alpha)+P\cdot\cos(\alpha)=0;\ \alpha=-arctg(1)=\dfrac{-\pi}{4}.\end{array}\right\}. \tag{34}$$

This result contradicts the experimental data. Therefore, the strength criterion Z.T. Benevsky is unacceptable for calculating slopes from loose soil.

2.3. Hoek - Brown Strength Criterion.

In this case, for loose soil from condition (18) we have:

$$T' \le 0. \tag{35}$$

Repeating the reasoning presented in section 2.1., we find:

$$\left.\begin{array}{c} N = P \cdot \cos(\alpha);\ T = P \cdot \sin(\alpha);\ T' = 0; \\ T = T' \Rightarrow P \cdot \sin(\alpha) - 0 = 0;\ \alpha = \arcsin(0) = 0. \end{array}\right\} \tag{36}$$

Therefore, the Hoek-Brown strength criterion is unacceptable for calculating slopes from loose soil.

2.4. Strength criterion A.N. Shashenko.

In this case, for loose soil from condition (25) we have:

$$T' \le 0. \tag{37}$$

Repeating the reasoning presented in Section 2.3, we find

$$\alpha = arcsin(0) = 0. \tag{38}$$

Therefore, the strength criterion of A.N. Shashenko is unacceptable for calculating slopes from loose soil.

2.5. Modified strength criterion by A.N. Shashenko.

In this case, for loose soil from condition (31), we should put $c = c_{rez}$. As a result, we come to the Coulomb - Mohr strength criterion (Section 2.1), due to which:

$$\alpha = \varphi. \tag{39}$$

The data presented in paragraph 2 made it possible to conclude that for absolutely free-flowing rock (soil) the results obtained using the Coulomb - Mohr strength criteria and the modified A.N. Shashenko.

In this case, the strength criteria Z.T. Benyavsky, Hoek - Brown and A.N. Shashenko for calculating slopes from loose soil is absolutely unacceptable.

Stage 3. Features of the use of strength criteria when solving problems of ensuring the stability of vertical slopes. This problem arises when constructing trenches, foundation pits, open workings with vertical walls, etc.

The research task was formulated as follows:

A. The criterion of rock (or soil) strength is known.

B. Rock (or soil) has internal friction and specific adhesion.

C. It is required to determine the critical height of the slope, the design scheme of which is shown in Fig. 1, b, based on the fact that in this case the main normal stress σ_3 is equal to zero, and the main normal stress is: $\sigma_1 = \gamma \cdot z + q$

3.1. Coulomb-Mohr strength criterion. Putting in (1) $\sigma_3 = 0$ and solving the thus obtained inequality with respect to the principal stress σ_1 we find:

$$\left.\sigma_1 \le \frac{2 \cdot c \cdot \cos(\varphi)}{1 - \sin(\varphi)}\right\}. \tag{40}$$

Taking into account that in this case $\sigma_1 = \gamma \cdot z + q$ we finally find:

$$h_{\max} = z_{\max} \le \frac{2 \cdot c}{\gamma} \cdot \frac{\cos(\varphi)}{1 - \sin(\varphi)} - \frac{q}{\gamma}. \tag{41}$$

Analysis (41) made it possible to draw the following conclusions:

1. When the specific adhesion is equal to zero (c=0), the critical slope height is equal to zero $h_{max} \le 0$.
2. When the angle of internal friction is equal to zero (φ=0), the critical height of the vertical slope is equal to $h_{max} = \frac{2 \cdot c}{\gamma} - \frac{q}{\gamma}$. Thus, solution (41) is acceptable for perfectly connected (plastic) rocks and rocks with specific cohesion and internal friction. The solution to problem (41) is known from the literature and is presented, for example, in [4, 5].

3.2. Strength criterion Z.T. Benyavsky. Putting $\sigma_3 = 0$ in (7) and solving the inequality thus obtained with respect to the principal stress σ_1, we find $\sigma_1 \le R_c$. Taking into account (8) and explanations for formulas (7), we arrive at the

equality $\sigma_1 \le 2 \cdot c \cdot tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)$. Since on the interval $\varphi \in \left(0,\frac{\pi}{4}\right)$ (this is the range of variation of the angle of internal friction of the rock), we will come to (40) and (41) in succession. Thus, the use of the strength criterion Z.T. Benyavsky (7) to solve this problem made it possible to obtain exactly the same results as using the Coulomb - Mohr strength criterion (1). Therefore, the strength criterion Z.T. Benyavsky is quite acceptable for determining the critical height of vertical slopes.

3.3. Hoek - Brown Strength Criterion. Putting $\sigma_3 = 0$ in (15) and solving the thus obtained inequality with respect to the principal stress σ_1, we find $\sigma_1 \le R_c$. Taking into account (8) and explanations for formulas (7), we arrive at the equality $\sigma_1 \le 2 \cdot c \cdot tg\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)$. Since on the interval $\varphi \in \left(0,\frac{\pi}{4}\right)$ we come to (40) and (41) in succession. Thus, the use of the Hoek - Brown strength criterion (15) for solving this problem made it possible to obtain exactly the same results as the use of the Coulomb - Mohr strength criterion (1).

3.4. Strength criterion A.N. Shashenko. Putting $\sigma_3 = 0$ in (21) and solving the thus obtained inequality with respect to the principal stress σ_1, find $\sigma_1 \le 2\cdot c\cdot(\sin(\varphi)+1) / \cos(\varphi) = 2\cdot c\cdot\cos(\varphi) / (1-\sin(\varphi))$.

Thus, the use of the strength criterion of A.N. Shashenko (21) to solve this problem made it possible to obtain exactly the same results as using the Coulomb - Mohr strength criterion (1). Therefore, the strength criterion of A.N. Shashenko is quite acceptable for determining the critical height of vertical slopes.

3.5. Modified strength criterion by A.N. Shashenko. In this case, the results of solving the problem are completely identical to the data presented in Section 3.4. Therefore, the modified strength criterion by A.N. Shashenko is quite acceptable for determining the critical height of vertical slopes.

3. Conclusions

In general, it was concluded that all the strength criteria considered at stage 3 in relation to solving the problem of the stability of a vertical slope are quite acceptable and give the same result.

1. Using the strength criteria of Coulomb - Mora, Z.T. Benyavsky, Hoek - Brown, A.N. Shashenko and our proposed modification of the strength criterion A.N. Shashenko obtained analytical solutions to the following problems:
   1.1. Determination of the critical slope height from ideally free-flowing rock.
   1.2. Determination of the critical slope height from ideally plastic rock.
   1.3. Determination of the critical slope height from rock with specific adhesion and internal friction.

Using the strength criteria Z.T. Benyavsky, Hoek - Brown, A.N. Shashenko and the modified strength criterion A.N. Shashenko solutions of the listed problems were obtained for the first time.

2. The generalization of the well-known strength criterion by A.N. Shashenko in case of perfect loose soil. The modification is based on the fact that the asymptotic expansion of the classical strength criterion by A.N. Shashenko in the region of low pressures completely coincides with the Coulomb - Mohr strength criterion.
3. It is shown that using all the listed strength criteria is only possible to solve one problem - to determine the critical height of the vertical slope (slope).
4. It has been established that the use of the strength criteria Z.T. Benyavsky, Hoek - Brown and A.N. Shashenko to determine the critical angle of the slopes from ideally free-flowing rock is absolutely impossible.
5. It is shown that the strength criteria of Coulomb - Mohr and A.N. Shashenko (modified) are acceptable for solving all of the above problems.
6. It is shown that the results of solving the considered problems obtained using the modified strength criterion A.N. Shashenko and Coulona - Mora practically coincide at: low stress values; small values of the angles of internal friction; zero and low values of specific cohesion; very high specific cohesion values.

In this case, at large values of stress, there is a significant discrepancy.

In general, it was concluded that when designing overground and underground transport communications, attention should be paid to nonlinear strength criteria, of which the most promising is the modified strength criterion of A. Shashenko.

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