

THE REVISED METHOD FOR CALCULATING OF THE OPTIMAL TRAIN CONTROL MODE

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Abstract:

Development of a method for calculating the optimal mode of conducting a train in terms of energy saving meet the safety requirements and schedules. The method of calculation must solve the assigned tasks without significant time spent on the calculation. To implement this method of calculation was used a simplified model of the train as a controlled system. The existing mathematical and algorithmic methods for solving isoperimetric problems of finding the optimal solution in the presence of restrictions on resources were the information base for methodology development. Scientific works of domestic and foreign scientists, professional periodicals, materials of scientific and practical conferences, methodical and normative materials, currently in force on Ukrainian Railways. The results of these studies were used to create simulators on the basis of computer technology for the training of locomotive drivers. The scientific novelty of the proposed calculation method consists in applying the simplified calculations of the status of the train as a controlled system, without the use of differential equations of motion that allows to significantly increase the speed of the calculations. This, in turn, will solve the problems of finding optimal control in real time, taking into account changing conditions during the movement of the train. The practical significance of the obtained results is the use of such a calculation method that does not require significant time for its implementation and can be used as a subsystem of the on-board train control system capable of performing calculations taking into account changes in the current train situation.

Keywords: optimal control, modes of conducting a train, running profile of the train, driving simulators, electric traction motor.

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1. Introduction

Improving energy efficiency and achieving energy independence is one of the key priorities for Ukraine today. Energy independence is a matter of Ukraine's national security, and it is necessary to envisage mechanisms for reducing energy consumption, as well as implementing mechanisms aimed at stimulating energy conservation and the use of renewable energy sources (Domin, R., et al., 2017; Burdzik, R., et al., 2017; Jacyna, M., et al., 2018).

Given the high level of energy dependence of the Ukrainian economy, the need to introduce energy conservation policies is associated, first and foremost, with a deficit of own fuel and energy resources, dependence on gas-oil exporting countries, the growing cost of their extraction, and global environmental problems. The greatest challenge for modern industrial enterprises is to improve energy efficiency at all stages of production, so much attention is paid to the development of modern technologies and conduct organizational, technical and economic activities to improve energy efficiency, as well as to develop the energy-efficient technologies (Opala, M., et al., 2016; Cole, C., et al., 2017; Conti, R., et al., 2015).

Effective use of energy is one of the indicators of the economy development, science and the socio-cultural development of the nation. According to this indicator, Ukraine is among the countries where the current situation can provoke a serious economic crisis with the following large-scale social upheavals. The transport complex is closely connected with almost all branches of production and social sphere, and therefore the development trends of transport closely follow the general dynamics of Ukrainian economic development. In addition, transport, like some other sectors of the economy, has many inherited problems from the former USSR, such as irrational structure and high energy intensity. With all the variety of conditions and specifics of the work in various branches of the transport complex, the energy efficiency in general remains quite low (Akulov, A. S., et al., 2017).

One of the priority directions for ensuring stable and profitable operation of the railway transport, its development and improvement is the transition to resource-saving technologies. Administration of Ukrainian Railways implements a long-term strategy for the development of the energy management

system. Railway engineers introduce new technologies, work out standards and methods for controlling the energy resources, apply methods to reduce the energy intensity of the performed work. Each regional branch of the Ukrainian Railways is managed by energy management service, which develop and organize the implementation of organizational and technical energy saving measures; provide control over the use of energy and rationing of various directions of their consumption.

The optimization of train running profile is one of the most important measures for solving the current problem of saving energy resources on traction (Sablina, O., et al., 2017; Sun, B., et al., 2018; Wang, X., et al., 2017). This paper presents the method of energy-optimal traction calculation taking into account the plan and track profile, characteristics of the cars, the traction and braking characteristics of the locomotive and the speed limits.

2. Objective

The purpose of the publication is to develop a methodology for calculating the optimal modes of conducting the train in terms of energy consumption. This methodology should allow the calculations to be performed quickly and without significant loss of accuracy, and the results of the calculations must meet the criteria for optimality, safety and observance of the train schedule (Blochin, E., et al., 2016; Shvets, A. A., et al., 2016; Ursulyak, L., et al., 2017). Obviously, the use of traditional train models in the form of a system of differential equations of its motion are highly time consuming to perform the calculation. On the other hand, simplification of mathematical models can lead to significant losses in the accuracy of the results of the solution and even to the ineffectiveness of the train trajectories. The train trajectory means variation of train speed from its coordinate. In this regard, one of the given tasks was to investigate the possible risk of accuracy loss and determine the best ways to eliminate them.

3. The methodology of train running trajectory optimization

3.1. Building a grid

When calculating the energy-optimal trajectory of a train, the method of dynamic programming can be used (Bellman, R. E., 1960; Blokhin, Y. P., et al., 2007). The application of this method assumes that the train can only have discrete values of phase

states. Here, the phase states are understood as the speed of the train and its coordinate along the track. To implement this method, a grid is constructed in the Speed-Track coordinates. Further we'll call the intersection points of grid lines by nodes, and the set of grid nodes located vertically as a cross-section (Fig. 1). Obviously, only the first and last cross-sections of the grid have nodes with zero speed, because the first section corresponds to the departure point of the train from the station, and the last one corresponds to the stop point of the train at the destination station.

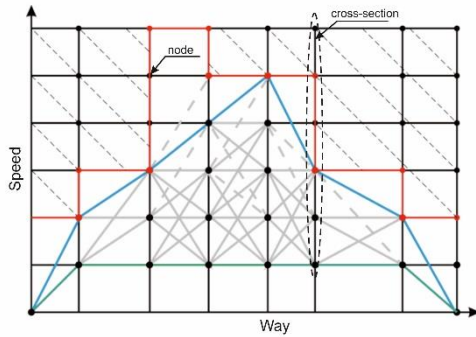


Fig. 1. Possible options of train trajectories

The grid spacing along the track and speed can be uniform or variable. The variability of the grid step along the track may be caused by presence of train speed limits. Indeed, in this case it is convenient to place the cross-sections of the grid at the beginning and at the end of the constraints, then it will not be necessary to check the permissibility of the train speed between grid sections. At Fig. 1 broken line of red color shows the speed limit of the train. Obviously, all the nodes lying on the constraint lines and above them are inaccessible for constructing the trajectory of the train. The zone of inaccessible nodes is marked with dashed lines.

In the process of constructing the optimal trajectory of motion, changes in the phase coordinates (speeds and coordinates) of the train are considered only between the available grid nodes. The method of dynamic programming, in fact, is a method of looking through the options for switching between grid nodes. If we consider all possible variants of the transitions between all grid nodes, we obtain all possible options of train trajectories. Each of these trajectories will have its own travel time between the

start and end points and its energy consumption (Fig. 1). Among the many options of the transitions between grid nodes, some are eliminated because of resource constraints in the traction and braking mode (the dashed lines on Fig. 1). As a result, only realizable transitions and realizable trajectories will remain. It is obvious that the trajectory shown in Fig. 1 by green line, will correspond to the maximum train travel time (T_{\max}), and the trajectory shown by the blue line – to the minimum (T_{\min}). For all other trajectories the following inequality holds:

$$T_{\min} \leq T_{xi} \leq T_{\max} \quad (1)$$

In this way, a finite set of train trajectories can be obtained; the number of phase states of the train is finite. Each of the obtained trajectories corresponds to its travel time and its energy consumption.

In practice, it is required to obtain one train trajectory, which ensures a minimum energy consumption for a given travel time. But because of the finiteness of the number of phase states of the train, it is impossible to obtain a trajectory, according to which the train's travel time will exactly conform with the specified travel time. Here we can only talk about the approximate conforming of the predetermined and estimated travel time, i.e.:

$$|T_{xi} - T_{xp}| \leq \delta T \quad (2)$$

It is obvious that the value δT directly depends on the grid step in speed and track. The smaller these steps are, the smaller variations in speeds and distances will be possible and the less inconsistencies between the set and calculated travel time can be achieved. Simply, the smaller the grid, the more accurate the result could be in terms of travel time. On the other hand, the smaller the grid, the longer the calculation time is. Thus, it is worthwhile choosing a reasonable compromise between the accuracy of achieving a given travel time and the speed of calculations.

In the process of constructing the train trajectory by the method of dynamic programming, several problems must be solved. The first of them is the determination of the cost of the transition between nodes of adjacent grid sections. The second one is to determine the possibility of a transition and the control

mode when moving between these nodes (traction/braking/idle running). The third is the determination of the train travel time between the nodes of adjacent grid sections. The fourth is the choice from the set of transitions the most suitable one from the point of view of the criterion for finding a solution. In this case, the criterion is:

$$Q = E_T + \lambda \cdot T_x \quad (3)$$

where: E_T – energy costs for the traction of the train; T_x – time of train travel; λ Is the Lagrange multiplier.

Energy consumption for electric traction means the amount of electricity consumed by the train during the travel time, for diesel traction – the amount of fuel consumed by the train during the travel time. The Lagrange multiplier for electric traction means the average power realized by an electric locomotive for the traction of the train during the travel time, and for diesel traction – the average fuel consumption realized by the locomotive for the train traction during the travel time. And, finally, the fifth task is to choose the implementation defined in the second stage of the control mode.

3.2. Determination of the consumption of transition between grid nodes

In the process of constructing the optimal train trajectory, it is necessary to determine the possibility of a transition between the grid nodes of adjacent sections. Up to now, this has been done by integrating the equation of train's motion by one of the numerical methods. This method allows to immediately determine: the possibility of transition; the mode of controlling the train when going from one node to another; as well as the cost of this transition (energy consumption for the transition). However, this method works very slowly, firstly, because the numerical integration process is not fast and, secondly, because in the process of searching for a transition it is necessary to sort out a sufficiently large number of options for controlling the train. Anyway, as a result, for almost every transition option, the finite speed is almost never equal to the speed at the final junction point. So, it will be necessary to introduce a certain threshold of the admissibility of the non-conformity between the finite transition speed and the speed at the final grid node.

To accelerate the process of determining the possibility of a transition between grid nodes, another way is suggested. It implies the determination the work that locomotive must perform (in traction and electric braking modes or train braking system for pneumatic braking) to overcome the train distance between adjacent sections of the grid and change of its speed from speed in the initial node up to speed in the final node of transition. i.e.:

$$A_L = \Delta E_k + \Delta E_p + A_{wo} \quad (4)$$

where: ΔE_k – change of the kinetic energy of the train; ΔE_p – change of the potential energy of the train; A_{wo} – the work of the forces of the main resistance to the movement of the train. The first 2 addends can be calculated exactly:

$$\Delta E_k = M_t \frac{v_f^2 - v_i^2}{2} \quad (5)$$

$$\Delta E_p = M_t \cdot g \cdot \Delta h \quad (6)$$

where: M_t – the mass of the train; v_f , v_i – speed in the initial and final nodes of the grid; Δh – the difference in the heights of the center of train's mass as it moves between the sections of the grid.

Regarding the work of the main resistance forces to the movement of the train, its precise definition is necessary to know train speed as a function of the distance covered between nodes of adjacent sections of the grid.

$$A_{wo} = M_t \int_0^S w_o(v(x)) dx \quad (7)$$

where: S – distance between adjacent sections of the grid; w_o – the main comparative resistance force of train. The force of the main comparative resistance of train, according to the (Grebenyuk, P. T., et al., 1985), is a second-degree polynomial of the speed: $w_o = av^2 + bv + c$. In this formula, the coefficients for the degrees of speed are determined by the type of track and rolling stock.

In the simplest case A_{wo} , the value can be defined as the product of the mean value of the force of the main train resistance at a speed at the initial junction

node and at the final junction node to the distance between adjacent grid sections, i.e. under the assumption that the magnitude of the force of the main train resistance is constant on the track $0 \div S$ section.

$$A_{wo} = M_t \frac{w_o(v_i) - w_o(v_f)}{2} S \quad (8)$$

Or as a product of the main comparative resistance of train at an average speed in the interval $v_i \div v_f$

($v_{av} = \frac{v_i + v_f}{2}$) by the mass of the train and at the same distance, i.e. the assumption of the constancy of the force of the main train resistance remains.

$$A_{wo} = M_t \cdot w_o(v_{av}) \cdot S \quad (9)$$

It's possible also to determine the value A_{wo} on the basis of the exact average value of the force of the main train resistance:

$$A_{wo} = M_t \cdot S \frac{1}{v_f - v_i} \int_{v_i}^{v_f} w_o(v) dv \quad (10)$$

It could be emphasized that all three variants assume that the magnitude of the force of the main train resistance is constant when the train moves between the nodes of adjacent grid sections.

Consider the options for calculating the main train resistance by the example of a train made up of loaded wagons on roller bearings on a CWR track. The main locomotive resistance, in comparison with the train, can be neglected. The main comparative resistance of train in such conditions, according to

the rules of technical work, is calculated by the equation (Grebenyuk, P. T., et al., 1985):

$$w_o = 0.7 + \frac{3 + 0.09v + 0.002v^2}{q_o} \quad (11)$$

Consider that the wagon weight is 80t and the train consists of 60 identical [four-axle](#) wagons, then the main train resistance of the entire train in [kN] (Fig. 2):

$$\begin{aligned} W_o^* &= (0.7 + \frac{3}{20} + \frac{0.09v}{20} + \frac{0.002v^2}{20}) \cdot \\ &\cdot 80 \cdot 9.8 \cdot 0.001 \cdot 60 = \\ &= 39.98 + 0.211v + 0.0047v^2. \end{aligned} \quad (12)$$

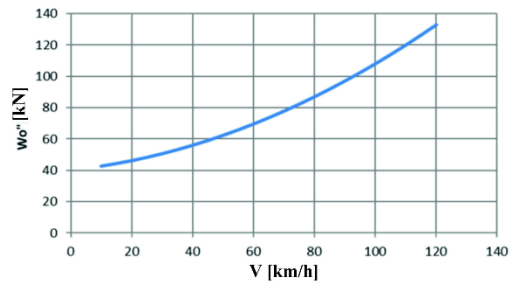


Fig. 2. Main resistance of train

Now we will estimate the difference in the results of determining the work of the forces of the main resistance of train according to formulas (8) and (9). In the calculations were considered the grid section lengths from 100 to 1000 m, which completely covers the possible range of these values. The results of calculations (in MJ) for different ranges of speed are given in Table. 1.

Table 1. Values of work made by main resistance force of train according to (8), (9), (10), MJ

S [m]	10 ÷ 20 [km/h]			30 ÷ 40 [km/h]			50 ÷ 60 [km/h]			60 ÷ 100 [km/h]		
	(8)	(9)	(10)	(8)	(9)	(10)	(8)	(9)	(10)	(8)	(9)	(10)
100	4.10	4.09	4.09	4.25	4.25	4.25	4.43	4.43	4.43	4.72	4.70	4.70
200	8.19	8.19	8.19	8.50	8.50	8.50	8.87	8.86	8.86	9.43	9.40	9.41
300	12.29	12.28	12.28	12.75	12.75	12.74	13.30	13.29	13.29	14.15	14.10	14.11
400	16.38	16.38	16.38	17.00	16.99	16.99	17.73	17.73	17.72	18.86	18.80	18.82
500	20.48	20.47	20.47	21.25	21.24	21.24	22.16	22.16	22.15	23.58	23.51	23.52
600	24.57	24.57	24.57	25.50	25.49	25.49	26.60	26.59	26.58	28.29	28.21	28.22
700	28.67	28.66	28.66	29.75	29.74	29.74	31.03	31.02	31.01	33.01	32.91	32.93
800	32.77	32.76	32.76	34.00	33.99	33.98	35.46	35.45	35.44	37.72	37.61	37.63
900	36.86	36.85	36.85	38.25	38.24	38.23	39.89	39.88	39.87	42.44	42.31	42.33
1000	40.96	40.95	40.94	42.50	42.49	42.48	44.33	44.32	44.30	47.16	47.01	47.04

As can be seen from the presented results, the values of the work of the main train resistance forces, calculated from formulas (8), (9) and (10), are practically the same in the speed range from 10 to 50 km/h when the speed is changed to 10 km/h and are very slightly different when the speed is changed to 40 km/h (from 60 to 100 km/h). This insignificant difference can be neglected. So, considering the fact that the approximate (in the sense of the assumption of the constancy of the main train resistance on the interval $0 \div S$) value of the work of the resistance forces can be determined by the simplest formula (9).

Now let's try to find out how the form of the speed curve affects the amount of work of the main train resistance forces, which in this case must be calculated by formula (7). There could be 3 cases: The curve of the speed of the train passing between the nodes of adjacent sections has a linear form:

$$v_{\text{lin}}(x) = v_i + \frac{v_f - v_i}{S} \cdot x. \quad (13)$$

The curve of the speed of the train passing between the nodes of adjacent sections has a convex form:

$$v_{\text{conv}}(x) = v_i + \frac{v_f - v_i}{\sqrt{S}} \cdot \sqrt{x}. \quad (14)$$

The curve of the speed of the train passing between the nodes of adjacent sections has a linear form:

$$v_{\text{conc}}(x) = v_i + \frac{v_f - v_i}{S^2} \cdot x^2. \quad (15)$$

The curves of Fig. 3 are shown for the case of a speed change from 40 to 60 km/h at a distance of 500 m.

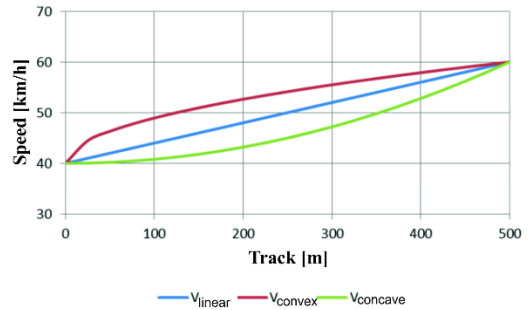


Fig. 3. Speed curves from 40 to 60 km/h on distance 500 m

Do not focus on how convex or concave the speed curves can be, but qualitatively check the effect of the speed curve shape on the amount of work of the main train resistance forces. The results of the calculations (in MJ) in comparison with the approximate calculations, which do not take into account the nature of the speed change, given in formula (9) are in Table 2.

In this table the first 2 columns show the results for small (10-20 km/h) and big (60 ÷ 70 km/h) speeds for small (10 km/h) speed' change. Assessment of deviations of the values of the work forces of the main resistance to the motion of the train received taking into account the nature of the changes in the curve of speed, from the values obtained in an approximate formula (9) shows that with a small change in speed, these results are very close to each other at low and at high speeds.

Table 2. Values of work made by main train resistance force (in MJ) according to (9) for different speed curves

S [m]	10 ÷ 20 [km/h]				60 ÷ 70 [km/h]				0 ÷ 50 [km/h]			
	$Aw_r(V_r)$	$Aw_r(\text{lin})$	$Aw_r(\text{conc})$	$Aw_r(\text{conv})$	$Aw_r(V_r)$	$Aw_r(\text{lin})$	$Aw_r(\text{conc})$	$Aw_r(\text{conv})$	$Aw_r(V_r)$	$Aw_r(\text{lin})$	$Aw_r(\text{conc})$	$Aw_r(\text{conv})$
100	4.09	4.09	4.08	4.11	4.53	4.53	4.51	4.55	4.17	4.17	4.11	4.24
200	8.19	8.19	8.17	8.21	9.06	9.06	9.03	9.10	8.33	8.35	8.23	8.48
300	12.28	12.28	12.25	12.32	13.60	13.60	13.54	13.65	12.50	12.52	12.34	12.72
400	16.38	16.38	16.33	16.42	18.13	18.13	18.06	18.20	16.67	16.70	16.46	16.95
500	20.47	20.47	20.41	20.53	22.66	22.66	22.57	22.75	20.84	20.87	20.57	21.19
600	24.56	24.57	24.50	24.64	27.19	27.19	27.09	27.30	25.00	25.05	24.68	25.43
700	28.66	28.66	28.58	28.74	31.73	31.73	31.60	31.85	29.17	29.22	28.80	29.67
800	32.75	32.76	32.66	32.85	36.26	36.26	36.12	36.40	33.34	33.40	32.91	33.91
900	36.85	36.85	36.75	36.95	40.79	40.79	40.63	40.95	37.50	37.57	37.02	38.15
1000	40.94	40.94	40.83	41.06	45.32	45.32	45.15	45.50	41.67	41.75	41.14	42.39
Difference [%]	0.007		-0.273	0.289		0.007	-0.381	0.394		0.181	-1.297	1.687

The third column shows the results for a large speed increment. The results shown in the 3-th column shows that even in this case, the variance of the results obtained, taking into account the nature of the curve speed, results in an approximate formula (9) differ by less than 2%.

Let's consider one more option – transition between the nodes having the same speed. If the speed remains constant during the transition between such nodes, the expression w_o becomes a constant and the legitimacy of applying expression (9) to determine the work of the forces of the main resistance to motion is beyond doubts. But the speed between the grid nodes can vary so that at the initial and final nodes of the transition it is the same, and between the nodes the velocity curve can be convex or concave. Consider these two cases in detail for the purpose of elucidating the effect of the curvature of the speed curve on the calculation results A_{w_o} when moving between nodes with the same speed.

Let the speed variation curve between the grid nodes be described by the expression:

$$v(x) = \left(\frac{4a}{S^2} \cdot x^2 - \frac{4a}{S} \cdot x + 1 \right) \cdot v_s \quad (16)$$

where: a – coefficient determining the speed deviation in the center of the interval $0 \div S$ of the speed in the initial and final junction node. With a negative value the a speed curve is convex, with a positive value curve – concave. For the value $a = \pm 0.1$ and

$v_c = 60$ km/h on Fig. 4, a speed variation curve is constructed. With these parameters, the largest value of the speed achieved in the middle of the interval is equal to 66 km/h, and the smallest – 54 km/h.

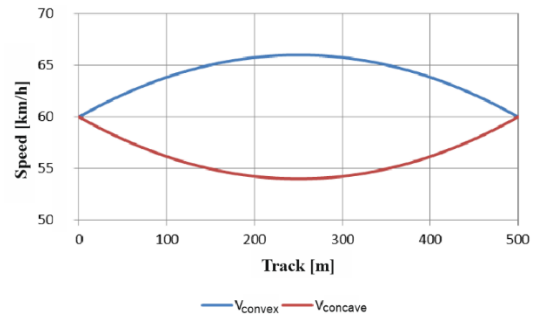


Fig. 4. The speed curve for the value $a = \pm 0.1$ and $V_c = 60$ km/h

Now let's determine the amount of work of the force of the main resistance to the movement of the train according to expression (7), taking into account the curvature of the velocity curve (Table. 3).

As you can see from the results, in this case the greatest deviation in calculation results of the main resistance forces to motion, obtained taking into account the curvature of the curve speed results in an approximate formula (9) does not exceed 2%.

Thus, if it meets a possible deviation of the results of about 2%, the work of the resistance forces to motion can be determined from the approximate formula (9).

Table 3. Values of work made by main train resistance force (in MJ) according to expression (7) for different speed curves

S [m]	20 [km/h]			60 [km/h]			100 [km/h]		
	$Aw_0(V_{av})$	$Aw_0(\text{conc})$	$Aw_0(\text{conv})$	$Aw_0(V_{av})$	$Aw_0(\text{conc})$	$Aw_0(\text{conv})$	$Aw_0(V_{av})$	$Aw_0(\text{conc})$	$Aw_0(\text{conv})$
100	4.13	4.12	4.14	4.48	4.44	4.52	4.95	4.86	5.04
200	8.26	8.24	8.28	8.96	8.88	9.04	9.89	9.72	10.07
300	12.39	12.36	12.42	13.44	13.32	13.57	14.84	14.58	15.11
400	16.52	16.48	16.56	17.92	17.76	18.09	19.79	19.45	20.14
500	20.65	20.60	20.70	22.40	22.20	22.61	24.73	24.31	25.18
600	24.78	24.72	24.84	26.88	26.64	27.13	29.68	29.17	30.22
700	28.91	28.84	28.98	31.36	31.08	31.65	34.63	34.03	35.25
800	33.04	32.96	33.12	35.84	35.52	36.17	39.57	38.89	40.29
900	37.17	37.08	37.26	40.32	39.96	40.70	44.52	43.75	45.33
1000	41.30	41.20	41.40	44.80	44.40	45.22	49.47	48.61	50.36
Difference [%]		-0.235	0.237		-0.904	0.919		-1.759	1.774

3.3. Determine the ability of transition between grid nodes and the control mode

The possibility of a transition between nodes of adjacent grid sections should be determined at the first stage of solving the problem of obtaining an energy-optimal train trajectory. The possibility of transition means feasibility. Each locomotive has limited train control resources, so the feasibility of control means the application of such a control that does not exceed the resource of the locomotive.

3.3.1. Restrictions on the train control

Firstly, consider the limitations concerning train control. The control of the train's movement means the application of the traction mode and the braking mode by the driver, i.e. an artificial and purposeful change of the train's phase state. The idling mode (idle running) also changes the phase state of the train, but is neither artificial, nor purposeful, because in this case the change of the phase state occurs without the intervention of the driver, due to the action of natural causes. So, only 2 modes of train movement control are possible – traction mode and braking mode. The main braking mode is pneumatic braking of the train, although some locomotives have an electric brake (rheostatic or regenerative). Thus, it is possible to construct the zones of train control limitations in coordinates Force-Speed. For DC locomotives the traction force limitation zone is divided into parts in accordance with the traction motor connection diagrams (S – series, SP – series-parallel and P – parallel motor connection).

Admissible control in the traction mode can be limited from above, for example, by constraints on coupling and weakening of the field. It is possible to construct a curve limiting the traction force from below based on the minimum traction current at which the efficiency has not too small value. And it is possible, using electromechanical characteristics of the locomotive traction electric motor, to allocate some acceptable level of motor efficiency and on the curve of the dependence of the efficiency from the motor traction current to determine the range of traction current ($I_{dmin} \div I_{dmax}$), and according to the dependence of the motor traction force from current – the range of the motor traction force ($F_{dmin} \div F_{dmax}$) and, then, the range of the total traction force of locomotive.

It should be noted at once that while the control of

the locomotive traction in the zone with big values of the traction motor efficiency reduces the losses of energy, but does not guarantee a reduction of power consumption for the traction of the train throughout the track section, because in this case, it will be necessary to use for the acceleration of the train either longer or more frequent traction modes.

During driving of trains equipped with pneumatic braking to necessary extent in order to adjust the speed of the train it is applied almost every time (except braking on stops and steep descents) the brake step with inhibition of braking system in the summer time for loaded trains – 0.7-0.6 ATM, for empty ones – 0.6-0.5 ATM (Grebnyuk, P. T., et al., 1985). Using these ranges of brake system discharge, it is possible to calculate the curves that limit the zone of possible braking forces.

Electric braking is mainly used to stabilize or limit the growth of the train's speed on long descents. The use of pneumatic braking in such situations, especially for loaded freight trains, leads to a significant reduction in speed due to the fact that the processes occurring in the braking system of the train are rather slow. Consequently, the use of pneumatic braking to stabilize the speed of the train leads to a useless and excessive loss of its kinetic energy. The use of electric braking of the locomotive (especially regenerative) allows to achieve a smooth reduction or stabilization of the speed and can reduce the power consumption. Therefore, it should be used in those cases where there is no need to significantly reduce the speed of the train.

3.3.2. Train control mode

Then, consider the choice of the train control mode during navigation between nodes of adjacent grid sections. Earlier it was shown (see formula 4) how to obtain the value A_L of the locomotive work or the brake system of the train for the transition between the grid nodes. Obviously, this work is connected with the traction and braking forces:

$$A_L = \begin{cases} \int_0^s F_t(x) dx, & \text{traction} \\ \int_0^s F_b(x) dx, & \text{braking} \end{cases} \quad (17)$$

From this equation one can determine the traction or

braking force as a function of the track:

$$F(x) = \frac{dA_L}{dx} \quad (18)$$

or the average value of the force on the track segment $0 \div S$:

$$F_{av} = \frac{A_L}{S} \quad (19)$$

Concerning running mode. In this mode, expression (4) should take the following form:

$$\Delta E_k + \Delta E_p + A_{wo} = 0 \quad (20)$$

So, the sum of the changes of the kinetic and potential energies should be compensated by the work of the forces of the main resistance to the movement of the train. Nevertheless, this condition cannot be fulfilled practically never. Therefore, in order to implement the idle running mode, it is necessary to introduce a certain threshold on force $\pm F_0$. And, if the force obtained from the expression (19) does not reach the threshold, the idle running mode can be used. In this case, the deviation of the final speed from the velocity in the final grid node should not exceed $\pm \delta v_0$.

When performing a transition between nodes of adjacent grid sections with equal speed the $v_i \div v_f$ ratio between speeds is equal to:

$$v_f^2 = v_i^2 + 2aS \quad (21)$$

If the transition is possible in the idle running mode, the acceleration (deceleration) is equal to:

$$a = \frac{W_i + W_o}{M_t} \quad (22)$$

where: W_i – resistance force to train movement from the slope of the track or downhill, W_o – the main resistance force.

Takin this into consideration:

$$v_f^2 = v_i^2 + 2S \frac{W_i + W_o}{M_t} \quad (23)$$

Now suppose that in addition to the 2 mentioned

forces, another force acts on the train F_0 . Then the final speed should change:

$$v_f^{*2} = v_i^2 + 2S \frac{W_i + W_o + F_0}{M_t} \quad (24)$$

$$v_f^* = v_f + \delta v$$

After some transformations finally, we can get:

$$F_o = \frac{\delta v \cdot (2v_f + \delta v)}{2S} M_t \quad (25)$$

The deviation of the final speed from the speed at the end point can be either upward or downward. Hence, the force F_0 can be both positive and negative:

$$F_o^+ = \frac{\delta v \cdot (2v_f + \delta v)}{2S} M_t \quad (26)$$

$$F_o^- = \frac{-\delta v \cdot (2v_f - \delta v)}{2S} M_t \quad (27)$$

Thus, if as a result of the transition between the two nodes, an average force is obtained $F_{av} \leq |F_o|$, then the deviation of the final speed (v_f^*) of the speed in the final grid node (v_f) does not exceed the value δv_0 , i.e.:

$$|v_f^* - v_f| \leq \delta v \quad (28)$$

Thus, one can determine the feasibility of the transition between the grid nodes, and the control mode during this transition. If the average force does not fall into any of the zones of admissible controls, such a transition is not considered as possible. If F_{av} the value falls into the zone of pneumatic braking, in this case the control is reduced to the application of the brake step, for which the range of forces of pneumatic braking is calculated. If F_{av} the value falls into the zone of electric braking, it is necessary to check additionally whether it can be realized in the entire speed range ($v_i \div v_f$). If the speed falls into

the run-out zone, then the idle mode of the locomotive should be applied. If F_{av} the value falls into the traction zone, it is also necessary to check the possibility of realizing the traction force in the speed range ($v_i \div v_f$). As the majority of locomotives use step regulation of traction force, once the question of the application of the traction mode become resolved positively, we should separately decide on the application of certain steps of force regulation.

3.4. Determination of the train travel time between the grid nodes

The question of the time spent on the transition between nodes of neighboring grid sections, which at first sight seems simple, actually requires special attention, due to direct connection with evaluation of the criterion of finding a solution (see formula 3). Providing the grid were built in coordinates *Speed-Time*, then the problem of determining the transition time would be solved automatically. But in this case, when searching for possible transitions between nodes of adjacent grid sections, only the initial and final speed and the distance between the grid sections are known.

In the simplest case, the transition time can be determined from the average transition speed:

$$t_{ij}^k = \frac{S}{v_i + v_f/2} \quad (29)$$

where: t_{ij}^k – time of transition from i node k section in the j node of the next section. However, the nature of the speed change is not taken into account here (see Fig. 1).

Let's estimate the effect of the curvature of the speed curve on the transition time between the nodes of adjacent grid sections. Let the speed at the transition from i th node in j varies from v_i before v_f on a segment of the track between adjacent grid sections of length S . The speed variation curve is described by the expressions (30)-(32). These are illustrative examples that used to show the influence of the form of speed curve on motion time between two nodes: – the speed curve has a linear form:

$$v_{lin}(x) = v_i + \frac{v_f - v_i}{S} \cdot x, \quad (30)$$

– the speed curve has a convex shape:

$$v_{conv}(x) = v_i + \frac{v_f - v_i}{\sqrt{S}} \cdot \sqrt{x}, \quad (31)$$

– the speed curve has a concave shape:

$$v_{conc}(x) = v_i + \frac{v_f - v_i}{S^2} \cdot x^2. \quad (32)$$

The transition time between nodes is determined from the following considerations: $\frac{dx}{dt} = v$, from where:

$$t = \int_0^S \frac{dx}{v(x)}. \quad (33)$$

Let's compare the values of the transition time between nodes with a change in speed in accordance with expressions (30-32) with values obtained from the average speed of movement, i.e.:

$$t = \frac{2S}{v_i + v_f}. \quad (34)$$

After integrating expression (33), we obtain formulas for calculating the travel times, taking into account the curvature of the speed curve:

$$T_{lin} = \frac{S}{v_i - v_f} \ln \frac{v_f}{v_i}. \quad (35)$$

$$T_{conc} = \begin{cases} \frac{S}{\sqrt{v_i(v_f - v_i)}} \arctg\left(\sqrt{\frac{v_f - v_i}{v_i}}\right), & v_f > v_i \\ \frac{S}{\sqrt{v_i(v_f - v_i)}} \operatorname{arcth}\left(\sqrt{\frac{v_i - v_f}{v_f}}\right), & v_f < v_i \end{cases} \quad (36)$$

$$T_{conv} = \frac{S}{(v_i - v_f)^2} \left(v_f - v_i \cdot \left(1 + \ln \frac{v_f}{v_i} \right) \right). \quad (37)$$

The train travel times was determined using the formulas (34) and (35-37), when distances varied from 100 to 1000 m and with various changes in speed. Results are given in tab. 4.

These results show that only at high speed of train and with a small change of it (column 2) results of the transition time on the basis of an approximate formula (34) and formulas, taking into account the

curvature of the speed are close to each other. In other cases, the results are very different.

Tabl. 5 shows the results of determining the train travel times at the initial speed 50 km/h and finite, varying from 0 to 90 km/h on distance 500 m.

In the 3 right-hand columns of this table, the deviations of train travel times are calculated taking into account the curvature of the speed curve with respect to the results obtained from the approximate formula (34). On the basis of the deviation values, it is seen that only when the speed is ± 10 km/h relatively to the initial speed, deviations of the travel times are within $\pm 5\%$. With large changes in speed, deviations become unacceptable.

Thus, for small train speeds and for large changes, the train travel time between the grid nodes can not be determined by the approximate formula (34). In the region of low speeds, the grid speed step can be

smaller than at large ones, i.e., to improve the accuracy of calculations it is expedient to use a grid with variable step along the speed axis. On the other hand, in order to prevent the occurrence of significant changes in the speed of movement, we should reduce the grid step along the track.

4. Results of the developed method application

Training of the locomotive drivers for safe and efficient operation is an important aspect for assuring the of freight carriage that greatly affects their cost. The largest networks of railways use the simulators quite actively to achieve this goal. On the Simulator we can specify and analyze modes of conducting train on section, create emergency situations in the train work as well as gain a range of skills that can only be obtained on Simulator.

Table 4. The travel times of the train moving over a distance of 100 to 1000 m with various changes in speed

S [m]	10 ÷ 20 [km/h]				60 ÷ 70 [km/h]				10 ÷ 60 [km/h]				40 ÷ 90 [km/h]			
	$T(v_{av})$ [s]	T_{lin} [s]	T_{conc} [s]	T_{conv} [s]	$T(v_{av})$ [s]	T_{lin} [s]	T_{conc} [s]	T_{conv} [s]	$T(v_{av})$ [s]	T_{lin} [s]	T_{conc} [s]	T_{conv} [s]	$T(v_{av})$ [s]	T_{lin} [s]	T_{conc} [s]	T_{conv} [s]
100	24.0	25.0	28.3	22.1	5.5	5.5	5.7	5.4	10.3	12.9	18.5	9.2	5.5	5.8	6.8	5.1
200	48.0	49.9	56.5	44.2	11.1	11.1	11.4	10.8	20.6	25.8	37.0	18.5	11.1	11.7	13.5	10.1
300	72.0	74.9	84.8	66.3	16.6	16.6	17.1	16.2	30.9	38.7	55.6	27.7	16.6	17.5	20.3	15.2
400	96.0	99.8	113.1	88.4	22.2	22.2	22.8	21.6	41.1	51.6	74.1	37.0	22.2	23.4	27.1	20.2
500	120.0	124.8	141.4	110.5	27.7	27.7	28.5	27.0	51.4	64.5	92.6	46.2	27.7	29.2	33.9	25.3
600	144.0	149.7	169.6	132.6	33.2	33.3	34.2	32.4	61.7	77.4	111.1	55.4	33.2	35.0	40.6	30.3
700	168.0	174.7	197.9	154.7	38.8	38.8	39.9	37.8	72.0	90.3	129.6	64.7	38.8	40.9	47.4	35.4
800	192	199.6	226.2	176.7	44.3	44.4	45.6	43.3	82.3	102.2	148.2	73.9	44.3	46.7	54.2	40.5
900	216.0	224.6	254.5	198.8	49.8	49.9	51.3	48.7	92.6	116.1	166.7	83.2	49.8	52.5	60.9	45.5
1000	240	249.5	282.7	220.9	55.4	55.5	57.0	54.1	102.9	129.0	185.2	92.4	55.4	58.4	67.7	50.6
Difference [%]		3.97	17.81	-7.94		0.20	2.85	-2.38		25.42	80.04	10.17		5.42	22.24	-8.67

Table 5. The train travel times at the initial speed 50 km/h and finite, varying from 0 to 90 km/h on distance 500 m

v_f [km/h]	$v_i = 50$ [km/h]						
	$T(v_{av})$ [s]	T_{lin} [s]	T_{conc} [s]	T_{conv} [s]	ΔT_{lin} [%]	ΔT_{conc} [%]	ΔT_{conv} [%]
0	70.6	96.2	219.8	143.7	36.2	211.4	103.6
10	60.0	58.1	91.1	72.4	-3.9	51.8	20.7
20	51.4	47.9	63.3	55.0	6.8	23.0	6.9
30	45.0	42.4	49.9	46.0	-5.7	10.8	2.6
40	40.0	38.7	41.7	40.2	-3.2	4.2	0.4
50	36.0	36.0	36.0	36.0	0.0	0.0	0.0
60	32.7	33.9	31.8	32.8	3.4	-2.8	0.3
70	30.0	32.1	28.6	30.3	7.0	4.7	0.9
80	27.7	30.6	26.0	28.2	10.6	6.1	1.8
90	25.7	29.4	23.9	26.5	14.2	-7.2	2.9

Calculation method described here, in terms of optimum energy consumption, the trajectory of the moving of train and control modes implemented is one of the subsystems of the Simulator. This technique made it possible to determine well-implemented train control modes with insignificant deviations from a given train time.

During training task an apprentice (Driver) sees energy-saving travel path, which appears at the bottom of the screen (only in the mode of study travel) (Fig. 5).

Before the trip, the instructor can calculate the modes for the selected train and section. The calculation is carried out taking into account information

about the traffic area, the train, the locomotive and the traffic schedule. And, if all the stations are listed in the schedule, the calculation is carried out with observance of all the section running times. If the schedule part of the stations (or missing everything except the departure station and the station of destination), the span between these timetabling stations optimized to generate additional savings of energy. The results of the calculations are the modes of trains on the section and changes in the speed of movement, which are presented in a graphical form (Fig. 6). Control modes are displayed on the chart in different colors. Blue colour – draft, green – run-out, red – pneumatic braking.

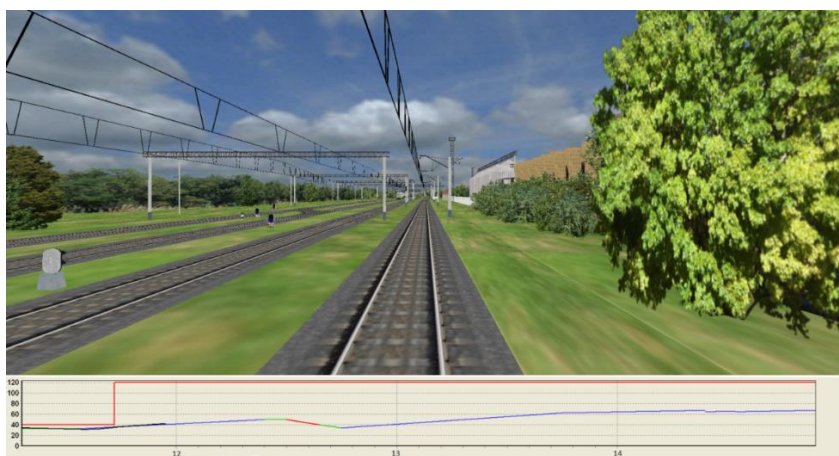


Fig. 5. The screen of the monitor during the travel

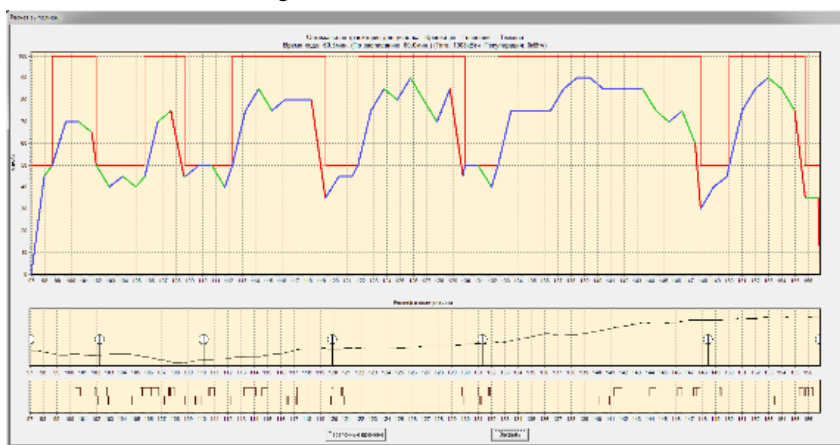


Fig. 6. The result of calculation of train conducting modes

According recommended trajectory of graph (Fig. 6), the broken line shows the speed limits. The bottom part of the window shows the relief, location of stations and curves in the profile of the railway section selected for training ride. The header of the window shows (Fig. 6) the obtained travel time, the travel time according to the schedule and the energy consumption. During the study trip, the driver, adhering to the trajectory of the train, has the opportunity to improve his experience in saving energy spent on traction

5. Conclusions

The scientific novelty of the proposed calculation method consists in applying the simplified calculations of the state of the train as a controlled system, without differential equations of movement that allows to significantly increase the speed of the calculations. This, in turn, will solve the problems of finding optimal control in real time, taking into account changing conditions during the movement of the train. The practical significance of the obtained results is the use of such a calculation method that does not require significant time for its implementation and can be used as a subsystem of the on-board train control system capable of performing calculations taking into account changes in the current train situation. The results formed the basis of the hardware and software complex "Driver Simulator".

Based on the study on the selection of energy-optimal modes of train operation, the following conclusions can be made:

- 1) The use of simplified mathematical models of the train movement is fully applicable for solving the posed problems.
- 2) The refusal to use the systems of differential equations describing the movement of the train made it possible to substantially reduce the time spent on performing the calculations.
- 3) Taking into account the peculiarities of calculating the components of the total energy consumption for the movement of the train does not lead to significant losses in the accuracy of the calculations and allows to obtain realizable modes of train conducting.

This technique can be used to perform calculations of energy-optimal modes of conducting a train in training systems such as "Driver simulator".

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